Binomial Theorem

Exercise 10A

Q. 1. Using binomial theorem, expand each of the following:

$(1 - 2x)^5$

Answer : To find: Expansion of $(1 - 2x)^5$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(1 - 2x)^5$

$\Rightarrow [{}^5C_0(1)^5] + [{}^5C_1(1)^{5-1}(-2x)^1] + [{}^5C_2(1)^{5-2}(-2x)^2] + [{}^5C_3(1)^{5-3}(-2x)^3] + [{}^5C_4(1)^{5-4}(-2x)^4] + [{}^5C_5(-2x)^5]$

$$\Rightarrow \left[\frac{5!}{0!(5-0)!}(1)^5\right] - \left[\frac{5!}{1!(5-1)!}(1)^4(2x)\right] + \left[\frac{5!}{2!(5-2)!}(1)^3(4x^2)\right]$$

$$-\left[\frac{5!}{3!(5-3)!}(1)^2(8x^3)\right] + \left[\frac{5!}{4!(5-4)!}(1)^1(16x^4)\right] - \left[\frac{5!}{5!(5-5)!}(32x^5)\right]$$

$\Rightarrow 1 - 5(2x) + 10(4x^2) - 10(8x^3) + 5(16x^4) - 1(32x^5)$

$\Rightarrow 1 - 10x + 40x^2 - 80x^3 + 80x^4 - 32x^5$

On rearranging

Ans) $-32x^5 + 80x^4 - 80x^3 + 40x^2 - 10x + 1$

Q. 2. Using binomial theorem, expand each of the following:

$(2x - 3)^6$

Answer : To find: Expansion of $(2x - 3)^6$

Formula used: **(i)** $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(2x - 3)^6$

$\Rightarrow [{}^6C_0(2x)^6]+[{}^6C_1(2x)^{6-1}(-3)^1]+[{}^6C_2(2x)^{6-2}(-3)^2]+[{}^6C_3(2x)^{6-3}(-3)^3]+[{}^6C_4(2x)^{6-4}(-3)^4]+[{}^6C_5(2x)^{6-5}(-3)^5]+[{}^6C_6(-3)^6]$

$$\Rightarrow \left[\frac{6!}{0!(6-0)!}(2x)^6\right] - \left[\frac{6!}{1!(6-1)!}(2x)^5(3)\right] + \left[\frac{6!}{2!(6-2)!}(2x)^4(9)\right]$$

$$-\left[\frac{6!}{3!(6-3)!}(2x)^3(27)\right] + \left[\frac{6!}{4!(6-4)!}(2x)^2(81)\right]$$

$$-\left[\frac{6!}{5!(6-5)!}(2x)^1(243)\right] + \left[\frac{6!}{6!(6-6)!}(729)\right]$$

$\Rightarrow [(1)(64x^6)] - [(6)(32x^5)(3)] + [15(16x^4)(9)] - [20(8x^3)(27)] + [15(4x^2)(81)] - [(6)(2x)(243)] + [(1)(729)]$

$\Rightarrow 64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

Ans) $64x^6 - 576x^5 + 2160x^4 - 4320x^3 + 4860x^2 - 2916x + 729$

Q. 3. Using binomial theorem, expand each of the following:

$(3x + 2y)^5$

Answer : To find: Expansion of $(3x + 2y)^5$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(3x + 2y)^5$

$\Rightarrow [{}^5C_0(3x)^{5-0}] + [{}^5C_1(3x)^{5-1}(2y)^1] + [{}^5C_2(3x)^{5-2}(2y)^2] + [{}^5C_3(3x)^{5-3}(2y)^3] + [{}^5C_4(3x)^{5-4}(2y)^4] + [{}^5C_5(2y)^5]$

$\Rightarrow \left[\frac{5!}{0!(5-0)!}(243x^5)\right] + \left[\frac{5!}{1!(5-1)!}(81x^4)(2y)\right] + \left[\frac{5!}{2!(5-2)!}(27x^3)(4y^2)\right] + \left[\frac{5!}{3!(5-3)!}(9x^2)(8y^3)\right] + \left[\frac{5!}{4!(5-4)!}(3x)(16y^4)\right] + \left[\frac{5!}{5!(5-5)!}(32y^5)\right]$

$\Rightarrow [1(243x^5)] + [5(81x^4)(2y)] + [10(27x^3)(4y^2)] + [10(9x^2)(8y^3)] + [5(3x)(16y^4)] + [1(32y^5)]$

$\Rightarrow 243x^5 + 810x^4y + 1080x^3y^2 + 720x^2y^3 + 240xy^4 + 32y^5$

Ans) $243x^5 + 810x^4y + 1080x^3y^2 + 720x^2y^3 + 240xy^4 + 32y^5$

Q. 4. Using binomial theorem, expand each of the following:

$(2x – 3y)^4$

Answer : To find: Expansion of $(2x – 3y)^4$

Formula used: **(i)** ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(2x – 3y)^4$

$\Rightarrow [{}^4C_0(2x)^{4-0}] + [{}^4C_1(2x)^{4-1}(-3y)^1] + [{}^4C_2(2x)^{4-2}(-3y)^2] + [{}^4C_3(2x)^{4-3}(-3y)^3] + [{}^4C_4(-3y)^4]$

$\left[\frac{4!}{0!(4-0)!}(2x)^4\right] - \left[\frac{4!}{1!(4-1)!}(2x)^3(3y)\right] + \left[\frac{4!}{2!(4-2)!}(2x)^2(9y^2)\right] - \left[\frac{4!}{3!(4-3)!}(2x)^1(27y^3)\right] + \left[\frac{4!}{4!(4-4)!}(81y^4)\right]$

$\Rightarrow [1(16x^4)] – [4(8x^3)(3y)] + [6(4x^2)(9y^2)] – [4(2x)(27y^3)] + [1(81y^4)]$

$\Rightarrow 16x^4 – 96x^3y + 216x^2y^2 – 216xy^3 + 81y^4$

Ans) $16x^4 – 96x^3y + 216x^2y^2 – 216xy^3 + 81y^4$

Q. 5. Using binomial theorem, expand each of the following:

$$\left(\frac{2x}{3}-\frac{3}{2x}\right)^6$$

Answer : To find: Expansion of $\left(\frac{2x}{3}-\frac{3}{2x}\right)^6$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(\frac{2x}{3}-\frac{3}{2x}\right)^6$

$$\Rightarrow \left[{}^6C_0\left(\frac{2x}{3}\right)^{6-0}\right] + \left[{}^6C_1\left(\frac{2x}{3}\right)^{6-1}\left(-\frac{3}{2x}\right)^1\right] + \left[{}^6C_2\left(\frac{2x}{3}\right)^{6-2}\left(-\frac{3}{2x}\right)^2\right] + \left[{}^6C_3\left(\frac{2x}{3}\right)^{6-3}\left(-\frac{3}{2x}\right)^3\right] + \left[{}^6C_4\left(\frac{2x}{3}\right)^{6-4}\left(-\frac{3}{2x}\right)^4\right]$$

$$+ \left[{}^6C_5\left(\frac{2x}{3}\right)^{6-5}\left(-\frac{3}{2x}\right)^5\right] + \left[{}^6C_6\left(-\frac{3}{2x}\right)^6\right]$$

$$\Rightarrow \left[\frac{6!}{0!(6-0)!}\left(\frac{2x}{3}\right)^6\right] - \left[\frac{6!}{1!(6-1)!}\left(\frac{2x}{3}\right)^5\left(\frac{3}{2x}\right)\right] + \left[\frac{6!}{2!(6-2)!}\left(\frac{2x}{3}\right)^4\left(\frac{9}{4x^2}\right)\right] - \left[\frac{6!}{3!(6-3)!}\left(\frac{2x}{3}\right)^3\left(\frac{27}{8x^3}\right)\right] + \left[\frac{6!}{4!(6-4)!}\left(\frac{2x}{3}\right)^2\left(\frac{81}{16x^4}\right)\right] - \left[\frac{6!}{5!(6-5)!}\left(\frac{2x}{3}\right)^1\left(\frac{243}{32x^5}\right)\right]$$

$$+ \left[\frac{6!}{6!(6-6)!}\left(\frac{729}{64x^6}\right)\right]$$

$$\Rightarrow \left[1\left(\frac{64x^6}{729}\right)\right] - \left[6\left(\frac{32x^5}{243}\right)\left(\frac{3}{2x}\right)\right] + \left[15\left(\frac{16x^4}{81}\right)\left(\frac{9}{4x^2}\right)\right] - \left[20\left(\frac{8x^3}{27}\right)\left(\frac{27}{8x^3}\right)\right] + \left[15\left(\frac{4x^2}{9}\right)\left(\frac{81}{16x^4}\right)\right] - \left[6\left(\frac{2x}{3}\right)\left(\frac{243}{32x^5}\right)\right] + \left[1\left(\frac{729}{64x^6}\right)\right]$$

$$\Rightarrow \frac{64}{729}x^6 - \frac{32}{27}x^4 + \frac{20}{3}x^2 - 20 + \frac{135}{4}\frac{1}{x^2} - \frac{243}{8}\frac{1}{x^4} + \frac{729}{64}\frac{1}{x^6}$$

Ans) $\frac{64}{729}x^6 - \frac{32}{27}x^4 + \frac{20}{3}x^2 - 20 + \frac{135}{4}\frac{1}{x^2} - \frac{243}{8}\frac{1}{x^4} + \frac{729}{64}\frac{1}{x^6}$

Q. 6. Using binomial theorem, expand each of the following:

$$\left(x^2 - \frac{3}{x}\right)^7$$

Answer : To find: Expansion of $\left(x^2 - \frac{3x}{7}\right)^7$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(x^2 - \frac{3x}{7}\right)^7$

$$\Rightarrow [{}^7C_0(x^2)^{7-0}] + \left[{}^7C_1(x^2)^{7-1}\left(-\frac{3x}{7}\right)^1\right] + \left[{}^7C_2(x^2)^{7-2}\left(-\frac{3x}{7}\right)^2\right] + \left[{}^7C_3(x^2)^{7-3}\left(-\frac{3x}{7}\right)^3\right] + \left[{}^7C_4(x^2)^{7-4}\left(-\frac{3x}{7}\right)^4\right] + \left[{}^7C_5(x^2)^{7-5}\left(-\frac{3x}{7}\right)^5\right] + \left[{}^7C_6(x^2)^{7-6}\left(-\frac{3x}{7}\right)^6\right] + \left[{}^7C_7\left(-\frac{3x}{7}\right)^7\right]$$

$$\Rightarrow \left[\frac{7!}{0!(7-0)!}(x^2)^7\right] - \left[\frac{7!}{1!(7-1)!}(x^2)^6\left(\frac{3x}{7}\right)\right] + \left[\frac{7!}{2!(7-2)!}(x^2)^5\left(\frac{9x^2}{49}\right)\right] - \left[\frac{7!}{3!(7-3)!}(x^2)^4\left(\frac{27x^3}{343}\right)\right] + \left[\frac{7!}{4!(7-4)!}(x^2)^3\left(\frac{81x^4}{2401}\right)\right] - \left[\frac{7!}{5!(7-5)!}(x^2)^2\left(\frac{243x^5}{16807}\right)\right] + \left[\frac{7!}{6!(7-6)!}(x^2)^1\left(\frac{729x^6}{117649}\right)\right] - \left[\frac{7!}{7!(7-7)!}\left(\frac{2187x^7}{823543}\right)\right]$$

$$\Rightarrow [1(x^{14})] - \left[7(x^{12})\left(\frac{3x}{7}\right)\right] + \left[21(x^{10})\left(\frac{9x^2}{49}\right)\right] - \left[35(x^8)\left(\frac{27x^3}{343}\right)\right] + \left[35(x^6)\left(\frac{81x^4}{2401}\right)\right] - \left[21(x^4)\left(\frac{243x^5}{16807}\right)\right] + \left[7(x^2)\left(\frac{729x^6}{117649}\right)\right] - \left[1\left(\frac{2187x^7}{823543}\right)\right]$$

$$\Rightarrow x^{14} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} - \left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 - \left(\frac{2187}{823543}\right)x^7$$

Ans)

$$x^{14} - 3x^{13} + \left(\frac{27}{7}\right)x^{12} - \left(\frac{135}{49}\right)x^{11} + \left(\frac{405}{343}\right)x^{10} - \left(\frac{729}{2401}\right)x^9 + \left(\frac{729}{16807}\right)x^8 - \left(\frac{2187}{823543}\right)x^7$$

Q. 7. Using binomial theorem, expand each of the following:

$$\left(x - \frac{1}{y}\right)^5$$

Answer : To find: Expansion of $\left(x - \frac{1}{y}\right)^5$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(x - \frac{1}{y}\right)^5$

$\Rightarrow {}^5C_0(x)^{5-0} + {}^5C_1(x)^{5-1}\left(-\frac{1}{y}\right)^1 + {}^5C_2(x)^{5-2}\left(-\frac{1}{y}\right)^2 + {}^5C_3(x)^{5-3}\left(-\frac{1}{y}\right)^3 + {}^5C_4(x)^{5-4}\left(-\frac{1}{y}\right)^4 + {}^5C_5\left(-\frac{1}{y}\right)^5$

$$\Rightarrow \left[\frac{5!}{0!(5-0)!}(x^5)\right] - \left[\frac{5!}{1!(5-1)!}(x^4)\left(\frac{1}{y}\right)^1\right] + \left[\frac{5!}{2!(5-2)!}(x^3)\left(\frac{1}{y^2}\right)\right]$$

$$-\left[\frac{5!}{3!(5-3)!}(x^2)\left(\frac{1}{y^3}\right)\right] + \left[\frac{5!}{4!(5-4)!}(x)\left(\frac{1}{y^4}\right)\right] - \left[\frac{5!}{5!(5-5)!}\left(\frac{1}{y^5}\right)\right]$$

$$\Rightarrow [1(x^5)] - \left[5\left(\frac{x^4}{y}\right)\right] + \left[10\left(\frac{x^3}{y^2}\right)\right] - \left[10\left(\frac{x^2}{y^3}\right)\right] + \left[5\left(\frac{x}{y^4}\right)\right] - [1(y^5)]$$

$$\Rightarrow x^5 - 5\frac{x^4}{y} + 10\frac{x^3}{y^2} - 10\frac{x^2}{y^3} + 5\frac{x}{y^4} - y^5$$

Ans) $x^5 - 5\frac{x^4}{y} + 10\frac{x^3}{y^2} - 10\frac{x^2}{y^3} + 5\frac{x}{y^4} - y^5$

Q. 8. Using binomial theorem, expand each of the following:

$\left(\sqrt{x} + \sqrt{y}\right)^8$

Answer : To find: Expansion of $\left(\sqrt{x} + \sqrt{y}\right)^8$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(\sqrt{x}+\sqrt{y}\right)^8$

We can write $\sqrt{x}$ as $x^{\frac{1}{2}}$ and $\sqrt{y}$ as $y^{\frac{1}{2}}$

Now, we have to solve for $\left(x^{\frac{1}{2}}+y^{\frac{1}{2}}\right)^8$

$$\Rightarrow \left[{}^8C_0\left(x^{\frac{1}{2}}\right)^{8-0}\right] + \left[{}^8C_1\left(x^{\frac{1}{2}}\right)^{8-1}\left(y^{\frac{1}{2}}\right)^1\right] + \left[{}^8C_2\left(x^{\frac{1}{2}}\right)^{8-2}\left(y^{\frac{1}{2}}\right)^2\right] + \left[{}^8C_3\left(x^{\frac{1}{2}}\right)^{8-3}\left(y^{\frac{1}{2}}\right)^3\right] + \left[{}^8C_4\left(x^{\frac{1}{2}}\right)^{8-4}\left(y^{\frac{1}{2}}\right)^4\right] + \left[{}^8C_5\left(x^{\frac{1}{2}}\right)^{8-5}\left(y^{\frac{1}{2}}\right)^5\right] + \left[{}^8C_6\left(x^{\frac{1}{2}}\right)^{8-6}\left(y^{\frac{1}{2}}\right)^6\right] + \left[{}^8C_7\left(x^{\frac{1}{2}}\right)^{8-7}\left(y^{\frac{1}{2}}\right)^7\right] + \left[{}^8C_8\left(y^{\frac{1}{2}}\right)^8\right]$$

$$\Rightarrow \left[\frac{8!}{0!(8-0)!}\left(x^{\frac{8}{2}}\right)\right] + \left[\frac{8!}{1!(8-1)!}\left(x^{\frac{7}{2}}\right)\left(y^{\frac{1}{2}}\right)\right] + \left[\frac{8!}{2!(8-2)!}\left(x^{\frac{6}{2}}\right)\left(y^{\frac{2}{2}}\right)\right] + \left[\frac{8!}{3!(8-3)!}\left(x^{\frac{5}{2}}\right)\left(y^{\frac{3}{2}}\right)\right] + \left[\frac{8!}{4!(8-4)!}\left(x^{\frac{4}{2}}\right)\left(y^{\frac{4}{2}}\right)\right] + \left[\frac{8!}{5!(8-5)!}\left(x^{\frac{3}{2}}\right)\left(y^{\frac{5}{2}}\right)\right] + \left[\frac{8!}{6!(8-6)!}\left(x^{\frac{2}{2}}\right)\left(y^{\frac{6}{2}}\right)\right] + \left[\frac{8!}{7!(8-7)!}\left(x^{\frac{1}{2}}\right)\left(y^{\frac{7}{2}}\right)\right] + \left[\frac{8!}{8!(8-8)!}\left(y^{\frac{8}{2}}\right)\right]$$

$$\Rightarrow [1(x^4)] + \left[8\left(x^{\frac{7}{2}}\right)\left(y^{\frac{1}{2}}\right)\right] + [28(x^3)(y)] + \left[56\left(x^{\frac{5}{2}}\right)\left(y^{\frac{3}{2}}\right)\right]$$

$$+ [70(x^2)(y^2)] + \left[56\left(x^{\frac{3}{2}}\right)\left(y^{\frac{5}{2}}\right)\right] + [28(x^1)(y^3)] + \left[8\left(x^{\frac{1}{2}}\right)\left(y^{\frac{7}{2}}\right)\right] + [1(y^4)]$$

Ans) $(x^4) + 8\left(x^{7/2}\right)\left(y^{1/2}\right) + 28(x^3)(y) + 56\left(x^{5/2}\right)\left(y^{3/2}\right) + 70(x^2)(y^2) + 56\left(x^{3/2}\right)\left(y^{5/2}\right) + 28(x)^1(y)^3 + 8\left(x^{1/2}\right)\left(y^{7/2}\right) + (y)^4$

Q. 9. Using binomial theorem, expand each of the following:

$\left(\sqrt[3]{x} - \sqrt[3]{y}\right)^6$

Answer : To find: Expansion of $\left(\sqrt[3]{x}-\sqrt[3]{y}\right)^6$

Formula used: (i) $^{n}C_{r}=\frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(\sqrt[3]{x}-\sqrt[3]{y}\right)^6$

We can write $\sqrt[3]{x}$ as $x^{\frac{1}{3}}$ and $\sqrt[3]{y}$ as $y^{\frac{1}{3}}$

Now, we have to solve for $\left(x^{\frac{1}{3}}-y^{\frac{1}{3}}\right)^6$

$$\Rightarrow\left[{}^6C_0\left(x^{\frac{1}{3}}\right)^{6-0}\right]+\left[{}^6C_1\left(x^{\frac{1}{3}}\right)^{6-1}\left(-y^{\frac{1}{3}}\right)^1\right]+\left[{}^6C_2\left(x^{\frac{1}{3}}\right)^{6-2}\left(-y^{\frac{1}{3}}\right)^2\right]+$$
$$\left[{}^6C_3\left(x^{\frac{1}{3}}\right)^{6-3}\left(-y^{\frac{1}{3}}\right)^3\right]+\left[{}^6C_4\left(x^{\frac{1}{3}}\right)^{6-4}\left(-y^{\frac{1}{3}}\right)^4\right]+\left[{}^6C_5\left(x^{\frac{1}{3}}\right)^{6-5}\left(-y^{\frac{1}{3}}\right)^5\right]+$$
$$\left[{}^6C_6\left(-y^{\frac{1}{3}}\right)^6\right]$$

$$\Rightarrow\left[{}^6C_0\left(x^{\frac{6}{3}}\right)\right]-\left[{}^6C_1\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right]+\left[{}^6C_2\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right]-\left[{}^6C_3\left(x^{\frac{3}{3}}\right)\left(y^{\frac{3}{3}}\right)\right]+$$
$$\left[{}^6C_4\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{3}}\right)\right]-\left[{}^6C_5\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right]+\left[{}^6C_6\left(y^{\frac{6}{3}}\right)\right]$$

$$\Rightarrow\left[\frac{6!}{0!(6-0)!}(x^2)\right]-\left[\frac{6!}{1!(6-1)!}\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right]+\left[\frac{6!}{2!(6-2)!}\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right]$$

$$-\left[\frac{6!}{3!(6-3)!}(x)(y)\right]+\left[\frac{6!}{4!(6-4)!}\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{3}}\right)\right]-\left[\frac{6!}{5!(6-5)!}\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right]$$

$$+\left[\frac{6!}{6!(6-6)!}(y^2)\right]$$

$\Rightarrow [1(x^2)] - \left[6\left(x^{\frac{5}{3}}\right)\left(y^{\frac{1}{3}}\right)\right] + \left[15\left(x^{\frac{4}{3}}\right)\left(y^{\frac{2}{3}}\right)\right] - [20(x)(y)] + \left[15\left(x^{\frac{2}{3}}\right)\left(y^{\frac{4}{3}}\right)\right] - \left[6\left(x^{\frac{1}{3}}\right)\left(y^{\frac{5}{3}}\right)\right] + [1(y^2)]$

$\Rightarrow x^2 - 6x^{\frac{5}{3}}y^{\frac{1}{3}} + 15x^{\frac{4}{3}}y^{\frac{2}{3}} - 20xy + 15x^{\frac{2}{3}}y^{\frac{4}{3}} - 6x^{\frac{1}{3}}y^{\frac{5}{3}} + y^2$

Ans) $x^2 - 6x^{5/3}y^{1/3} + 15x^{4/3}y^{2/3} - 20xy + 15x^{2/3}y^{4/3} - 6x^{1/3}y^{5/3} + y^2$

Q. 10. Using binomial theorem, expand each of the following:

$(1 + 2x – 3x^2)^4$

Answer : To find: Expansion of $(1 + 2x – 3x^2)^4$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(1 + 2x – 3x^2)^4$

Let $(1+2x) = a$ and $(-3x^2) = b$ … (i)

Now the equation becomes $(a + b)^4$

$\Rightarrow [{}^4C_0(a)^{4-0}] + [{}^4C_1(a)^{4-1}(b)^1] + [{}^4C_2(a)^{4-2}(b)^2] + [{}^4C_3(a)^{4-3}(b)^3] + [{}^4C_4(b)^4]$

$\Rightarrow [{}^4C_0(a)^4] + [{}^4C_1(a)^3(b)^1] + [{}^4C_2(a)^2(b)^2] + [{}^4C_3(a)(b)^3] + [{}^4C_4(b)^4]$

(Substituting value of b from eqn. i)

$$\Rightarrow \left[\frac{4!}{0!(4-0)!}(a)^4\right] + \left[\frac{4!}{1!(4-1)!}(a)^3(-3x^2)^1\right] + \left[\frac{4!}{2!(4-2)!}(a)^2(-3x^2)^2\right]$$

$$+ \left[\frac{4!}{3!(4-3)!}(a)(-3x^2)^3\right] + \left[\frac{4!}{4!(4-4)!}(-3x^2)^4\right]$$

(Substituting value of b from eqn. i)

$\Rightarrow [1(1+2x)^4] - [4(1+2x)^3(3x^2)] + [6(1+2x)^2(9x^4)] - [4(1+2x)(27x^6)^3] + [1(81x^8)^4]$ ...(ii)

We need the value of a^4,a^3 and a^2, where a = (1+2x)

For $(1+2x)^4$, Applying Binomial theorem

$(1+2x)^4 \Rightarrow$

$$^4C_0(1)^{4-0} + {}^4C_1(1)^{4-1}(2x)^1 + {}^4C_2(1)^{4-2}(2x)^2 + {}^4C_3(1)^{4-3}(2x)^3 + {}^4C_4(2x)^4$$

$$\Rightarrow \frac{4!}{0!(4-0)!}(1)^4 + \frac{4!}{1!(4-1)!}(1)^3(2x)^1 + \frac{4!}{2!(4-2)!}(1)^2(2x)^2$$

$$+ \frac{4!}{3!(4-3)!}(1)(2x)^3 + \frac{4!}{4!(4-4)!}(2x)^4$$

$$\Rightarrow [1] + [4(1)(2x)] + [6(1)(4x^2)] + [4(1)(8x^3)] + [1(16x^4)]$$

$\Rightarrow 1 + 8x + 24x^2 + 32x^3 + 16x^4$

We have $(1+2x)^4 = 1 + 8x + 24x^2 + 32x^3 + 16x^4$ ... (iii)

For $(a+b)^3$, we have formula $a^3+b^3+3a^2b+3ab^2$

For, $(1+2x)^3$, substituting a = 1 and b = 2x in the above formula

$\Rightarrow 1^3 + (2x)^3 + 3(1)^2(2x) + 3(1)(2x)^2$

$\Rightarrow 1 + 8x^3 + 6x + 12x^2$

$\Rightarrow 8x^3 + 12x^2 + 6x + 1$ … (iv)

For $(a+b)^2$, we have formula $a^2+2ab+b^2$

For, $(1+2x)^2$, substituting a = 1 and b = 2x in the above formula

$\Rightarrow (1)^2 + 2(1)(2x) + (2x)^2$

$\Rightarrow 1 + 4x + 4x^2$

$\Rightarrow 4x^2 + 4x + 1$ … (v)

Putting the value obtained from eqn. (iii),(iv) and (v) in eqn. (ii)

$\Rightarrow 1(1 + 8x + 24x^2 + 32x^3 + 16x^4) - 4(8x^3 + 12x^2 + 6x + 1)(3x^2)$

$+ 6(4x^2 + 4x + 1)(9x^4) - 4(1+2x)(27x^6)^3 + 1(81x^8)$

$\Rightarrow 1(1 + 8x + 24x^2 + 32x^3 + 16x^4) - 4(24x^5 + 36x^4 + 18x^3 + 3x^2)$

$+ 6(36x^6 + 36x^5 + 9x^4) - 4(27x^6 + 54x^7) + 1(81x^8)$

$\Rightarrow 1 + 8x + 24x^2 + 32x^3 + 16x^4 - 96x^5 - 144x^4 - 72x^3 - 12x^2 + 216x^6 + 216x^5 + 54x^4 - 108x^6 - 216x^7 + 81x^8$

On rearranging

Ans) $81x^8 - 216x^7 + 108x^6 + 120x^5 - 74x^4 - 40x^3 + 12x^2 + 8x + 1$

Q. 11. Using binomial theorem, expand each of the following:

$\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4, x \neq 0$

Answer : To find: Expansion of $\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4, x \neq 0$

Formula used: **(i)** ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $\left(1 + \frac{x}{2} - \frac{2}{x}\right)^4, x \neq 0$

Let $\left(1 + \frac{x}{2}\right) = a$ and $\left(-\frac{2}{x}\right) = b$ … (i)

Now the equation becomes $(a + b)^4$

$\Rightarrow [{}^4C_0(a)^{4-0}] + [{}^4C_1(a)^{4-1}(b)^1] + [{}^4C_2(a)^{4-2}(b)^2] + [{}^4C_3(a)^{4-3}(b)^3] + [{}^4C_4(b)^4]$

$\Rightarrow [{}^4C_0(a)^4] + [{}^4C_1(a)^3(b)^1] + [{}^4C_2(a)^2(b)^2] + [{}^4C_3(a)(b)^3] + [{}^4C_4(b)^4]$

(Substituting value of b from eqn. i)

$$\Rightarrow \left[\frac{4!}{0!(4-0)!}(a)^4\right] + \left[\frac{4!}{1!(4-1)!}(a)^3\left(-\frac{2}{x}\right)^1\right] + \left[\frac{4!}{2!(4-2)!}(a)^2\left(-\frac{2}{x}\right)^2\right] +$$
$$\left[\frac{4!}{3!(4-3)!}(a)^1\left(-\frac{2}{x}\right)^3\right] + \left[\frac{4!}{4!(4-4)!}\left(-\frac{2}{x}\right)^4\right]$$

(Substituting value of a from eqn. i)

$$\Rightarrow \left[1\left(1+\frac{x}{2}\right)^4\right] - \left[4\left(1+\frac{x}{2}\right)^3\left(\frac{2}{x}\right)\right] + \left[6\left(1+\frac{x}{2}\right)^2\left(\frac{4}{x^2}\right)\right]$$
$$-\left[4\left(1+\frac{x}{2}\right)^1\left(\frac{8}{x^3}\right)\right] + \left[1\left(\frac{16}{x^4}\right)\right] \quad \ldots\text{(ii)}$$

We need the value of a^4,a^3 and a^2, where a = $\left(1+\frac{x}{2}\right)$

For $\left(1+\frac{x}{2}\right)^4$, Applying Binomial theorem

$\left(1+\frac{x}{2}\right)^4$ =

$$[{}^4C_0(1)^{4-0}] + \left[{}^4C_1(1)^{4-1}\left(\frac{x}{2}\right)^1\right] + \left[{}^4C_2(1)^{4-2}\left(\frac{x}{2}\right)^2\right] + \left[{}^4C_3(1)^{4-3}\left(\frac{x}{2}\right)^3\right] + \left[{}^4C_4\left(\frac{x}{2}\right)^4\right]$$

$$\Rightarrow \left[\frac{4!}{0!(4-0)!}(1)^4\right] + \left[\frac{4!}{1!(4-1)!}(1)^3\left(\frac{x}{2}\right)^1\right] + \left[\frac{4!}{2!(4-2)!}(1)^2\left(\frac{x}{2}\right)^2\right]$$

$$+ \left[\frac{4!}{3!(4-3)!}(1)\left(\frac{x}{2}\right)^3\right] + \left[\frac{4!}{4!(4-4)!}\left(\frac{x}{2}\right)^4\right]$$

$$\Rightarrow [\,1\,] + \left[4(1)\left(\frac{x}{2}\right)\right] + \left[6(1)\left(\frac{x^2}{4}\right)\right] + \left[4(1)\left(\frac{x^3}{8}\right)\right] + \left[1\left(\frac{x^4}{16}\right)\right]$$

$\Rightarrow 1 + 2x + \frac{3}{2}x^2 + \frac{x^3}{2} + \frac{x^4}{16}$

On rearranging the above eqn.

$\Rightarrow \frac{1}{16}x^4 + \frac{1}{2}x^3 + \frac{3}{2}x^2 + 2x + 1 \ldots$ (iii)

We have, $\left(1+\frac{x}{2}\right)^4 = \frac{1}{16}x^4 + \frac{1}{2}x^3 + \frac{3}{2}x^2 + 2x + 1$

For, $(a+b)^3$, we have formula $a^3+b^3+3a^2b+3ab^2$

For, $\left(1+\frac{x}{2}\right)^3$, substituting a = 1 and b = $\frac{x}{2}$ in the above formula

$\Rightarrow 1^3 + \left(\frac{x}{2}\right)^3 + 3(1)^2\left(\frac{x}{2}\right) + 3(1)\left(\frac{x}{2}\right)^2$

$\Rightarrow 1 + \left(\frac{x^3}{8}\right) + \left(\frac{3x}{2}\right) + \left(\frac{3x^2}{4}\right)$

$\Rightarrow \left(\frac{x^3}{8}\right) + \left(\frac{3x^2}{4}\right) + \left(\frac{3x}{2}\right) + 1 \ldots$ (iv)

For, $(a+b)^2$, we have formula $a^2+2ab+b^2$

For, $\left(1+\frac{x}{2}\right)^2$, substituting a = 1 and b = $\frac{x}{2}$ in the above formula

$\Rightarrow (1)^2 + 2(1)\left(\frac{x}{2}\right) + \left(\frac{x}{2}\right)^2$

$\Rightarrow 1 + x + \left(\frac{x^2}{4}\right)$

$\Rightarrow \frac{x^2}{4} + x + 1 \ldots$ (v)

Putting the value obtained from eqn. (iii),(iv) and (v) in eqn. (ii)

$$\Rightarrow \left[1\left(\frac{1}{16}x^4+\frac{1}{2}x^3+\frac{3}{2}x^2+2x+1\right)\right] - \left[4\left(\frac{x^3}{8}+\frac{3x^2}{4}+\frac{3x}{2}+1\right)\left(\frac{2}{x}\right)\right]$$

$$\left[6\left(\frac{x^2}{4}+x+1\right)\left(\frac{4}{x^2}\right)\right] - \left[4\left(1+\frac{x}{2}\right)\left(\frac{8}{x^3}\right)\right] + \left[1\left(\frac{16}{x^4}\right)\right]$$

$$\Rightarrow \frac{1}{16}x^4+\frac{1}{2}x^3+\frac{3}{2}x^2+2x+1-x^2-6x-12-\frac{8}{x}+6+\frac{24}{x}+\frac{24}{x^2}$$

$$-\frac{32}{x^3}-\frac{16}{x^2}+\frac{16}{x^4}$$

On rearranging

Ans) $\frac{1}{16}x^4+\frac{1}{2}x^3+\frac{1}{2}x^2-4x-5+\frac{16}{x}+\frac{8}{x^2}-\frac{32}{x^3}+\frac{16}{x^4}$

Q. 12. Using binomial theorem, expand each of the following:

$(3x^2 - 2ax + 3a^2)^3$

Answer : To find: Expansion of $(3x^2 - 2ax + 3a^2)^3$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

We have, $(3x^2 - 2ax + 3a^2)^3$

Let, $(3x^2 - 2ax) = p$ … (i)

The equation becomes $(p + 3a^2)^3$

$$\Rightarrow [{}^3C_0(p)^{3-0}] + [{}^3C_1(p)^{3-1}(3a^2)^1] + [{}^3C_2(p)^{3-2}(3a^2)^2] + [{}^3C_3(3a^2)^3]$$

$$\Rightarrow [{}^3C_0(p)^3] + [{}^3C_1(p)^2(3a^2)] + [{}^3C_2(p)(9a^4)] + [{}^3C_3(27a^6)]$$

Substituting the value of p from eqn. (i)

$$\Rightarrow \left[\frac{3!}{0!(3-0)!}(3x^2-2ax)^3\right] + \left[\frac{3!}{1!(3-1)!}(3x^2-2ax)^2(3a^2)\right]$$

$$+\left[\frac{3!}{2!(3-2)!}(3x^2-2ax)(9a^4)\right]+\left[\frac{3!}{3!(3-3)!}(27a^6)\right]$$

$\Rightarrow [1(3x^2-2ax)^3]+[3(3x^2-2ax)^2(3a^2)]+[3(3x^2-2ax)(9a^4)]+[1(27a^6)^3]$ ... (ii)

We need the value of p^3 and p^2, where $p = 3x^2 - 2ax$

For, $(a+b)^3$, we have formula $a^3+b^3+3a^2b+3ab^2$

For, $(3x^2 - 2ax)^3$, substituting $a = 3x^2$ and $b = -2ax$ in the above formula

$\Rightarrow [(3x^2)^3]+[(-2ax)^3]+[3(3x^2)^2(-2ax)]+[3(3x^2)(-2ax)^2]$

$\Rightarrow 27x^6 - 8a^3x^3 - 54ax^5 + 36a^2x^4$ ... (iii)

For, $(a+b)^2$, we have formula $a^2+2ab+b^2$

For, $(3x^2 - 2ax)^3$, substituting $a = 3x^2$ and $b = -2ax$ in the above formula

$\Rightarrow [(3x^2)^2]+[2(3x^2)(-2ax)]+[(-2ax)^2]$

$\Rightarrow 9x^4 - 12x^3a + 4a^2x^2$ ... (iv)

Putting the value obtained from eqn. (iii) and (iv) in eqn. (ii)

$\Rightarrow [1(27x^6 - 8a^3x^3 - 54ax^5 + 36a^2x^4)]+[3(9x^4 - 12x^3a + 4a^2x^2)(3a^2)]+[3(3x^2-2ax)(9a^4)]+[1(27a^6)]$

$\Rightarrow 27x^6 - 8a^3x^3 - 54ax^5 + 36a^2x^4 + 81a^2x^4 - 108x^3a^3 + 36a^4x^2 + 81a^4x^2 - 54a^5x + 27a^6$

On rearranging

Ans) $27x^6 - 54ax^5 + 117a^2x^4 - 116x^3a^3 + 117a^4x^2 - 54a^5x + 27a^6$

Q. 13. Evaluate :

$$\left(\sqrt{2}+1\right)^6+\left(\sqrt{2}-1\right)^6$$

Answer : To find: Value of $(\sqrt{2}+1)^6+(\sqrt{2}-1)^6$

Formula used: (i) $^nC_r= \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+1)^6 =$
$[^6C_0a^6] +[^6C_1a^{6-1}1] + [^6C_2a^{6-2}1^2] + [^6C_3a^{6-3}1^3] + [^6C_4a^{6-4}1^4]+ [^6C_5a^{6-5}1^5] + [^6C_61^6]$

$\Rightarrow {}^6C_0a^6 + {}^6C_1a^5 + {}^6C_2a^4 + {}^6C_3a^3 + {}^6C_4a^2 + {}^6C_5a + {}^6C_6$ … (i)

$(a – 1)^6 =$

$[^6C_0a^6]+ [^6C_1a^{6-1}(-1)^1]+ [^6C_2a^{6-2}(-1)^2] + [^6C_3a^{6-3}(-1)^3] + [^6C_4a^{6-4}(-1)^4] + [^6C_5a^{6-5}(-1)^5] + [^6C_6(-1)^6]$

$\Rightarrow {}^6C_0a^6 - {}^6C_1a^5 + {}^6C_2a^4 - {}^6C_3a^3 + {}^6C_4a^2 - {}^6C_5a + {}^6C_6$ … (ii)

Adding eqn. (i) and (ii)

$(a+1)^6 + (a-1)^6 = [^6C_0a^6 + {}^6C_1a^5 + {}^6C_2a^4 + {}^6C_3a^3 + {}^6C_4a^2 + {}^6C_5a + {}^6C_6] + [^6C_0a^6- {}^6C_1a^5 + {}^6C_2a^4 - {}^6C_3a^3 + {}^6C_4a^2 - {}^6C_5a + {}^6C_6]$

$\Rightarrow 2[^6C_0a^6 + {}^6C_2a^4 + {}^6C_4a^2 + {}^6C_6]$

$\Rightarrow 2\left[\left(\frac{6!}{0!(6-0)!}a^6\right) + \left(\frac{6!}{2!(6-2)!}a^4\right) + \left(\frac{6!}{4!(6-4)!}a^2\right)+ \left(\frac{6!}{6!(6-6)!}\right)\right]$

$\Rightarrow 2[(1)a^6 + (15)a^4 + (15)a^2 + (1)]$

$\Rightarrow 2[a^6 + 15a^4 + 15a^2 + 1] = (a+1)^6 + (a-1)^6$

Putting the value of a = $\sqrt{2}$ in the above equation

$(\sqrt{2}+1)^6+(\sqrt{2}-1)^6 = 2[(\sqrt{2})_6 + 15(\sqrt{2})_4 + 15(\sqrt{2})_2 + 1]$

$\Rightarrow 2[8 + 15(4) + 15(2) + 1]$

$\Rightarrow 2[8 + 60 + 30 + 1]$

$\Rightarrow 2[99]$

$\Rightarrow 198$

Ans) 198

Q. 14. Evaluate :

$$\left(\sqrt{3}+1\right)^5 - \left(\sqrt{3}-1\right)^5$$

Answer : To find: Value of $(\sqrt{3}+1)^5 - (\sqrt{3}-1)^5$

Formula used: (I) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+1)^5 = {}^5C_0a^5 + {}^5C_1a^{5-1}1 + {}^5C_2a^{5-2}1^2 + {}^5C_3a^{5-3}1^3 + {}^5C_4a^{5-4}1^4 + {}^5C_51^5$

$\Rightarrow {}^5C_0a^5 + {}^5C_1a^4 + {}^5C_2a^3 + {}^5C_3a^2 + {}^5C_4a + {}^5C_5 \ldots$ (i)

$(a-1)^5$

$= [{}^5C_0a^5] + [{}^5C_1a^{5-1}(-1)^1] + [{}^5C_2a^{5-2}(-1)^2] + [{}^5C_3a^{5-3}(-1)^3] + [{}^5C_4a^{5-4}(-1)^4] + [{}^5C_5(-1)^5]$

$\Rightarrow {}^5C_0a^5 - {}^5C_1a^4 + {}^5C_2a^3 - {}^5C_3a^2 + {}^5C_4a - {}^5C_5 \ldots$ (ii)

Subtracting (ii) from (i)

$(a+1)^5 - (a-1)^5 = [{}^5C_0a^5 + {}^5C_1a^4 + {}^5C_2a^3 + {}^5C_3a^2 + {}^5C_4a + {}^5C_5] - [{}^5C_0a^5 - {}^5C_1a^4 + {}^5C_2a^3 - {}^5C_3a^2 + {}^5C_4a - {}^5C_5]$

$\Rightarrow 2[{}^5C_1a^4 + {}^5C_3a^2 + {}^5C_5]$

$\Rightarrow 2\left[\left(\frac{5!}{1!(5-1)!}a^4\right) + \left(\frac{5!}{3!(5-3)!}a^2\right) + \left(\frac{5!}{5!(5-5)!}\right)\right]$

$\Rightarrow 2[(5)a^4 + (10)a^2 + (1)]$

$\Rightarrow 2[5a^4 + 10a^2 + 1] = (a+1)^5 - (a-1)^5$

Putting the value of a = $\sqrt{3}$ in the above equation

$(\sqrt{3}+1)^5-(\sqrt{3}-1)^5 = 2[5(\sqrt{3})^4 + 10(\sqrt{3})^2 + 1]$

$\Rightarrow 2[(5)(9) + (10)(3) + 1]$

$\Rightarrow 2[45+30+1]$

$\Rightarrow 152$

Ans) 152

Q. 15. Evaluate :

$$\left(2+\sqrt{3}\right)^7+\left(2-\sqrt{3}\right)^7$$

Answer: To find: Value of $(2+\sqrt{3})^7+(2-\sqrt{3})^7$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+b)^7 = [{}^7C_0a^7] + [{}^7C_1a^{7-1}b] + [{}^7C_2a^{7-2}b^2] + [{}^7C_3a^{7-3}b^3] + [{}^7C_4a^{7-4}b^4] + [{}^7C_5a^{7-5}b^5] + [{}^7C_6a^{7-6}b^6] + [{}^7C_7b^7]$

$\Rightarrow {}^7C_0a^7 + {}^7C_1a^6b + {}^7C_2a^5b^2 + {}^7C_3a^4b^3 + {}^7C_4a^3b^4 + {}^7C_5a^2b^5 + {}^7C_6a^1b^6 + {}^7C_7b^7 \ldots$ (i)

$(a-b)^7 = [{}^7C_0a^7] + [{}^7C_1a^{7-1}(-b)] + [{}^7C_2a^{7-2}(-b)^2] + [{}^7C_3a^{7-3}(-b)^3] + [{}^7C_4a^{7-4}(-b)^4] + [{}^7C_5a^{7-5}(-b)^5] + [{}^7C_6a^{7-6}(-b)^6] + [{}^7C_7(-b)^7]$

$\Rightarrow {}^7C_0a^7 - {}^7C_1a^6b + {}^7C_2a^5b^2 - {}^7C_3a^4b^3 + {}^7C_4a^3b^4 - {}^7C_5a^2b^5 + {}^7C_6a^1b^6 - {}^7C_7b^7 \ldots$ (ii)

Adding eqn. (i) and (ii)

$(a+b)^7 + (a-b)^7 = [{}^7C_0a^7 + {}^7C_1a^6b + {}^7C_2a^5b^2 + {}^7C_3a^4b^3 + {}^7C_4a^3b^4 + {}^7C_5a^2b^5 + {}^7C_6a^1b^6 + {}^7C_7b^7] + [{}^7C_0a^7 - {}^7C_1a^6b + {}^7C_2a^5b^2 - {}^7C_3a^4b^3 + {}^7C_4a^3b^4 - {}^7C_5a^2b^5 + {}^7C_6a^1b^6 - {}^7C_7b^7]$

$\Rightarrow 2[{}^7C_0a^7 + {}^7C_2a^5b^2 + {}^7C_4a^3b^4 + {}^7C_6a^1b^6]$

$\Rightarrow 2\left[\left[\frac{7!}{o!(7-0)!}a^7\right] + \left[\frac{7!}{2!(7-2)!}a^{5b^2}\right] + \left[\frac{7!}{4!(7-4)!}a^3b^4\right] + \left[\frac{7!}{6!(7-6)!}a^1b^6\right]\right]$

$\Rightarrow 2[(1)a^7 + (21)a^5b^2 + (35)a^3b^4 + (7)ab^6]$

$\Rightarrow 2[a^7 + 21a^5b^2 + 35a^3b^4 + 7ab^6] = (a+b)^7 + (a-b)^7$

Putting the value of a = 2 and b = $\sqrt{3}$ in the above equation

$\left(2+\sqrt{3}\right)^7 + \left(2-\sqrt{3}\right)^7$

$= 2\left[\{2^7\} + \{21(2)^5(\sqrt{3})^2\} + \{35(2)^3(\sqrt{3})^4\} + \{7(2)(\sqrt{3})^6\}\right]$

= 2[128 + 21(32)(3)+ 35(8)(9) + 7(2)(27)]

= 2[128 + 2016 + 2520 + 378]

= 10084

Ans) 10084

Q. 16. Evaluate :

$\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$

Answer : To find: Value of $(\sqrt{3}+\sqrt{2})^6 - (\sqrt{3}-\sqrt{2})^6$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+b)^6 = {}^6C_0a^6 + {}^6C_1a^{6-1}b + {}^6C_2a^{6-2}b^2 + {}^6C_3a^{6-3}b^3 + {}^6C_4a^{6-4}b^4 + {}^6C_5a^{6-5}b^5 + {}^6C_6b^6$

$\Rightarrow {}^6C_0a^6 + {}^6C_1a^5b + {}^6C_2a^4b^2 + {}^6C_3a^3b^3 + {}^6C_4a^2b^4 + {}^6C_5ab^5 + {}^6C_6b^6 \ldots$ (i)

$(a-b)^6 =$

$= [^6C_0a^6] + [^6C_1a^{6-1}(-b)] + [^6C_2a^{6-2}(-b)^2] + [^6C_3a^{6-3}(-b)^3] + [^6C_4a^{6-4}(-b)^4] + [^6C_5a^{6-5}(-b)^5] + [^6C_6(-b)^6]$

$\Rightarrow {}^6C_0a^6 - {}^6C_1a^5b + {}^6C_2a^4b^2 - {}^6C_3a^3b^3 + {}^6C_4a^2b^4 - {}^6C_5ab^5 + {}^6C_6b^6$ ... (ii)

Substracting (ii) from (i)

$(a+b)^6 - (a-b)^6 = [{}^6C_0a^6 + {}^6C_1a^5b + {}^6C_2a^4b^2 + {}^6C_3a^3b^3 + {}^6C_4a^2b^4 + {}^6C_5ab^5 + {}^6C_6b^6] - [{}^6C_0a^6 - {}^6C_1a^5b + {}^6C_2a^4b^2 - {}^6C_3a^3b^3 + {}^6C_4a^2b^4 - {}^6C_5ab^5 + {}^6C_6b^6]$

$= 2[{}^6C_1a^5b + {}^6C_3a^3b^3 + {}^6C_5ab^5]$

$= 2\left[\left\{\frac{6!}{1!(6-1)!}a^5a\right\} + \left\{\frac{6!}{3!(6-3)!}a^3b^3\right\} + \left\{\frac{6!}{5!(6-5)!}ab^5\right\}\right]$

$= 2[(6)a^5b + (20)a^3b^3 + (6)ab^5]$

$\Rightarrow (a+b)^6 - (a-b)^6 = 2[(6)a^5b + (20)a^3b^3 + (6)ab^5]$

Putting the value of $a = \sqrt{3}$ and $b = \sqrt{2}$ in the above equation

$\left(\sqrt{3}+\sqrt{2}\right)^6 - \left(\sqrt{3}-\sqrt{2}\right)^6$

$\Rightarrow 2\left[(6)(\sqrt{3})^5(\sqrt{2}) + (20)(\sqrt{3})^3(\sqrt{2})^3 + (6)(\sqrt{3})(\sqrt{2})^5\right]$

$\Rightarrow 2[54(\sqrt{6}) + 120(\sqrt{6}) + 24(\sqrt{6})]$

$\Rightarrow 396\sqrt{6}$

Ans) $396\sqrt{6}$

Q. 17. Prove that

$$\sum_{r=0}^{n} {}^nC_r.3^r = 4^n$$

Answer :

To prove: $\sum_{r=0}^{n} {}^nC_r \cdot 3^r = 4^n$

Formula used: $\sum_{r=0}^{n} {}^nC_r \cdot a^{n-r} b^r = (a+b)^n$

Proof: In the above formula if we put a = 1 and b = 3, then we will ge

$$\sum_{r=0}^{n} {}^nC_r \cdot 1^{n-r} 3^r = (1+3)^n$$

Therefore,

$$\sum_{r=0}^{n} {}^nC_r \cdot 3^r = (4)^n$$

Hence Proved.

Q. 18. Using binominal theorem, evaluate each of the following :

(i) $(101)^4$ (ii) $(98)^4$
(iii)$(1.2)^4$

Answer : (i) $(101)^4$

To find: Value of $(101)^4$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0 a^n + {}^nC_1 a^{n-1} b + {}^nC_2 a^{n-2} b^2 + \ldots\ldots + {}^nC_{n-1} a b^{n-1} + {}^nC_n b^n$

101 = (100+1)

Now $(101)^4 = (100+1)^4$

$(100+1)^4 =$
$[{}^4C_0(100)^{4-0}] + [{}^4C_1(100)^{4-1}(1)^1] + [{}^4C_2(100)^{4-2}(1)^2] +$
$[{}^4C_3(100)^{4-3}(1)^3] + [{}^4C_4(1)^4]$

$\Rightarrow [{}^4C_0(100)^4] + [{}^4C_1(100)^3(1)^1] + [{}^4C_2(100)^2(1)^2] + [{}^4C_3(100)^1(1)^3] + [{}^4C_4(1)^4]$

$\Rightarrow \left[\frac{4!}{0!(4-0)!}(100000000)\right] + \left[\frac{4!}{1!(4-1)!}(1000000)\right] + \left[\frac{4!}{2!(4-2)!}(10000)\right] + \left[\frac{4!}{3!(4-3)!}(100)\right] + \left[\frac{4!}{4!(4-4)!}(1)\right]$

$\Rightarrow [(1)(100000000)] + [(4)(1000000)] + [(6)(10000)] + [(4)(100)] + [(1)(1)]$

= 104060401

Ans) 104060401

(ii) $(98)^4$

To find: Value of $(98)^4$

Formula used: (I) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

98 = (100-2)

Now $(98)^4 = (100-2)^4$

$(100-2)^4$
$= [{}^4C_0(100)^{4-0}] + [{}^4C_1(100)^{4-1}(-2)^1] + [{}^4C_2(100)^{4-2}(-2)^2] + [{}^4C_3(100)^{4-3}(-2)^3] + [{}^4C_4(-2)^4]$

$\Rightarrow [{}^4C_0(100)^4] - [{}^4C_1(100)^3(2)] + [{}^4C_2(100)^2(4)] - [{}^4C_3(100)^1(8)] + [{}^4C_4(16)]$

$\Rightarrow \left[\frac{4!}{0!(4-0)!}(100000000)\right] - \left[\frac{4!}{1!(4-1)!}(1000000)(2)\right] + \left[\frac{4!}{2!(4-2)!}(10000)(4)\right] - \left[\frac{4!}{3!(4-3)!}(100)(8)\right] + \left[\frac{4!}{4!(4-4)!}(16)\right]$

$\Rightarrow [(1)(100000000)] - [(4)(1000000)(2)] + [(6)(10000)(4)] - [(4)(100)(8)] + [(1)(16)]$

= 92236816

Ans) 92236816

(iii) $(1.2)^4$

To find: Value of $(1.2)^4$

Formula used: (i) $^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

1.2 = (1 + 0.2)

Now $(1.2)^4 = (1 + 0.2)^4$

$(1+0.2)^4$
$= [{}^4C_0(1)^{4-0}] + [{}^4C_1(1)^{4-1}(0.2)^1] + [{}^4C_2(1)^{4-2}(0.2)^2] + [{}^4C_3(1)^{4-3}(0.2)^3] + [{}^4C_4(0.2)^4]$

$\Rightarrow [{}^4C_0(1)^4] + [{}^4C_1(1)^3(0.2)^1] + [{}^4C_2(1)^2(0.2)^2] + [{}^4C_3(1)^1(0.2)^3] + [{}^4C_4(0.2)^4]$

$\Rightarrow \left[\frac{4!}{0!(4-0)!}(1)\right] + \left[\frac{4!}{1!(4-1)!}(1)(0.2)\right] + \left[\frac{4!}{2!(4-2)!}(1)(0.04)\right] + \left[\frac{4!}{3!(4-3)!}(1)(0.008)\right] + \left[\frac{4!}{4!(4-4)!}(0.0016)\right]$

$\Rightarrow [(1)(1)] + [(4)(1)(0.2)] + [(6)(1)(0.04)] + [(4)(1)(0.008)] + [(1)(0.0016)]$

= 2.0736

Ans) 2.0736

Q. 19. Using binomial theorem, prove that (2^{3n} - 7n -1) is divisible by 49, where n N.

Answer : To prove: $(2^{3n} - 7n - 1)$ is divisible by 49, where n N

Formula used: $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(2^{3n} - 7n - 1) = (2^3)^n - 7n - 1$

$\Rightarrow 8^n - 7n - 1$

$\Rightarrow (1+7)^n - 7n - 1$

$\Rightarrow {}^nC_01^n + {}^nC_11^{n-1}7 + {}^nC_21^{n-2}7^2 + \ldots\ldots + {}^nC_{n-1}7^{n-1} + {}^nC_n7^n - 7n - 1$

$\Rightarrow {}^nC_0 + {}^nC_17 + {}^nC_27^2 + \ldots\ldots + {}^nC_{n-1}7^{n-1} + {}^nC_n7^n - 7n - 1$

$\Rightarrow 1 + 7n + 7^2[{}^nC_2 + {}^nC_37 + \ldots + {}^nC_{n-1}\,7^{n-3} + {}^nC_n\,7^{n-2}] - 7n - 1$

$\Rightarrow 7^2[{}^nC_2 + {}^nC_37 + \ldots + {}^nC_{n-1}\,7^{n-3} + {}^nC_n\,7^{n-2}]$

$\Rightarrow 49[{}^nC_2 + {}^nC_37 + \ldots + {}^nC_{n-1}\,7^{n-3} + {}^nC_n\,7^{n-2}]$

$\Rightarrow$ 49K, where $K = ({}^nC_2 + {}^nC_37 + \ldots + {}^nC_{n-1}\,7^{n-3} + {}^nC_n7^{n-2})$

Now, $(2^{3n} - 7n - 1) = 49K$

Therefore $(2^{3n} - 7n - 1)$ is divisible by 49

Q. 20. Prove that $\left(2+\sqrt{x}\right)^4 + \left(2-\sqrt{x}\right)^4 = 2(16+24x+x^2)$

Answer : To prove: $(2+\sqrt{x})^4 + (2-\sqrt{x})^4 = 2(16+24x+x^2)$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $(a+b)^n = {}^nC_0a^n + {}^nC_1a^{n-1}b + {}^nC_2a^{n-2}b^2 + \ldots\ldots + {}^nC_{n-1}ab^{n-1} + {}^nC_nb^n$

$(a+b)^4 = {}^4C_0a^4 + {}^4C_1a^{4-1}b + {}^4C_2a^{4-2}b^2 + {}^4C_3a^{4-3}b^3 + {}^4C_4b^4$

$\Rightarrow {}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3a^1b^3 + {}^4C_4b^4 \ldots$ (i)

$(a-b)^4 = {}^4C_0a^4 + {}^4C_1a^{4-1}(-b) + {}^4C_2a^{4-2}(-b)^2 + {}^4C_3a^{4-3}(-b)^3 + {}^4C_4(-b)^4$

$\Rightarrow {}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4 \ldots$ (ii)

Adding (i) and (ii)

$(a+b)^4 + (a-b)^7 = [{}^4C_0a^4 + {}^4C_1a^3b + {}^4C_2a^2b^2 + {}^4C_3a^1b^3 + {}^4C_4b^4] + [{}^4C_0a^4 - {}^4C_1a^3b + {}^4C_2a^2b^2 - {}^4C_3ab^3 + {}^4C_4b^4]$

$\Rightarrow 2[{}^4C_0a^4 + {}^4C_2a^2b^2 + {}^4C_4b^4]$

$\Rightarrow 2\left[\left(\frac{4!}{0!(4-0)!}a^4\right) + \left(\frac{4!}{2!(4-2)!}a^2b^2\right) + \left(\frac{4!}{4!(4-4)!}b^4\right)\right]$

$\Rightarrow 2[(1)a^4 + (6)a^2b^2 + (1)b^4]$

$\Rightarrow 2[a^4 + 6a^2b^2 + b^4]$

Therefore, $(a+b)^4 + (a-b)^7 = 2[a^4 + 6a^2b^2 + b^4]$

Now, putting a = 2 and b = $(\sqrt{x})$ in the above equation.

$(2+\sqrt{x})^4+(2-\sqrt{x})^4 = 2[(2)^4 + 6(2)^2 (\sqrt{x})^2 + (\sqrt{x})^4]$

$= 2(16+24x+x^2)$

Hence proved.

Q. 21. Find the 7^{th} term in the expansion of $\left(\frac{4x}{5}+\frac{5}{2x}\right)^8$.

Answer : To find: 7^{th} term in the expansion of $\left(\frac{4x}{5}+\frac{5}{2x}\right)^8$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

For 7^{th} term, r+1=7

$\Rightarrow r = 6$

In, $\left(\frac{4x}{5}+\frac{5}{2x}\right)^8$

7^{th} term = T_{6+1}

$\Rightarrow {}^{8}C_{6}\left(\frac{4x}{5}\right)^{8-6}\left(\frac{5}{2x}\right)^{6}$

$\Rightarrow \frac{8!}{6!(8-6)!}\left(\frac{4x}{5}\right)^{2}\left(\frac{5}{2x}\right)^{6}$

$\Rightarrow (28)\left(\frac{16x^2}{25}\right)\left(\frac{15625}{64x^6}\right)$

$\Rightarrow \frac{4375}{x^4}$

Ans) $\frac{4375}{x^4}$

Q. 22. Find the 9th term in the expansion of $\left(\frac{a}{b}-\frac{b}{2a^2}\right)^{12}$.

Answer : To find: 9^{th} term in the expansion of $\left(\frac{a}{b}-\frac{b}{2a^2}\right)^{12}$

Formula used: (i) ${}^{n}C_{r} = \frac{n!}{(n-r)!(r)!}$

(ii) $T_{r+1} = {}^{n}C_{r}\, a^{n-r} b^{r}$

For 9^{th} term, r+1=9

$\Rightarrow r = 8$

In, $\left(\frac{a}{b}-\frac{b}{2a^2}\right)^{12}$

9^{th} term = T_{8+1}

$\Rightarrow {}^{12}C_{8}\left(\frac{a}{b}\right)^{12-8}\left(\frac{-b}{2a^2}\right)^{8}$

$\Rightarrow \frac{12!}{8!(12-8)!}\left(\frac{a}{b}\right)^{4}\left(\frac{-b}{2a^2}\right)^{8}$

$\Rightarrow 495\left(\frac{a^4}{b^4}\right)\left(\frac{b^8}{256a^{16}}\right)$

$\Rightarrow \left(\frac{495b^4}{256a^{12}}\right)$

Ans) $\left(\frac{495\,b^4}{256\,a^{12}}\right)$

Q. 23. Find the 16th term in the expansion of $\left(\sqrt{x}-\sqrt{y}\right)^{17}$ **.**

Answer : To find: 16th term in the expansion of $\left(\sqrt{x}-\sqrt{y}\right)^{17}$

Formula used: (i) ${}^nC_r = \frac{n!}{(n-r)!(r)!}$

(ii) $T_{r+1} = {}^nC_r\, a^{n-r}\, b^r$

For 16th term, r+1=16

$\Rightarrow$ r = 15

In, $\left(\sqrt{x}-\sqrt{y}\right)^{17}$

16th term = T_{15+1}

$\Rightarrow {}^{17}C_{15}\left(\sqrt{x}\right)^{17-15}\left(-\sqrt{y}\right)^{15}$

$\Rightarrow \frac{17!}{15!(17-15)!}\left(\sqrt{x}\right)^2\left(-\sqrt{y}\right)^{15}$

$\Rightarrow 136(x)(-y)^{\frac{15}{2}}$

$\Rightarrow -136x\,y^{\frac{15}{2}}$

Ans) $-136\,y^{\frac{15}{2}}$

Q. 24. Find the 13th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}, x \neq 0$.

Answer : To find: 13th term in the expansion of $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$

Formula used: (i) ${}^{n}C_{r}=\frac{n!}{(n-r)!(r)!}$

(ii) $T_{r+1} = {}^{n}C_{r}\, a^{n-r}\, b^{r}$

For 13th term, r+1=13

⇒ r = 12

In, $\left(9x-\frac{1}{3\sqrt{x}}\right)^{18}$

13th term = T_{12+1}

$\Rightarrow {}^{18}C_{12}\,(9x)^{18-12}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$\Rightarrow \frac{18!}{12!(18-12)!}(9x)^{6}\left(-\frac{1}{3\sqrt{x}}\right)^{12}$

$\Rightarrow 18564(531441x^{6})\left(\frac{1}{531441x^{6}}\right)$

⇒ 18564

Q. 25. Find the coefficients of x^7 and x^8 in the expansion of $\left(2+\frac{x}{3}\right)^{n}$.

Answer : To find : coefficients of x^7 and x^8

Formula : $t_{r+1} = \binom{n}{r} a^{n-r} b^{r}$

Here, a=2, $b = \frac{x}{3}$

We have, $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

$\therefore t_{r+1} = \binom{n}{r} (2)^{n-r} \left(\frac{x}{3}\right)^r$

$= \binom{n}{r} \frac{2^{n-r}}{3^r} x^r$

To get a coefficient of x^7, we must have,

$x^7 = x^r$

• r = 7

Therefore, the coefficient of $x^7 = \binom{n}{7} \frac{2^{n-7}}{3^7}$

And to get the coefficient of x^8 we must have,

$x^8 = x^r$

• r = 8

Therefore, the coefficient of $x^8 = \binom{n}{8} \frac{2^{n-8}}{3^8}$

Conclusion :

• Coefficient of $x^7 = \binom{n}{7} \frac{2^{n-7}}{3^7}$

• Coefficient of $x^8 = \binom{n}{8} \frac{2^{n-8}}{3^8}$

Q. 26. Find the ratio of the coefficient of x^{15} to the term independent of x in the expansion of $\left(x^2 + \frac{2}{x}\right)^{15}$ **.**

Answer : To Find: the ratio of the coefficient of x^{15} to the term independent of x

Formula : $t_{r+1} = \binom{n}{r} a^{n-r} b^r$

Here, $a=x^2$, $b=\frac{2}{x}$ and $n=15$

We have a formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{15}{r} (x^2)^{15-r} \left(\frac{2}{x}\right)^r$$

$$= \binom{15}{r} (x)^{30-2r} (2)^r (x)^{-r}$$

$$= \binom{15}{r} (x)^{30-2r-r} (2)^r$$

$$= \binom{15}{r} (2)^r (x)^{30-3r}$$

To get coefficient of x^{15} we must have,

$(x)^{30-3r} = x^{15}$

• 30 - 3r = 15

• 3r = 15

• r = 5

Therefore, coefficient of $x^{15} = \binom{15}{5} (2)^5$

Now, to get coefficient of term independent of x that is coefficient of x^0 we must have,

$(x)^{30-3r} = x^0$

• 30 - 3r = 0

• 3r = 30

• r = 10

Therefore, coefficient of $x^0 = \binom{15}{10}(2)^{10}$

But $\binom{15}{10} = \binom{15}{5}$ .... $[\because \binom{n}{r} = \binom{n}{n-r}]$

Therefore, the coefficient of $x^0 = \binom{15}{5}(2)^{10}$

Therefore,

$$\frac{\text{coefficient of } x^{15}}{\text{coefficient of } x^0} = \frac{\binom{15}{5}(2)^5}{\binom{15}{5}(2)^{10}}$$

$$= \frac{1}{(2)^5}$$

$$= \frac{1}{32}$$

Hence, coefficient of x^{15} : coefficient of x^0 = 1:32

Conclusion : The ratio of coefficient of x^{15} to coefficient of x^0 = 1:32

Q. 27. Show that the ratio of the coefficient of x^{10} in the expansion of $(1 – x2)^{10}$and the term independent of x in the expansion of $\left(x - \frac{2}{x}\right)^{10}$ is 1 : 32.

Answer : To Prove : coefficient of x^{10} in $(1-x^2)^{10}$: coefficient of x^0 in $\left(x - \frac{2}{x}\right)^{10}$ = 1:32

For $(1-x^2)^{10}$,

Here, a=1, $b=-x^2$ and n=15

We have formula,

$$t_{r+1} = \binom{n}{r} a^{n-r} b^r$$

$$= \binom{10}{r} (1)^{10-r}(-x^2)^r$$