NCERT Solutions for Class-XI Maths

Chapter-11 Exercise-11.1

NCERT Math Class 11

1. Find the equation of the circle with centre $(0,2)$ and radius 2

1. The equation of a circle with centre (h,k) and radius r is given as

$(x-h)^2+(y-k)^2=r^2$

It is given that centre $(h,k)=(0,2)$ and radius $(r)=2$.

Therefore, the equation of the circle is

$(x-0)^2+(y-2)^2=2^2$

$\Rightarrow x^2+y^2+4-4y=4$

$\Rightarrow x^2+y^2-4y=0$

2. Centre $(-2,3)$ and radius 4

2. The equation of a circle with centre (h,k) and radius r is given as

$(x-h)^2+(y-k)^2=r^2$

It is given that centre (h,K) = (-2,3) and radius (r) =4

Therefore, the equation of the circle is

$(x+2)^2+(y-3)^2=(4)^2$

$\Rightarrow x^2+4x+4+y^2-6y+9=16$

$\Rightarrow x^2+y^2+4x-6y-3=0$

Therefore, the equation of the circle is $x^2+y^2+4x-6y-3=0$

3. Find the equation of the circle with centre $\left(\frac{1}{2},\frac{1}{4}\right)$ and radius $\frac{1}{12}$

3. The equation of a circle with centre (h,k) and radius r is given as $(x-h)^2+(y-k)^2=r^2$

It is given that centre $(h,k)=\left(\frac{1}{2},\frac{1}{4}\right)$ and radius $(r)=\frac{1}{12}$

Therefore, the equation of the circle is

$$\left(x-\frac{1}{2}\right)^2+\left(y-\frac{1}{4}\right)^2=\left(\frac{1}{12}\right)^2$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} = \frac{1}{144}$$

$$x^2 - x + \frac{1}{4} + y^2 - \frac{y}{2} + \frac{1}{16} - \frac{1}{144} = 0$$

$$144x^2 - 144x + 36 + 144y^2 - 72y + 9 - 1 = 0$$

$$144x^2 - 144x + 144y^2 - 72y + 44 = 0$$

$$36x^2 - 36x + 36y^2 - 18y + 11 = 0$$

$$36x^2 + 36y^2 - 36x - 18y + 11 = 0$$

4. Centre (1,1) and radius $\sqrt{2}$

4. The equation of a circle with centre (h,k) and radius r is given as

$(x-h)^2 + (y-k)^2 = r^2$

It is given that centre (h,k) =(1, 1) and radius (r) = $\sqrt{2}$.

Therefore, the equation of the circle is

$(x-1)^2 + (y-1)^2 = \left(\sqrt{2}\right)^2$

$\Rightarrow x^2 - 2x + 1 + y^2 - 2y + 1 = 2$

$\Rightarrow {}^2 + y^2 - 2x - 2y = 0$

5. Find the equation of the circle with centre $(-a, -b)$ and radius $\sqrt{a^2 - b^2}$.

5. The equation of a circle with centre (h, k) and radius r is given as

$$(x - h)^2 + (y - k)^2 = r^2$$

It is given that centre $(h, k) = (-a, -b)$ and radius $(r) = \sqrt{a^2 - b^2}$.

Therefore, the equation of the circle is

$$(x + a)^2 + (y + b)^2 = \left(\sqrt{a^2 - b^2}\right)^2$$

$$x^2 + 2ax + a^2 + y^2 + 2by + b^2 = a^2 - b^2$$

$$x^2 + y^2 + 2ax + 2by + 2b^2 = 0$$

6. In each of the following Exercises 6 to 9, find the centre and radius of the circles.

$(x + 5)^2 + (y - 3)^2 = 36$

6. The equation of the given circle is $(x+5)^2 + (y-3)^2 = 36$.

$(x+5)^2 + (y-3)^2 = 36$

$\Rightarrow \{x - (-5)\}^2 + (y - 3)^2 = 6^2$, which is of the form $(x - h)^2 + (y - k)^2 = r^2$,

Where, h =-5, k =3 and r= 6.
Thus, the centre of the given circle is (-5, 3), while its radius is 6.

7. Find the centre and radius of the circle $x^2+y^2-4x-8y-45=0$

7. The equation of the given circle is $x^2+y^2-4x-8y-45=0$.

$x^2+y^2-4x-8y-45=0$

$\Rightarrow\left(x^2-4x\right)+\left(y^2-8y\right)=45$

$\Rightarrow\left\{x^2-2(x)(2)+2^2\right\}+\left\{y^2-2(y)(4)+4^2\right\}-4-16=45$

$\Rightarrow(x-2)^2+(y-4)^2=65$

$\Rightarrow(x-2)^2+(y-4)^2=\sqrt{65},$

Which is of the form $(x-h)^2+(y-k)^2=r^2$, where $h=2,k=4$ and $r=\sqrt{65}$, Thus, the centre of the given circle is $(2,4)$, while its radius is $\sqrt{65}$.

8. $x^2 + y^2 – 8x + 10y – 12 = 0$

8. The equation of the given circle is $x^2 + y^2 -8x + 10y -12 =0$.

$x^2 + y^2 -8x + 10y -12 =0$

$\Rightarrow$ $(x^2 – 8x) + (y^2+10y) = 12$

$\Rightarrow$ $\left\{x^2-2(x)(4)+4^2\right\}$ + $\left\{y^2-2(y)(5)+5^2\right\}$ – 16 – 25 = 12

$\Rightarrow$ $(x - 4)^2 + (y + 5)^2 =53$

$\Rightarrow$ $(x - 4)^2 + \left\{y-(-5)\right\}^2 =\left(\sqrt{53}\right)^2$, which is form $(x-h)^2 +(y-k)^2 = r^2$, where h=4,

K= -5 and r= $\sqrt{53}$

9. Find the centre and radius of the circle $2x^2+2y^2-x=0$

9. The equation of the given circle is $2x^2+2y^2-x=0$.

$2x^2+2y^2-x=0$

$\Rightarrow\left(2x^2-x\right)+2y^2=0$

$\Rightarrow 2\left[\left(x^2-\frac{x}{2}\right)+y^2\right]=0$

$\Rightarrow\left\{x^2-2\cdot x\left(\frac{1}{4}\right)+\left(\frac{1}{4}\right)^2\right\}+y^2-\left(\frac{1}{4}\right)^2=0$

$\Rightarrow \left(x-\frac{1}{4}\right)^2+(y-0)^2=\left(\frac{1}{4}\right)^2$

which is of the form $(x-h)^2+(y-k)^2=r^2$, where $h=1/4, k=0$ and $r=1/4$. Thus, the centre of the given circle is $(1/4, 0)$, while its radius is $1/4$.

10. Find the equation of the circle passing through the points (4,1) and (6,5) and whose centre is on the line 4x + y = 16.

10. Let the equation of the required circle be $(x-h)^2 + (y-k)^2 = r^2$

Since, the circle passes through points (4,1) and (6,5),

$(4-h)^2 + (1-k)^2 = r^2$ ....(1)

$(x-h)^2 + (y-k)^2 = r^2$ ....(2)

Since, the centre (h,k) of the circle lies on line 4x+y = 16,

$4h + k = 16$..................... (3)

From the equation (1) and (2), we obtain

$(4-h)^2 + (1-k)^2 = (6-h)^2 + (5-k)^2$

$\Rightarrow 16 - 8h + h^2 + 1 - 2k + k^2 = 36 - 12h + h^2 + 15 - 10k + k^2$

$\Rightarrow 16 - 8h + 1 - 2k + 12h - 25 - 10k$

$\Rightarrow 4h + 8k = 44$

$\Rightarrow h + 2k = 11$................ (4)

On solving equations (3) and (4), we obtain h=3 and k= 4.

On substituting the values of h and k in equation (1), we obtain

$(4-3)^2 + (1-4)^2 = r^2$

$\Rightarrow (1)^2 + (-3)^2 = r^2$

$\Rightarrow 1+9 = r^2$

$\Rightarrow r = \sqrt{10}$

Thus, the equation of the requires circle is

$(x-3)^2 + (y-4)^2 = \left(\sqrt{10}\right)^2$

$\Rightarrow x^2 - 6x + 9 + y^2 - 8y + 16 = 10$

$\Rightarrow x^2 + y^2 - 6x - 8y = 15 = 0$

11. Find the equation of the circle passing through the points $(2,3)$ and $(-1,1)$ and whose centre is on the line $x-3y-11=0$.

11. Let the equation of the required circle be $(x-h)^2+(y-k)^2=r^2$.

Since the circle passes through points $(2,3)$ and $(-1,1)$,

$(2-h)^2+(3-k)^2=r^2$

$(-1-h)^2+(1-k)^2=r^2$

Since the centre (h,k) of the circle lies on line $x-3y-11=0$,
$h-3k=11$

From equations (1) and (2), we obtain
$(2-h)^2+(3-k)^2=(-1-h)^2+(1-k)^2$

$\Rightarrow 4-4h+h^2+9-6k+k^2=1+2h+h^2+1-2k+k^2$

$\Rightarrow 4-4h+9-6k=1+2h+1-2k$

$\Rightarrow 6h+4k=11\ldots$ (4)

On solving equations (3) and (4), we obtain $h=\frac{7}{2}$ and $k=\frac{-5}{2}$.

On substituting the values of h and k in equation (1), we obtain

$$\left(2-\frac{7}{2}\right)^2+\left(3+\frac{5}{2}\right)^2=r^2$$

$$\Rightarrow\left(\frac{4-7}{2}\right)^2+\left(\frac{6+5}{2}\right)^2=r^2$$

$$\Rightarrow\left(\frac{-3}{2}\right)^2+\left(\frac{11}{2}\right)^2=r^2$$

$$\Rightarrow\frac{9}{4}+\frac{121}{4}=r^2$$

$$\Rightarrow\frac{130}{4}=r^2$$

Thus, the equation of the required circle is

$$\left(x-\frac{7}{2}\right)^2+\left(y+\frac{5}{2}\right)^2=\frac{130}{4}$$

$$\left(\frac{2x-7}{2}\right)^2+\left(\frac{2y+5}{2}\right)^2=\frac{130}{4}$$

$$4x^2-28x+49+4y^2+20y+25=130$$

$$4x^2+4y^2-28x+20y-56=0$$

$$4\left(x^2+y^2-7x+5y-14\right)=0$$

$$x^2+y^2-7x+5y-14=0$$

12. Find the equation of the circle with radius 5 whose centre lies on x-axis and passes through the point (2,3).

12. Let the equation of the required circle be $(x - h)^2 + (y - k)^2 = r^2$

Since, the radius of the circle is 5 and its centre lies on the x- axis , k= 0 and r= 5.

Now, the equation of the circle becomes $(x - h)^2 + y^2 = 25$.

It is given that the circle passes through point (2, 3)

$\therefore (2 - h)^2 + 3^2 = 2$

$\Rightarrow (2 - h)^2 = 25-9$

$\Rightarrow (2 - h)^2 = 16$

$\Rightarrow 2 - h = \pm\sqrt{16} = \pm 4$

If 2-h =4, then h = -2

If 2-h = -4, then h =6.

When h= -2, the equation of the circle becomes

$(x - 2)^2 + y^2 = 25$

$\Rightarrow x^2 - 12x + 36 + y^2 = 25$

$\Rightarrow x^2 + y^2 - 4x + 21 = 0$

When h= 6, the equation of the circle becomes

$(x - 6)^2 + y^2 = 25$

$\Rightarrow x^2 - 12x + 36 + y^2 = 25$

$\Rightarrow x^2 + y^2 - 12x + 11 = 0$

13. Find the equation of the circle passing through $(0,0)$ and making intercepts a and b on the coordinate axes.

13. Let the equation of the required circle be $(x-h)^2 + (y-k)^2 = r^2$.

Since the centre of the circle passes through $(0,0)$,

$(0-h)^2 + (0-k)^2 = r^2$

$\Rightarrow h^2 + k^2 = r^2$

The equation of the circle now becomes $(x-h)^2 + (y-k)^2 = h^2 + k^2$.

It is given that the circle makes intercepts a and b on the coordinate axes. This means that the circle passes through points $(a,0)$ and $(0,b)$. Therefore,

$(a-h)^2 + (0-k)^2 = h^2 + k^2$

$(0-h)^2 + (b-k)^2 = h^2 + k^2$

From equation (1), we obtain $a^2 - 2ah + h^2 + k^2 = h^2 + k^2$

$\Rightarrow a^2 - 2ah = 0$

$\Rightarrow a(a-2h) = 0$

$\Rightarrow a = 0$ or $(a-2h) = 0$

However, $a \neq 0$; hence, $(a-2h) = 0 \Rightarrow h = a/2$.

From equation (2), we obtain $h^2 + b^2 - 2bk + k^2 = h^2 + k^2$

$\Rightarrow b^2 - 2bk = 0$

$\Rightarrow b(b-2k) = 0$

$\Rightarrow b = 0$ or $(b-2k) = 0$

However, $b \neq 0$; hence, $(b-2k) = 0 \Rightarrow k = b/2$.

Thus, the equation of the required circle is

$\left(x - \frac{a}{2}\right)^2 + \left(y - \frac{b}{2}\right)^2 = \left(\frac{a}{2}\right)^2 + \left(\frac{b}{2}\right)^2$

$\Rightarrow \left(\frac{2x-a}{2}\right)^2 + \left(\frac{2y-b}{2}\right)^2 = \frac{a^2+b^2}{4}$

$\Rightarrow 4x^2 - 4ax + a^2 + 4y^2 - 4by + b^2 = a^2 + b^2$

$\Rightarrow 4x^2 + 4y^2 - 4ax - 4by = 0$

$\Rightarrow x^2 + y^2 - ax - by = 0$

14. Find the equation of a circle with centre (2,2) and passes through the point (4,5).

14. The centre of the circle is given as (h,k) = (2,2).

Since, the circle passes through point (4,5), the radius (r) of the circle is the distance between the points (2,2) and (4,5).

$\therefore\ r = \sqrt{(2-4)^2 + (2-5)^2} = \sqrt{(-2)^2 + (-3)^2} = \sqrt{4+9} = \sqrt{13}$

Thus, the equation of the circle is

$(x-h)^2 + (y-k)^2 = r^2$

$\Rightarrow (x-h)^2 + (y-k)^2 = \sqrt{(13)^2}$

$\Rightarrow x^2 - 4x + 4 + y^2 - 4y + 4 = 13$

$\Rightarrow x^2 + y^2 - 4x - 4y - 5 = 0$

15. Does the point $(-2.5, 3.5)$ lie inside, outside or on the circle $x^2 + y^2 = 25$?

15. The equation of the given circle is $x^2 + y^2 = 25$.

$x^2 + y^2 = 25$

$\Rightarrow (x-0)^2 + (y-0)^2 = 5^2$

which is of the form $(x-h)^2 + (y-k)^2 = r^2$, where $h = 0, k = 0$, and $r = 5$.

$\therefore$ Centre $= (0,0)$ and radius $= 5$

Distance between point $(-2.5, 3.5)$ and centre $(0,0)$

$= \sqrt{(-2.5-0)^2 + (3.5-0)^2}$

$= \sqrt{6.25 + 12.25}$

$= \sqrt{18.5}$

$= 4.3 \text{ (approx.)} < 5$

Since the distance between point $(-2.5, 3.5)$ and centre $(0,0)$ of the circle is less than the radius of the circle, point $(-2.5, 3.5)$ lies inside the circle.