Exercise 2.1 Page No: 2.33

1. Find the zeros of each of the following quadratic polynomials and verify the relationship between the zeros and their coefficients:

(i) $f(x) = x^2 – 2x – 8$

Solution:

Given,

$f(x) = x^2 – 2x – 8$

To find the zeros, we put f(x) = 0

⇒ $x^2 – 2x – 8 = 0$

⇒ $x^2 - 4x + 2x - 8 = 0$

⇒ $x(x - 4) + 2(x - 4) = 0$

⇒ $(x - 4)(x + 2) = 0$

This gives us 2 zeros, for

x = 4 and x = -2

Hence, the zeros of the quadratic equation are 4 and -2.

Now, for verification

Sum of zeros = - coefficient of x / coefficient of x^2

4 + (-2)= - (-2) / 1

2 = 2

Product of roots = constant / coefficient of x^2

4 x (-2) = (-8) / 1

-8 = -8

Therefore, the relationship between zeros and their coefficients is verified.

(ii) $g(s) = 4s^2 – 4s + 1$

Solution:

Given,

$g(s) = 4s^2 – 4s + 1$

To find the zeros, we put g(s) = 0

⇒ $4s^2 – 4s + 1 = 0$

⇒ $4s^2 - 2s - 2s + 1= 0$

⇒ $2s(2s - 1) - (2s - 1) = 0$

⇒ $(2s - 1)(2s – 1) = 0$

This gives us 2 zeros, for

s = 1/2 and s = 1/2

Hence, the zeros of the quadratic equation are 1/2 and 1/2.

Now, for verification

Sum of zeros = - coefficient of s / coefficient of s^2

1/2 + 1/2 = - (-4) / 4

1 = 1

Product of roots = constant / coefficient of s^2

1/2 x 1/2 = 1/4

1/4 = 1/4

Therefore, the relationship between zeros and their coefficients is verified.

(iii) $h(t)=t^2 - 15$

Solution:

Given,

$h(t) = t^2 - 15 = t^2 + (0)t - 15$

To find the zeros, we put $h(t) = 0$

$\Rightarrow \quad t^2 - 15 = 0$

$\Rightarrow \quad (t + \sqrt{15})(t - \sqrt{15}) = 0$

This gives us 2 zeros, for

$t = \sqrt{15}$ and $t = -\sqrt{15}$

Hence, the zeros of the quadratic equation are $\sqrt{15}$ and $-\sqrt{15}$.

Now, for verification

Sum of zeros = - coefficient of t / coefficient of t^2

$\sqrt{15} + (-\sqrt{15}) = -(0) / 1$

$0 = 0$

Product of roots = constant / coefficient of t^2

$\sqrt{15} \times (-\sqrt{15}) = -15/1$

$-15 = -15$

Therefore, the relationship between zeros and their coefficients is verified.

(iv) $f(x) = 6x^2 - 3 - 7x$

Solution:

Given,

$f(x) = 6x^2 - 3 - 7x$

To find the zeros, we put $f(x) = 0$

$\Rightarrow \quad 6x^2 - 3 - 7x = 0$

$\Rightarrow \quad 6x^2 - 9x + 2x - 3 = 0$

$\Rightarrow \quad 3x(2x - 3) + 1(2x - 3) = 0$

$\Rightarrow \quad (2x - 3)(3x + 1) = 0$

This gives us 2 zeros, for

$x = 3/2$ and $x = -1/3$

Hence, the zeros of the quadratic equation are 3/2 and -1/3.

Now, for verification

Sum of zeros = - coefficient of x / coefficient of x^2

$3/2 + (-1/3) = -(-7) / 6$

$7/6 = 7/6$

Product of roots = constant / coefficient of x^2

$3/2 \times (-1/3) = (-3) / 6$

$-1/2 = -1/2$

Therefore, the relationship between zeros and their coefficients is verified.

(v) $p(x) = x^2 + 2\sqrt{2}x - 6$

Solution:

Given,
$p(x) = x^2 + 2\sqrt{2}x - 6$
To find the zeros, we put p(x) = 0
$\Rightarrow \quad x^2 + 2\sqrt{2}x - 6 = 0$
$\Rightarrow \quad x^2 + 3\sqrt{2}x - \sqrt{2}x - 6 = 0$
$\Rightarrow \quad x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2}) = 0$
$\Rightarrow \quad (x - \sqrt{2})(x + 3\sqrt{2}) = 0$
This gives us 2 zeros, for
$x = \sqrt{2}$ and $x = -3\sqrt{2}$
Hence, the zeros of the quadratic equation are $\sqrt{2}$ and $-3\sqrt{2}$.
Now, for verification
Sum of zeros = - coefficient of x / coefficient of x^2
$\sqrt{2} + (-3\sqrt{2}) = -(2\sqrt{2}) / 1$
$-2\sqrt{2} = -2\sqrt{2}$
Product of roots = constant / coefficient of x^2
$\sqrt{2} \text{ x } (-3\sqrt{2}) = (-6) / 2\sqrt{2}$
$-3 \text{ x } 2 = -6/1$
$-6 = -6$
Therefore, the relationship between zeros and their coefficients is verified.

(vi) $q(x)=\sqrt{3}x^2 + 10x + 7\sqrt{3}$

Solution:

Given,
$q(x) = \sqrt{3}x^2 + 10x + 7\sqrt{3}$
To find the zeros, we put q(x) = 0
$\Rightarrow \quad \sqrt{3}x^2 + 10x + 7\sqrt{3} = 0$
$\Rightarrow \quad \sqrt{3}x^2 + 3x + 7x + 7\sqrt{3}x = 0$
$\Rightarrow \quad \sqrt{3}x(x + \sqrt{3}) + 7(x + \sqrt{3}) = 0$
$\Rightarrow \quad (x + \sqrt{3})(\sqrt{3}x + 7) = 0$
This gives us 2 zeros, for
$x = -\sqrt{3}$ and $x = -7/\sqrt{3}$
Hence, the zeros of the quadratic equation are $-\sqrt{3}$ and $-7/\sqrt{3}$.
Now, for verification
Sum of zeros = - coefficient of x / coefficient of x^2
$-\sqrt{3} + (-7/\sqrt{3}) = -(10)/\sqrt{3}$
$(-3-7)/\sqrt{3} = -10/\sqrt{3}$
$-10/\sqrt{3} = -10/\sqrt{3}$
Product of roots = constant / coefficient of x^2
$(-\sqrt{3}) \text{ x } (-7/\sqrt{3}) = (7\sqrt{3}) / \sqrt{3}$
$7 = 7$
Therefore, the relationship between zeros and their coefficients is verified.

(vii) $f(x) = x^2 – (\sqrt{3} + 1)x + \sqrt{3}$

Solution:

Given,
$f(x) = x^2 – (\sqrt{3} + 1)x + \sqrt{3}$
To find the zeros, we put f(x) = 0
⇒ $x^2 – (\sqrt{3} + 1)x + \sqrt{3} = 0$
⇒ $x^2 - \sqrt{3}x - x + \sqrt{3} = 0$
⇒ $x(x - \sqrt{3}) - 1 (x - \sqrt{3}) = 0$
⇒ $(x - \sqrt{3})(x - 1) = 0$
This gives us 2 zeros, for
$x = \sqrt{3}$ and $x = 1$
Hence, the zeros of the quadratic equation are √3 and 1.
Now, for verification
Sum of zeros = - coefficient of x / coefficient of x^2
$\sqrt{3} + 1 = - (-(\sqrt{3} +1)) / 1$
$\sqrt{3} + 1 = \sqrt{3} +1$
Product of roots = constant / coefficient of x^2
$1 \text{ x } \sqrt{3} = \sqrt{3} / 1$
$\sqrt{3} = \sqrt{3}$
Therefore, the relationship between zeros and their coefficients is verified.

(viii) $g(x)=a(x^2+1)–x(a^2+1)$

Solution:
Given,
$g(x) = a(x^2+1)–x(a^2+1)$
To find the zeros, we put g(x) = 0
⇒ $a(x^2+1)–x(a^2+1) = 0$
⇒ $ax^2 + a − a^2x – x = 0$
⇒ $ax^2 − a^2x – x + a = 0$
⇒ $ax(x − a) − 1(x – a) = 0$
⇒ $(x – a)(ax – 1) = 0$
This gives us 2 zeros, for
$x = a$ and $x = 1/a$
Hence, the zeros of the quadratic equation are a and 1/a.
Now, for verification
Sum of zeros = - coefficient of x / coefficient of x^2
$a + 1/a = - (-(a^2 + 1)) / a$
$(a^2 + 1)/a = (a^2 + 1)/a$
Product of roots = constant / coefficient of x^2
$a \text{ x } 1/a = a / a$
$1 = 1$
Therefore, the relationship between zeros and their coefficients is verified.

(ix) $h(s) = 2s^2 – (1 + 2\sqrt{2})s + \sqrt{2}$

Solution:

Given,

$h(s) = 2s^2 – (1 + 2\sqrt{2})s + \sqrt{2}$

To find the zeros, we put $h(s) = 0$

$\Rightarrow \quad 2s^2 – (1 + 2\sqrt{2})s + \sqrt{2} = 0$

$\Rightarrow \quad 2s^2 – 2\sqrt{2}s – s + \sqrt{2} = 0$

$\Rightarrow \quad 2s(s – \sqrt{2}) -1(s - \sqrt{2}) = 0$

$\Rightarrow \quad (2s - 1)(s - \sqrt{2}) = 0$

This gives us 2 zeros, for

$x = \sqrt{2}$ and $x = 1/2$

Hence, the zeros of the quadratic equation are $\sqrt{3}$ and 1.

Now, for verification

Sum of zeros = - coefficient of s / coefficient of s^2

$\sqrt{2} + 1/2 = - (-(1 + 2\sqrt{2})) / 2$

$(2\sqrt{2} + 1)/2 = (2\sqrt{2} +1)/2$

Product of roots = constant / coefficient of s^2

$1/2 \times \sqrt{2} = \sqrt{2} / 2$

$\sqrt{2} / 2 = \sqrt{2} / 2$

Therefore, the relationship between zeros and their coefficients is verified.

(x) $f(v) = v^2 + 4\sqrt{3}v - 15$

Solution:

Given,

$f(v) = v^2 + 4\sqrt{3}v – 15$

To find the zeros, we put $f(v) = 0$

$\Rightarrow \quad v^2 + 4\sqrt{3}v – 15 = 0$

$\Rightarrow \quad v^2 + 5\sqrt{3}v - \sqrt{3}v - 15 = 0$

$\Rightarrow \quad v(v + 5\sqrt{3}) - \sqrt{3} (v + 5\sqrt{3}) = 0$

$\Rightarrow \quad (v - \sqrt{3})(v + 5\sqrt{3}) = 0$

This gives us 2 zeros, for

$v = \sqrt{3}$ and $v = -5\sqrt{3}$

Hence, the zeros of the quadratic equation are $\sqrt{3}$ and $-5\sqrt{3}$.

Now, for verification

Sum of zeros = - coefficient of v / coefficient of v^2

$\sqrt{3} + (-5\sqrt{3}) = - (4\sqrt{3}) / 1$

$-4\sqrt{3} = -4\sqrt{3}$

Product of roots = constant / coefficient of v^2

$\sqrt{3} \times (-5\sqrt{3}) = (-15) / 1$

$-5 \times 3 = -15$

$-15 = -15$

Therefore, the relationship between zeros and their coefficients is verified.

(xi) $p(y) = y^2 + (3\sqrt{5}/2)y – 5$

Solution:

Given,
$p(y) = y^2 + (3\sqrt{5}/2)y – 5$
To find the zeros, we put f(v) = 0
⇒ $y^2 + (3\sqrt{5}/2)y – 5 = 0$
⇒ $y^2 - \sqrt{5}/2\, y + 2\sqrt{5}y - 5 = 0$
⇒ $y(y - \sqrt{5}/2) + 2\sqrt{5}\,(y - \sqrt{5}/2) = 0$
⇒ $(y + 2\sqrt{5})(y - \sqrt{5}/2) = 0$
This gives us 2 zeros, for
$y = \sqrt{5}/2$ and $y = -2\sqrt{5}$
Hence, the zeros of the quadratic equation are $\sqrt{5}/2$ and $-2\sqrt{5}$.
Now, for verification
Sum of zeros = - coefficient of y / coefficient of y^2
$\sqrt{5}/2 + (-2\sqrt{5}) = - (3\sqrt{5}/2) / 1$
$-3\sqrt{5}/2 = -3\sqrt{5}/2$
Product of roots = constant / coefficient of y^2
$\sqrt{5}/2 \times (-2\sqrt{5}) = (-5) / 1$
$- (\sqrt{5})^2 = -5$
$-5 = -5$
Therefore, the relationship between zeros and their coefficients is verified.

(xii) $q(y) = 7y^2 - (11/3)y – 2/3$

Solution:

Given,
$q(y) = 7y^2 - (11/3)y – 2/3$
To find the zeros, we put q(y) = 0
⇒ $7y^2 - (11/3)y – 2/3 = 0$
⇒ $(21y^2 - 11y -2)/3 = 0$
⇒ $21y^2 - 11y - 2 = 0$
⇒ $21y^2 - 14y + 3y - 2 = 0$
⇒ $7y(3y - 2) – 1(3y + 2) = 0$
⇒ $(3y - 2)(7y + 1) = 0$
This gives us 2 zeros, for
$y = 2/3$ and $y = -1/7$
Hence, the zeros of the quadratic equation are 2/3 and -1/7.
Now, for verification
Sum of zeros = - coefficient of y / coefficient of y^2
$2/3 + (-1/7) = - (-11/3) / 7$
$-11/21 = -11/21$
Product of roots = constant / coefficient of y^2
$2/3 \times (-1/7) = (-2/3) / 7$
$- 2/21 = -2/21$
Therefore, the relationship between zeros and their coefficients is verified.

2. For each of the following, find a quadratic polynomial whose sum and product respectively of the zeros are as given. Also, find the zeros of these polynomials by factorization.

(i) -8/3 , 4/3

Solution:

A quadratic polynomial formed for the given sum and product of zeros is given by:
$f(x) = x^2 + -(\text{sum of zeros})\, x + (\text{product of roots})$
Here, the sum of zeros is = -8/3 and product of zero= 4/3
Thus,
The required polynomial f(x) is,
⇒ $x^2 - (-8/3)x + (4/3)$
⇒ $x^2 + 8/3x + (4/3)$
So, to find the zeros we put f(x) = 0
⇒ $x^2 + 8/3x + (4/3) = 0$
⇒ $3x^2 + 8x + 4 = 0$
⇒ $3x^2 + 6x + 2x + 4 = 0$
⇒ $3x(x + 2) + 2(x + 2) = 0$
⇒ $(x + 2)\,(3x + 2) = 0$
⇒ $(x + 2) = 0$ and, or $(3x + 2) = 0$
Therefore, the two zeros are -2 and -2/3.

(ii) 21/8 , 5/16

Solution:

A quadratic polynomial formed for the given sum and product of zeros is given by:
$f(x) = x^2 + -(\text{sum of zeros})\, x + (\text{product of roots})$
Here, the sum of zeros is = 21/8 and product of zero = 5/16
Thus,
The required polynomial f(x) is,
⇒ $x^2 - (21/8)x + (5/16)$
⇒ $x^2 - 21/8x + 5/16$
So, to find the zeros we put f(x) = 0
⇒ $x^2 - 21/8x + 5/16 = 0$
⇒ $16x^2 - 42x + 5 = 0$
⇒ $16x^2 - 40x - 2x + 5 = 0$
⇒ $8x(2x - 5) - 1(2x - 5) = 0$
⇒ $(2x - 5)\,(8x - 1) = 0$
⇒ $(2x - 5) = 0$ and, or $(8x - 1) = 0$
Therefore, the two zeros are 5/2 and 1/8.

(iii) -2√3, -9

Solution:

A quadratic polynomial formed for the given sum and product of zeros is given by:
$f(x) = x^2 + -(\text{sum of zeros})\, x + (\text{product of roots})$
Here, the sum of zeros is = $-2\sqrt{3}$ and product of zero = -9

Thus,
The required polynomial f(x) is,
⇒ $x^2 - (-2\sqrt{3})x + (-9)$
⇒ $x^2 + 2\sqrt{3}x – 9$
So, to find the zeros we put f(x) = 0
⇒ $x^2 + 2\sqrt{3}x – 9 = 0$
⇒ $x^2 + 3\sqrt{3}x - \sqrt{3}x – 9 = 0$
⇒ $x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$
⇒ $(x + 3\sqrt{3}) (x - \sqrt{3}) = 0$
⇒ $(x + 3\sqrt{3}) = 0$ and, or $(x - \sqrt{3}) = 0$
Therefore, the two zeros are $-3\sqrt{3}$and $\sqrt{3}$.

(iv) -3/2√5, -1/2

Solution:

A quadratic polynomial formed for the given sum and product of zeros is given by:
$f(x) = x^2$ + -(sum of zeros) x + (product of roots)
Here, the sum of zeros is = $-3/2\sqrt{5}$ and product of zero = -1/2
Thus,
The required polynomial f(x) is,
⇒ $x^2 - (-3/2\sqrt{5})x + (-1/2)$
⇒ $x^2 + 3/2\sqrt{5}x – 1/2$
So, to find the zeros we put f(x) = 0
⇒ $x^2 + 3/2\sqrt{5}x – 1/2 = 0$
⇒ $2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$
⇒ $2\sqrt{5}x^2 + 5x – 2x - \sqrt{5} = 0$
⇒ $\sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$
⇒ $(2x + \sqrt{5}) (\sqrt{5}x - 1) = 0$
⇒ $(2x + \sqrt{5}) = 0$ and, or $(\sqrt{5}x - 1) = 0$
Therefore, the two zeros are $-\sqrt{5}/2$ and $1/\sqrt{5}$.

3. If α and β are the zeros of the quadratic polynomial $f(x) = x^2 – 5x + 4$, find the value of 1/α + 1/β – 2αβ.

Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial f(x) where a = 1, b = -5 and c = 4
So, we can find
Sum of the roots = α+β = -b/a = - (-5)/1 = -5
Product of the roots = αβ = c/a = 4/1 = 4
To find, 1/α +1/β – 2αβ
⇒ [(α +β)/ αβ] – 2αβ
⇒ (-5)/ 4 – 2(4) = -5/4 – 8 = -27/ 4

4. If α and β are the zeros of the quadratic polynomial $p(y) = 5y^2 – 7y + 1$, find the value of 1/α+1/β.

Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial f(x) where a =5, b = -7 and c = 1
So, we can find
Sum of the roots = α+β = -b/a = - (-7)/5 = 7/5
Product of the roots = αβ = c/a = 1/5
To find, 1/α +1/β
⇒ (α +β)/ αβ
⇒ (7/5)/ (1/5) = 7

5. If α and β are the zeros of the quadratic polynomial f(x)=x^2 – x – 4, find the value of 1/α+1/β–αβ.
Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial f(x) where a = 1, b = -1 and c = - 4
So, we can find
Sum of the roots = α+β = -b/a = - (-1)/1 = 1
Product of the roots = αβ = c/a = -4 /1 = - 4
To find, 1/α +1/β – αβ
⇒ [(α +β)/ αβ] – αβ
⇒ [(1)/ (-4)] – (-4) = -1/4 + 4 = 15/ 4

6. If α and β are the zeroes of the quadratic polynomial f(x) = x^2 + x – 2, find the value of 1/α – 1/β.
Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial f(x) where a = 1, b = 1 and c = - 2
So, we can find
Sum of the roots = α+β = -b/a = - (1)/1 = -1
Product of the roots = αβ = c/a = -2 /1 = - 2
To find, 1/α - 1/β
⇒ [(β - α)/ αβ]
⇒ $\frac{\beta-\alpha}{\alpha\beta} = \frac{\beta-\alpha}{\alpha\beta} \times \frac{(\alpha-\beta)}{\alpha\beta} = \frac{\sqrt{(\alpha+\beta)^2-4\alpha\beta}}{\alpha\beta} = \frac{\sqrt{1+8}}{2} = \frac{\sqrt{9}}{2} = \frac{3}{2}$

7. If one of the zero of the quadratic polynomial f(x) = $4x^2$ – 8kx – 9 is negative of the other, then find the value of k.
Solution:

From the question, it's given that:
The quadratic polynomial f(x) where a = 4, b = -8k and c = - 9
And, for roots to be negative of each other, let the roots be α and – α.
So, we can find
Sum of the roots = α - α = -b/a = - (-8k)/1 = 8k = 0 [∵ α - α = 0]
⇒ k = 0

8. If the sum of the zeroes of the quadratic polynomial $f(t)=kt^2 + 2t + 3k$ is equal to their product, then find the value of k.
Solution:

Given,
The quadratic polynomial $f(t)=kt^2 + 2t + 3k$, where a = k, b = 2 and c = 3k.
And,
Sum of the roots = Product of the roots
⇒ (-b/a) = (c/a)
⇒ (-2/k) = (3k/k)
⇒ (-2/k) = 3
∴ k = -2/3

9. If α and β are the zeros of the quadratic polynomial $p(x) = 4x^2 – 5x – 1$, find the value of $\alpha^2\beta+\alpha\beta^2$.
Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial p(x) where a = 4, b = -5 and c = -1
So, we can find
Sum of the roots = α+β = -b/a = - (-5)/4 = 5/4
Product of the roots = αβ = c/a = -1/4
To find, $\alpha^2\beta+\alpha\beta^2$
⇒ αβ(α +β)
⇒ (-1/4)(5/4) = -5/16

10. If α and β are the zeros of the quadratic polynomial $f(t)=t^2– 4t + 3$, find the value of $\alpha^4\beta^3+\alpha^3\beta^4$.
Solution:

From the question, it's given that:
α and β are the roots of the quadratic polynomial f(t) where a = 1, b = -4 and c = 3
So, we can find
Sum of the roots = α+β = -b/a = - (-4)/1 = 4
Product of the roots = αβ = c/a = 3/1 = 3
To find, $\alpha^4\beta^3+\alpha^3\beta^4$
⇒ $\alpha^3\beta^3$ (α +β)
⇒ $(\alpha\beta)^3$ (α +β)
⇒ $(3)^3$ (4) = 27 x 4 = 108