EXERCISE 5.4

The sum of the first five terms of an AP and the sum of the first seven terms of the same AP is 167. If the sum of the first ten terms of this AP is 235, find the sum of its first twenty terms.

Solution:

We know that, in an A.P.,
First term = a
Common difference = d
Number of terms of an AP = n
According to the question,
We have,
$S_5 + S_7 = 167$
Using the formula for sum of n terms,
$S_n = (n/2)\ [2a + (n-1)d]$
So, we get,
$(5/2)\ [2a + (5-1)d] + (7/2)[2a + (7-1)d] = 167$
$5(2a + 4d) + 7(2a + 6d) = 334$
$10a + 20d + 14a + 42d = 334$
$24a + 62d = 334$
$12a + 31d = 167$
$12a = 167 - 31d$ …(1)
We have,
$S_{10} = 235$
$(10/2)\ [2a + (10-1)d] = 235$
$5[\ 2a + 9d] = 235$
$2a + 9d = 47$
Multiplying L.H.S and R.H.S by 6,
We get,
$12a + 54d = 282$
From equation (1)
$167 - 31d + 54d = 282$
$23d = 282 – 167$
$23d = 115$
$d = 5$
Substituting the value of d = 5 in equation (1)
$12a = 167 - 31(5)$
$12a = 167 – 155$
$12a = 12$
$a = 1$
We know that,
$S_{20} = (n/2)\ [2a + (20 – 1)d]$
$= 20/(2[2(1) + 19\ (5)])$
$= 10[\ 2 + 95]$
$= 970$
Therefore, the sum of first 20 terms is 970.

1. Find the

(i) Sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

(ii) Sum of those integers from 1 to 500 which are multiples of 2 as well as of 5 .

(iii) Sum of those integers from 1 to 500 which are multiples of 2 or 5.

[Hint (iii): These numbers will be: multiples of 2 + multiples of 5 – multiples of 2 as well as of 5]

Solution:

(i) Sum of those integers between 1 and 500 which are multiples of 2 as well as of 5.

We know that,
Multiples of 2 as well as of 5 = LCM of (2, 5) = 10
Multiples of 2 as well as of 5 between 1 and 500 = 10, 20, 30…, 490.
Hence,
We can conclude that 10, 20, 30…, 490 is an AP with common difference, d = 10
First term, a = 10
Let the number of terms in this AP = n
Using n^{th} term formula,
$a_n = a + (n - 1)d$
$490 = 10 + (n - 1)10$
$480 = (n - 1)10$
$n - 1 = 48$
$n = 49$
Sum of an AP,
$S_n = (n/2)\ [a + a_n]$, here a_n is the last term, which is given]
$= (49/2) \times [10 + 490]$
$= (49/2) \times [500]$
$= 49 \times 250$
$= 12250$
Therefore, sum of those integers between 1 and 500 which are multiples of 2 as well as of 5 = 12250

(ii) Sum of those integers from 1 to 500 which are multiples of 2 as well as of 5.

We know that,
Multiples of 2 as well as of 5 = LCM of (2, 5) = 10
Multiples of 2 as well as of 5 from 1 and 500 = 10, 20, 30…, 500.
Hence,
We can conclude that 10, 20, 30…, 500 is an AP with common difference, d = 10
First term, a = 10
Let the number of terms in this AP = n
Using n^{th} term formula,
$a_n = a + (n - 1)d$
$500 = 10 + (n - 1)10$
$490 = (n - 1)10$
$n - 1 = 49$
$n = 50$
Sum of an AP,
$S_n = (n/2)\ [\ a + a_n]$, here a_n is the last term, which is given]

= (50/2) ×[10+500]
= 25× [10 + 500]
= 25(510)
= 12750

Therefore, sum of those integers from 1 to 500 which are multiples of 2 as well as of 5= 12750

(iii) Sum of those integers from 1 to 500 which are multiples of 2 or 5.

We know that,
Multiples of 2 or 5 = Multiple of 2 + Multiple of 5 – Multiple of LCM (2, 5)
Multiples of 2 or 5 = Multiple of 2 + Multiple of 5 – Multiple of LCM (10)
Multiples of 2 or 5 from 1 to 500 = List of multiple of 2 from 1 to 500 + List of multiple of 5 from 1 to 500 - List of multiple of 10 from 1 to 500
= (2, 4, 6… 500) + (5, 10, 15… 500) - (10, 20, 30… 500)
Required sum = sum(2, 4, 6,…, 500) + sum(5, 10, 15,…, 500) - sum(10, 20, 30,., 500)
Consider the first series,

2, 4, 6, …., 500
First term, a = 2
Common difference, d = 2
Let n be no of terms
$a_n = a + (n - 1)d$
$500 = 2 + (n - 1)^2$
$498 = (n - 1)^2$
n - 1 = 249
n = 250
Sum of an AP, $S_n = (n/2)\ [\ a + a_n]$
Let the sum of this AP be S_1,
$S_1 = S_{250} = (250/2) \times [2+500]$
$S_1 = 125(502)$
$S_1 = 62750$ … (1)

Consider the second series,

5, 10, 15, …., 500
First term, a = 5
Common difference, d = 5
Let n be no of terms
By nth term formula
$a_n = a + (n - 1)d$
$500 = 5 + (n - 1)$
$495 = (n - 1)5$
n - 1 = 99
n = 100
Sum of an AP, $S_n = (n/2)\ [\ a + a_n]$
Let the sum of this AP be S_2,
$S_2 = S_{100} = (100/2) \times [5+500]$
$S_2 = 50(505)$

$S_2 = 25250$ … (2)

Consider the third series,

10, 20, 30, …., 500
First term, a = 10
Common difference, d = 10
Let n be no of terms
$a_n = a + (n - 1)d$
$500 = 10 + (n - 1)10$
$490 = (n - 1)10$
$n - 1 = 49$
$n = 50$
Sum of an AP, $S_n = (n/2)\ [a + a_n]$
Let the sum of this AP be S_3,
$S_3 = S_{50} = (50/2) \times [2+510]$
$S_3 = 25(510)$
$S_3 = 12750$ … (3)

Therefore, the required Sum, $S = S_1 + S_2 - S_3$
$S = 62750 + 25250 – 12750$
$= 75250$

2. The eighth term of an AP is half its second term and the eleventh term exceeds one third of its fourth term by 1. Find the 15th term.

Solution:

We know that,
First term of an AP = a
Common difference of AP = d
n^{th} term of an AP, $a_n = a + (n - 1)d$
According to the question,
$a_s = ½\ a_2$
$2a_8 = a_2$
$2(a + 7d) = a + d$
$2a + 14d = a + d$
$a = - 13d$ …(1)
Also,
$a_{11} = 1/3\ a_4 + 1$
$3(a + 10d) = a + 3d + 3$
$3a + 30d = a + 3d + 3$
$2a + 27d = 3$
Substituting $a = -13d$ in the equation,
$2(- 13d) + 27d = 3$
$d = 3$
Then,
$a = - 13(3) = - 39$
Now,

$a_{15} = a + 14d$
$= -39 + 14(3)$
$= -39 + 42$
$= 3$

So 15^{th} term is 3.

3. An AP consists of 37 terms. The sum of the three middle most terms is 225 and the sum of the last three is 429. Find the AP.

Solution:

We know that,
First term of an AP = a
Common difference of AP = d
n^{th} term of an AP, $a_n = a + (n - 1)d$
Since, n = 37 (odd),
Middle term will be $(n+1)/2 = 19^{th}$ term
Thus, the three middle most terms will be,
18^{th}, 19^{th} and 20^{th} terms
According to the question,
$a_{18} + a_{19} + a_{20} = 225$
Using $a_n = a + (n - 1)d$
$a + 17d + a + 18d + a + 19d = 225$
$3a + 54d = 225$
$3a = 225 - 54d$
$a = 75 - 18d$ … (1)
Now, we know that last three terms will be 35^{th}, 36^{th} and 37^{th} terms.
According to the question,
$a_{35} + a_{36} + a_{37} = 429$
$a + 34d + a + 35d + a + 36d = 429$
$3a + 105d = 429$
$a + 35d = 143$
Substituting $a = 75 - 18d$ from equation 1,
$75 - 18d + 35d = 143$ [using eqn1]
$17d = 68$
$d = 4$
Then,
$a = 75 - 18(4)$
$a = 3$
Therefore, the AP is a, a + d, a + 2d….
i.e. 3, 7, 11….

4. Find the sum of the integers between 100 and 200 that are

(i) divisible by 9
(ii) not divisible by 9
[Hint (ii): These numbers will be: Total numbers – Total numbers divisible by 9]

Solution:

(i) The number between 100 and 200 which is divisible by 9 = 108, 117, 126, …198

Let the number of terms between 100 and 200 which is divisible by 9 = n

$a_n = a + (n - 1)d$

$198 = 108 + (n - 1)9$

$90 = (n - 1)9$

$n - 1 = 10$

$n = 11$

Sum of an AP = $S_n = (n/2)\ [a + a_n]$

$S_n = (11/2) \times [108 + 198]$

$= (11/2) \times 306$

$= 11(153)$

$= 1683$

(ii) Sum of the integers between 100 and 200 which is not divisible by 9 = (sum of total numbers between 100 and 200) – (sum of total numbers between 100 and 200 which is divisible by 9)

Sum, $S = S_1 - S_2$

Here,

S_1 = sum of AP 101, 102, 103, - - - , 199

S_2 = sum of AP 108, 117, 126, - - - , 198

For AP 101, 102, 103, - - - , 199

First term, a = 101

Common difference, d = 199

Number of terms = n

Then,

$a_n = a + (n - 1)d$

$199 = 101 + (n - 1)1$

$98 = (n - 1)$

$n = 99$

Sum of an AP = $S_n = (n/2)\ [a + a_n]$

Sum of this AP,

$S_1 = (99/2) \times [199 + 101]$

$= (99/2) \times 300$

$= 99(150)$

$= 14850$

For AP 108, 117, 126,------- , 198

First term, a = 108

Common difference, d = 9

Last term, $a_n = 198$

Number of terms = n

Then,

$a_n = a + (n - 1)d$

$198 = 108 + (n - 1)9$

$10 = (n - 1)$

$n = 11$

Sum of an AP = $S_n = (n/2)$ [a + a_n]
Sum of this AP,

$S_2 = (11/2) \times [108 + 198]$
$= (11/2) \times (306)$
$= 11(153)$
$= 1683$

Substituting the value of S_1 and S_2 in the equation, $S = S_1 - S_2$

$S = S_1 + S_2$
$= 14850 – 1683$
$= 13167$