EXERCISE 1.5

If A and B are two sets such that A ⊂ B, then Find:
(i) A ∩ B
(ii) A ∪ B
Solution:
(i) A ∩ B
A ∩ B denotes A intersection B. Common elements of A and B consists in this set.
Given A ⊂ B, every element of A are already an element of B.
∴ A ∩ B = A

(ii) A ∪ B
A ∪ B denotes A union B. Elements of either A or B or in both A and B consist in this set.
Given A ⊂ B, B is having all elements including elements of A.
∴ A ∪ B = B

2. If A = {1, 2, 3, 4, 5}, B = {4, 5, 6, 7, 8}, C = {7, 8, 9, 10, 11} and D = {10, 11, 12, 13, 14}. Find:
(i) A ∪ B
(ii) A ∪ C
(iii) B ∪ C
(iv) B ∪ D
(v) A ∪ B ∪ C
(vi) A ∪ B ∪ D
(vii) B ∪ C ∪ D
(viii) A ∩ (B ∪ C)
(ix) (A ∩ B) ∩ (B ∩ C)
(x) (A ∪ D) ∩ (B ∪ C).
Solution:
In general X ∪ Y = {a: a ∈ X or a ∈ Y}
X ∩ Y = {a: a ∈ X and a ∈ Y}.

(i) A = {1, 2, 3, 4, 5}
B = {4, 5, 6, 7, 8}
A ∪ B = {x: x ∈ A or x ∈ B}
= {1, 2, 3, 4, 5, 6, 7, 8}

(ii) A = {1, 2, 3, 4, 5}
C = {7, 8, 9, 10, 11}
A ∪ C = {x: x ∈ A or x ∈ C}
= {1, 2, 3, 4, 5, 7, 8, 9, 10, 11}

(iii) B = {4, 5, 6, 7, 8}
C = {7, 8, 9, 10, 11}
B ∪ C = {x: x ∈ B or x ∈ C}
= {4, 5, 6, 7, 8, 9, 10, 11}

(iv) B = {4, 5, 6, 7, 8}
D = {10, 11, 12, 13, 14}
B ∪ D = {x: x ∈ B or x ∈ D}
= {4, 5, 6, 7, 8, 10, 11, 12, 13, 14}

(v) A = {1, 2, 3, 4, 5}
B = {4, 5, 6, 7, 8}
C = {7, 8, 9, 10, 11}
A ∪ B = {x: x ∈ A or x ∈ B}
= {1, 2, 3, 4, 5, 6, 7, 8}
A ∪ B ∪ C = {x: x ∈ A ∪ B or x ∈ C}
= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

(vi) A = {1, 2, 3, 4, 5}
B = {4, 5, 6, 7, 8}
D = {10, 11, 12, 13, 14}
A ∪ B = {x: x ∈ A or x ∈ B}
= {1, 2, 3, 4, 5, 6, 7, 8}
A ∪ B ∪ D = {x: x ∈ A ∪ B or x ∈ D}
= {1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14}

(vii) B = {4, 5, 6, 7, 8}
C = {7, 8, 9, 10, 11}
D = {10, 11, 12, 13, 14}
B ∪ C = {x: x ∈ B or x ∈ C}
= {4, 5, 6, 7, 8, 9, 10, 11}
B ∪ C ∪ D = {x: x ∈ B ∪ C or x ∈ D}
= {4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14}

(viii) A = {1, 2, 3, 4, 5}
B = {4, 5, 6, 7, 8}
C = {7, 8, 9, 10, 11}
B ∪ C = {x: x ∈ B or x ∈ C}
= {4, 5, 6, 7, 8, 9, 10, 11}
A ∩ B ∪ C = {x: x ∈ A and x ∈ B ∪ C}
= {4, 5}

(ix) A = {1, 2, 3, 4, 5}
B = {4, 5, 6, 7, 8}
C = {7, 8, 9, 10, 11}
(A ∩ B) = {x: x ∈ A and x ∈ B}
= {4, 5}
(B ∩ C) = {x: x ∈ B and x ∈ C}
= {7, 8}
(A ∩ B) ∩ (B ∩ C) = {x: x ∈ (A ∩ B) and x ∈ (B ∩ C)}
= ϕ

(x) A = {1, 2, 3, 4, 5}
B = {4, 5, 6, 7, 8}
C = {7, 8, 9, 10, 11}
D = {10, 11, 12, 13, 14}
A ∪ D = {x: x ∈ A or x ∈ D}
= {1, 2, 3, 4, 5, 10, 11, 12, 13, 14}
B ∪ C = {x: x ∈ B or x ∈ C}
= {4, 5, 6, 7, 8, 9, 10, 11}
(A ∪ D) ∩ (B ∪ C) = {x: x ∈ (A ∪ D) and x ∈ (B ∪ C)}
= {4, 5, 10, 11}

3. Let A = {x: x ∈ N}, B = {x: x = 2n, n ∈ N), C = {x: x = 2n – 1, n ∈ N} and, D = {x: x is a prime natural number} Find:
(i) A ∩ B
(ii) A ∩ C
(iii) A ∩ D
(iv) B ∩ C
(v) B ∩ D
(vi) C ∩ D
Solution:
A = All natural numbers i.e. {1, 2, 3…..}

B = All even natural numbers i.e. {2, 4, 6, 8…}
C = All odd natural numbers i.e. {1, 3, 5, 7……}
D = All prime natural numbers i.e. {1, 2, 3, 5, 7, 11, …}

(i) $A \cap B$
A contains all elements of B.
$\therefore B \subset A = \{2, 4, 6, 8\ldots\}$
$\therefore A \cap B = B$

(ii) $A \cap C$
A contains all elements of C.
$\therefore C \subset A = \{1, 3, 5\ldots\}$
$\therefore A \cap C = C$

(iii) $A \cap D$
A contains all elements of D.
$\therefore D \subset A = \{2, 3, 5, 7..\}$
$\therefore A \cap D = D$

(iv) $B \cap C$
$B \cap C = \phi$
There is no natural number which is both even and odd at same time.

(v) $B \cap D$
$B \cap D = 2$
{2} is the only natural number which is even and a prime number.

(vi) $C \cap D$
$C \cap D = \{1, 3, 5, 7\ldots\}$
$= D - \{2\}$
Every prime number is odd except {2}.