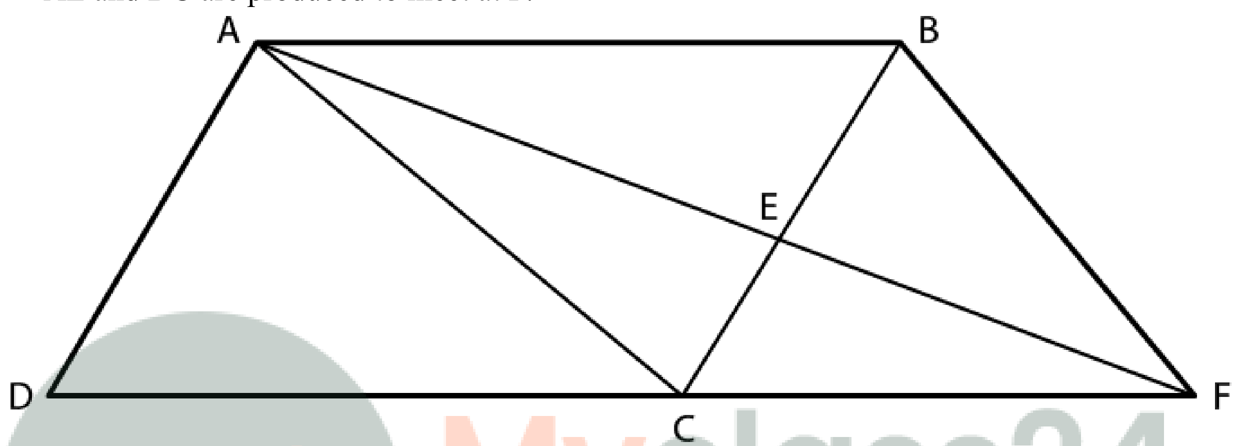
EXERCISE 9.4

1. A point E is taken on the side BC of a parallelogram ABCD. AE and DC are produced to meet at F. Prove that ar (ADF) = ar (ABFC)

Solution:

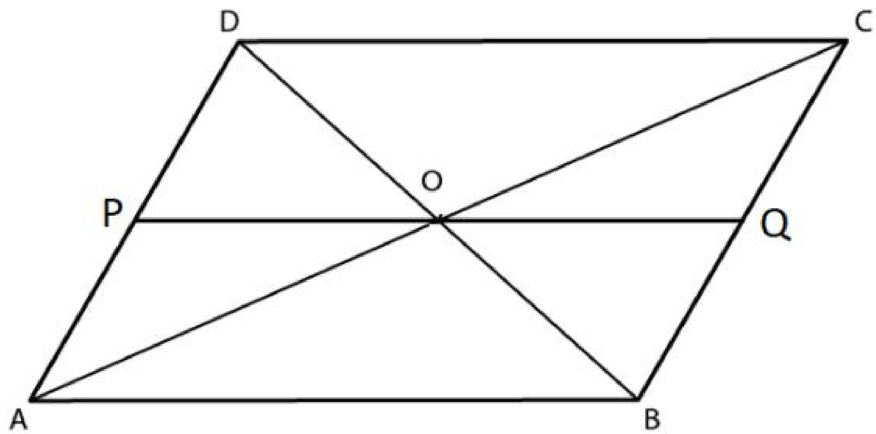
According to the question,
ABCD is a parallelogram.
E is a point on BC.
AE and DC are produced to meet at F.



To prove: area (ΔADF) = area (ABFC).
Proof:
We know that,
Triangles on the same base and between the same parallels are equal in area.
Here,
We have,
ΔABC and ΔABF are on same base AB and between same parallels, AB || CF.
Area (ΔABC) = area (ΔABF) ... (1)
We also know that,
Diagonal of a Parallelogram divides it into two triangles of equal area
So,
Area (ΔABC) = area (ΔACD) ... (2)
Now,
Area (ΔADF) = area (ΔACD) + area (ΔACF)
∴ Area (ΔADF) = area (ΔABC) + area (ΔACF) … (From equation (2))
⇒ Area (ΔADF) = area (ΔABF) + area (ΔACF) … (From equation (1))
⇒ Area (ΔADF) = area (ABFC**)**

2. The diagonals of a parallelogram ABCD intersect at a point O. Through O, a line is drawn to intersect AD at P and BC at Q. Show that PQ divides the parallelogram into two parts of equal area.

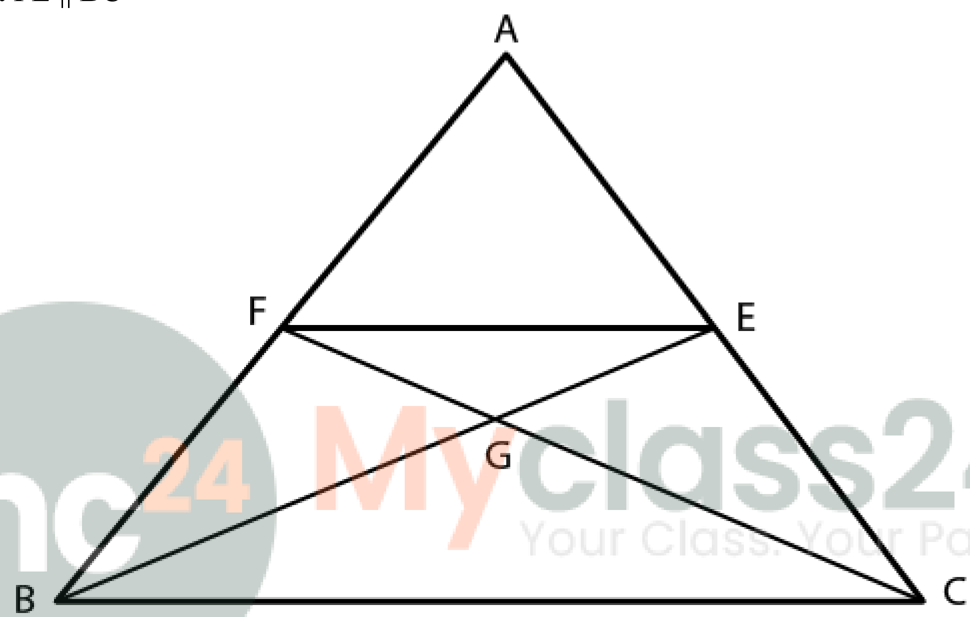
Solution:



According to the question,
The diagonals of a parallelogram ABCD intersect at a point O.
Through O, a line is drawn to intersect AD at P and BC at Q.
To Prove:
Ar (parallelogram PDCQ) = ar (parallelogram PQBA).
Proof:
AC is a diagonal of || gm ABCD
∴ ar(ΔABC) = ar(ΔACD)
= ½ ar (||gm ABCD) …(1)
In ΔAOP and ΔCOQ,
AO = CO
Since, diagonals of a parallelogram bisect each other,
We get,
∠AOP = ∠COQ
∠OAP = ∠OCQ (Vertically opposite angles)
∴ ΔAOP = ΔCOQ (Alternate interior angles)
∴ ar(ΔAOP) = ar(ΔCOQ) (By ASA Congruence Rule)
We know that,
Congruent figures have equal areas
So,
ar(ΔAOP) + ar(parallelogram OPDC) = ar(ΔCOQ) + ar(parallelogram OPDC)
⇒ ar(ΔACD) = ar(parallelogram PDCQ)
⇒ ½ ar(|| gm ABCD) = ar(parallelogram PDCQ)
From equation (1),
We get,
ar(parallelogram PQBA) = ar(parallelogram PDCQ)
⇒ ar(parallelogram PDCQ) = ar(parallelogram PQBA).
Hence Proved.

3. The medians BE and CF of a triangle ABC intersect at G. Prove that the area of △ GBC = area of the quadrilateral AFGE.
Solution:
According to the question,

We have,
BE & CF are medians
E is the midpoint of AC
F is the midpoint of AB
∴ ΔBCE = ΔBEA … (i)
ΔBCF = ΔCAF
Construct:
Join EF,
By midpoint theorem,
We get FE || BC



We know that,
Δ on the same base and between same parallels are equal in area
∴ ΔFBC = ΔBCE
ΔFBC - ΔGBC = ΔBCE - ΔGBC
⇒ ΔFBG = ΔCGE (ΔGBC is common)
⇒ ΔCGE = ΔFBG …(ii)
Subtracting equation (ii) from (i)
We get,
ΔBCE - ΔCGE = ΔBEA - ΔFBG
∴ ΔBGC =quadrilateral AFGE.

4. In Fig. 9.24, CD || AE and CY || BA. Prove that ar (CBX) = ar (AXY).

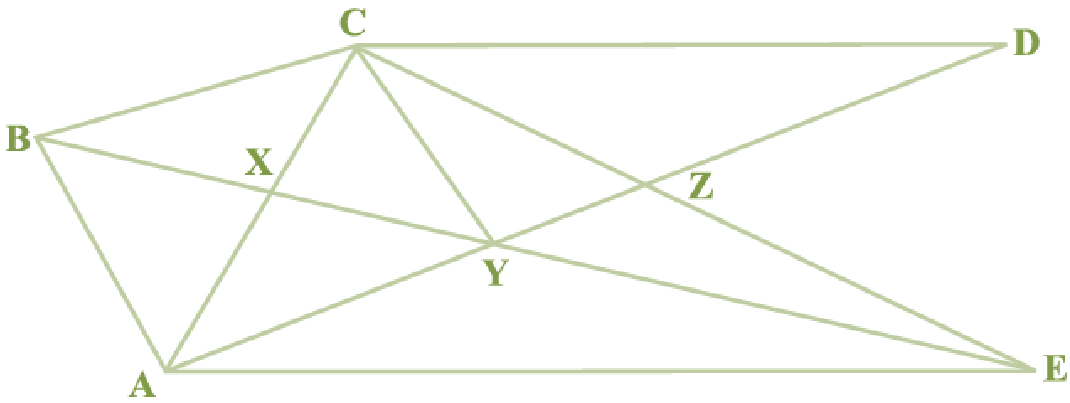


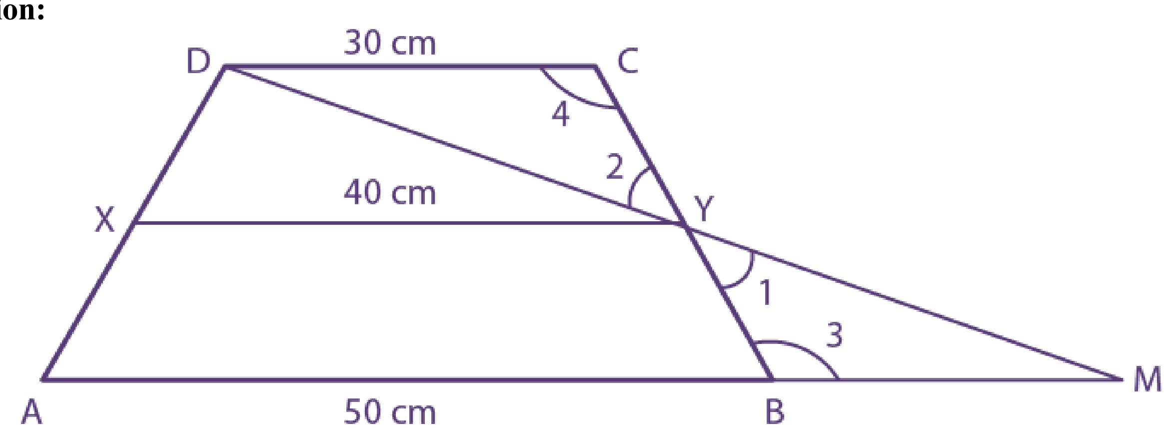
Fig. 9.24

Solution:

According to the question,
From figure,
We get,
CD||AE
and CY || BA
To prove:
ar (ΔCBX) = ar (ΔAXY) .
Proof:
We know that,
Triangles on the same base and between the same parallels are equal in areas.
Here,
ΔABY and ΔABC both lie on the same base AB and between the same parallels CY and BA.
ar (ΔABY) = ar (ΔABC)
⇒ ar (ABX) + ar (AXY) = ar (ABX) + ar (CBX)
Solving and cancelling ar (ABX),
We get,
⇒ ar (AXY) = ar (CBX)

5. ABCD is a trapezium in which AB || DC, DC = 30 cm and AB = 50 cm. If X and Y are, respectively the mid-points of AD and BC, prove that ar (DCYX) = 7/9 ar (XYBA)

Solution:



According to the question,
We have,
ABCD is a trapezium with AB || DC
Construction: Join DY and produce it to meet AB produced at P.
In ΔBYP and ΔCYD
∠BYP = ∠CYD (vertically opposite angles)
Since, alternate opposite angles of DC || AP and BC is the transversal
∠DCY = ∠PBY
Since, Y is the midpoint of BC,
BY = CY
Thus ΔBYP ≅ ΔCYD (by ASA cogence criterion)

So, DY = YP and DC = BP
⇒ Y is the midpoint of AD
∴ XY || AP and XY = ½ AP (by midpoint theorem)
⇒ XY = ½ AP
= ½ (AB + BP)
= ½ (AB + DC)
= ½ (50 + 30)
= ½ × 80 cm
= 40 cm

Since X is the midpoint of AD
And Y is the midpoint of BC
Hence, trapezium DCYX and ABYX are of same height, *h*, cm
Now

$$\frac{\text{area (DCYX)}}{\text{area (ABYX)}} = \frac{\frac{1}{2}(DC+XY)\times h}{\frac{1}{2}(AB \times XY)\ h} = \frac{30+40}{50+40} = \frac{70}{90} = \frac{7}{9}$$

⇒ 9 ar(DCXY) = 7 ar(XYBA)
⇒ ar(DCXY) = 7/9 ar(XYBA)
Hence Proved.