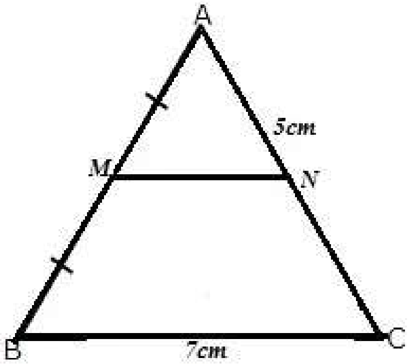
Chapter 12. Mid-point and Its Converse [Including Intercept Theorem]

Exercise 12(A)

Solution 1:

The triangle is shown below,



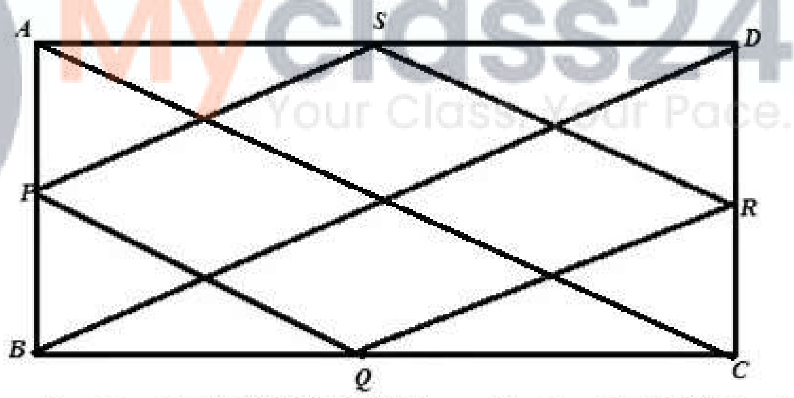
Since M is the midpoint of AB and MN||BC hence N is the midpoint of AC.Therefore

$MN = \frac{1}{2}BC = \frac{1}{2}\times 7 = 3.5cm$

And $AN = \frac{1}{2}AC = \frac{1}{2}\times 5 = 2.5cm$

Solution 2:

The figure is shown below,



Let ABCD be a rectangle where P,Q,R,S are the midpoint of AB,BC, CD, DA.We need to show that PQRS is a rhombus
For help we draw two diagonal BD and AC as shown in figure
Where BD=AC(Since diagonal of rectangle are equal)
Proof:
From ΔABD and ΔBCD

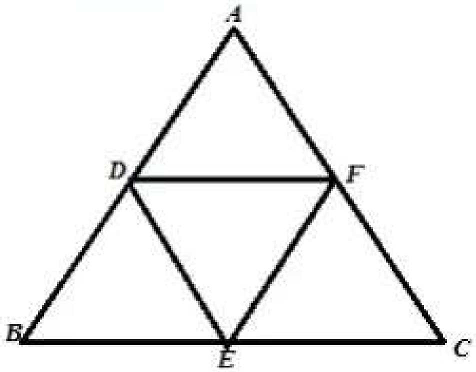
$PS = \frac{1}{2}BD = QR$ and PS||BD||QR

$2PS = 2QR$ =BD and PS||QR ------- (1)

Similarly 2PQ=2SR=AC and PQ||SR----- (2)
From (1) and (2) we get
PQ=QR=RS=PS
Therefore PQRS is a rhombus.
Hence proved

Solution 3:

The figure is shown below



Given that ABC is an isosceles triangle where AB=AC.
Since D,E,F are midpoint of AB,BC,CA therefore
2DE=AC and 2EF=AB this means DE=EF
Therefore DEF is an isosceles triangle an DE=EF.
Hence proved

Solution 4:

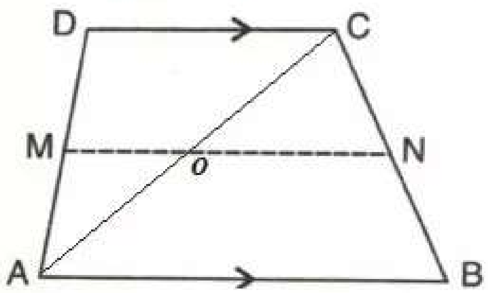
Here from triangle ABD P is the midpoint of AD and PR||AB, therefore Q is the midpoint of BD
Similarly R is the midpoint of BC as PR||CD||AB
From triangle ABD 2PQ=AB ....(1)
From triangle BCD 2QR=CD .....(2)
Now (1)+(2)=>
2(PQ+QR)=AB+CD

$$PR=\frac{1}{2}(AB+CD)$$

Hence proved

Solution 5:

Let we draw a diagonal AC as shown in the figure below,



(i)Given that AB=11cm,CD=8cm

From triangle ABC

$$ON = \frac{1}{2}AB = \frac{1}{2}\times 11 = 5.5cm$$

From triangle ACD

$$OM = \frac{1}{2}CD = \frac{1}{2}\times 8 = 4cm$$

Hence MN=OM+ON=(4+5.5)=9.5cm

(ii)Given that CD=20cm,MN=27cm

From triangle ACD

$$OM = \frac{1}{2}CD = \frac{1}{2}\times 20 = 10cm$$

Therefore ON=27-10=17cm

From triangle ABC

$$AB = 2ON = 2\times 17 = 34cm$$

(iii)Given that AB=23cm,MN=15cm

From triangle ABC

$$ON = \frac{1}{2}AB = \frac{1}{2}\times 23 = 11.5cm$$

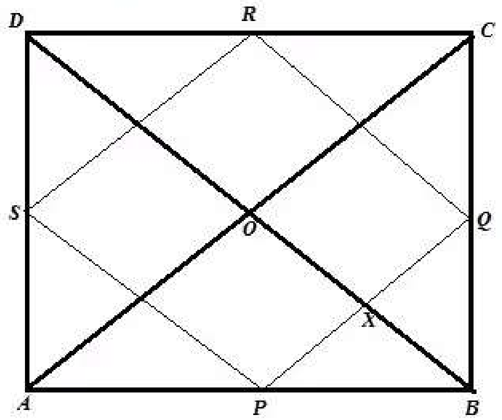
Therefore OM=15-11.5=3.5cm

From triangle ACD

$$CD = 2OM = 2\times 3.5 = 7cm$$

Solution 6:

The figure is shown below



Let ABCD be a quadrilateral where P,Q,R,S are the midpoint of AB,BC,CD,DA. Diagonal AC and BD intersects at right angle at point O. We need to show that PQRS is a rectangle

Proof:

From ΔABC and ΔADC

2PQ=AC and PQ||AC .....(1)

2RS=AC and RS||AC .....(2)

From (1) and (2) we get,

PQ=RS and PQ||RS

Similarly we can show that PS=RQ and PS||RQ

Therefore PQRS is a parallelogram.

Now PQ||AC, therefore $\angle AOD = \angle PXO = 90°$ $[Corresponding\ angle]$

Again BD||RQ, therefore $\angle PXO = \angle RQX = 90°$ $[Corresponding\ angle]$

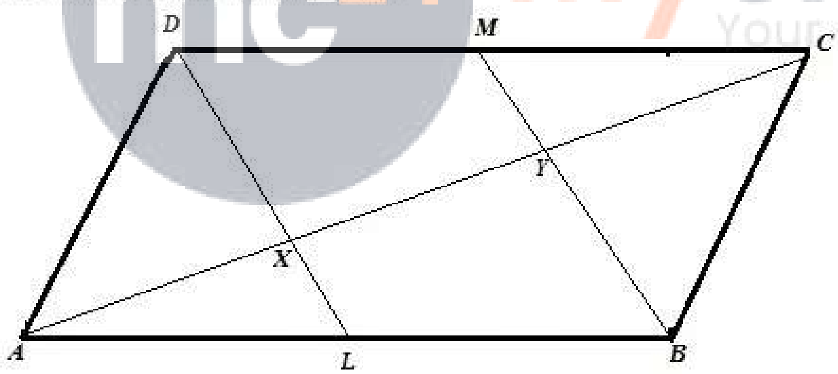
Similarly $\angle QRS = \angle RSP = \angle SPQ = 90°$

Therefore PQRS is a rectangle.

Hence proved

Solution 7:

The required figure is shown below



From figure,

BL=DM and BL||DM and BLMD is a parallelogram, therefore BM||DL

From triangle ABY

L is the midpoint of AB and XL||BY, therefore x is the midpoint of AY.ie AX=XY .....(1)

Similarly for triangle CDX

CY=XY .....(2)

From (1) and (2)

AX=XY=CY and AC=AX+XY+CY

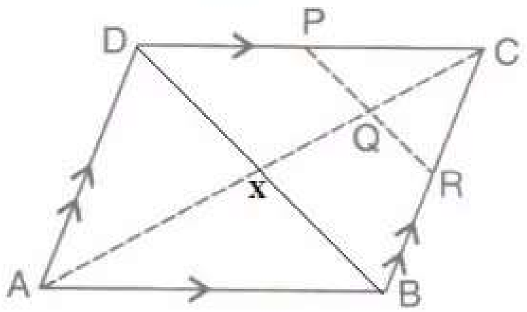
Hence proved

Solution 8:

Given that AD=BC .....(1)
From the figure,
For triangle ADC and triangle ABD
2GH=AD and 2EF=AD, therefore 2GH=2EF=AD .....(2)
For triangle BCD and triangle ABC
2GF=BC and 2EH=BC, therefore 2GF=2EH=BC .....(3)
From (1),(2),(3) we get,
2GH=2EF=2GF=2EH
GH=EF=GF=EH
Therefore EFGH is a rhombus.
Hence proved

Solution 9:

For help we draw the diagonal BD as shown below



The diagonal AC and BD cuts at point X.
We know that the diagonal of a parallelogram intersects equally each other. Therefore
AX=CX and BX=DX
Given,

$$CQ = \frac{1}{4}AC$$

$$CQ = \frac{1}{4} \times 2CX$$

$$CQ = \frac{1}{2}CX$$

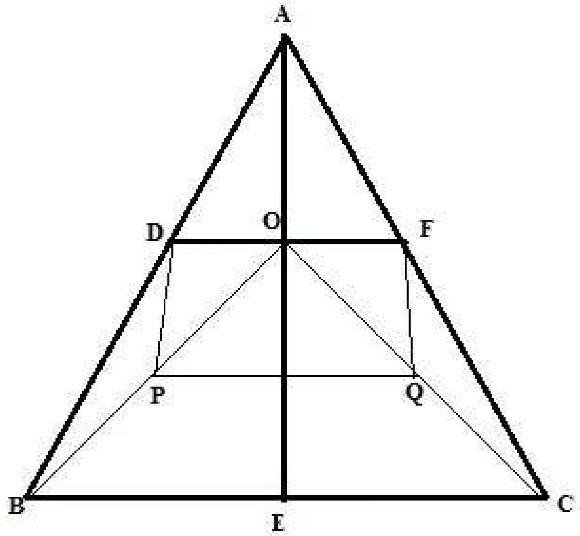
Therefore Q is the midpoint of CX.
(i)For triangle CDX PQ||DX or PR||BD

Since for triangle CBX
Q is the midpoint of CX and QR||BX. Therefore R is the midpoint of BC
(ii)For triangle BCD

As P and R are the midpoint of CD and BC ,therefore $PR = \frac{1}{2}DB$

Solution 10:

The required figure is shown below



For triangle ABC and OBC

2DE=BC and 2PQ=BC, therefore DE=PQ .....(1)

For triangle ABO and ACO

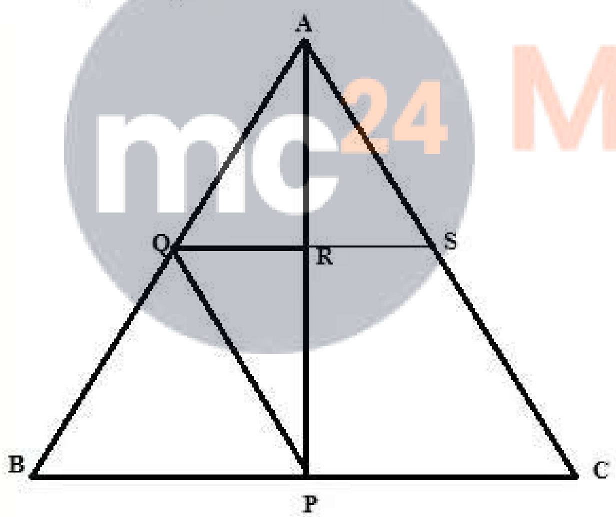
2PD=AO and 2FQ=AO, therefore PD=FQ .....(2)

From (1),(2) we get that PQFD is a parallelogram.

Hence proved

Solution 11:

The required figure is shown below



From the figure it is seen that P is the midpoint of BC and PQ||AC and QR||BC

Therefore Q is the midpoint of AB and R is the midpoint of AP

(i)Therefore AP=2AR

(ii)Here we increase QR so that it cuts AC at S as shown in the figure.

(iii)From triangle PQR and triangle ARS

$\angle PQR = \angle ARS \quad (Opposite\ angle)$

$PR = AR$

$PQ = AS \quad \left[PQ = AS = \frac{1}{2}AC\right]$

$\Delta PQR \cong \Delta ARS \quad$ (SAS Postulate)

Therefore QR=RS

Now

$BC = 2QS$

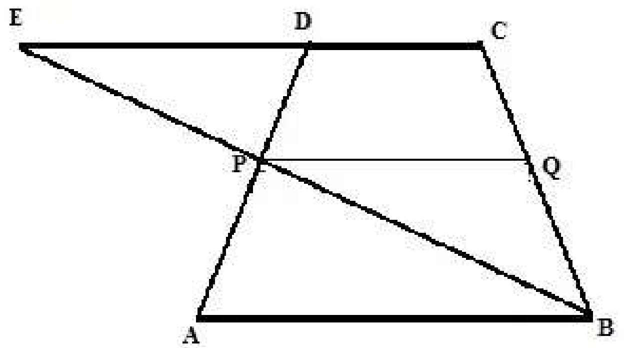
$BC = 2\times 2QR$

$BC = 4QR$

Hence proved

Solution 12:

The required figure is shown below



(i)

$From\ \Delta PED\ and\ \Delta ABP$

$PD = AP$ [P is the midpoint of AD]

$\angle DPE = \angle APB$ $[Opposite\ angle]$

$\angle PED = \angle PBA$ $[AB || CE]$

$\therefore$ $\Delta PED \cong \Delta ABP$ $[ASA$ postulate$]$

$\therefore$ $EP = BP$

(ii)For tiangle ECB PQ||CE

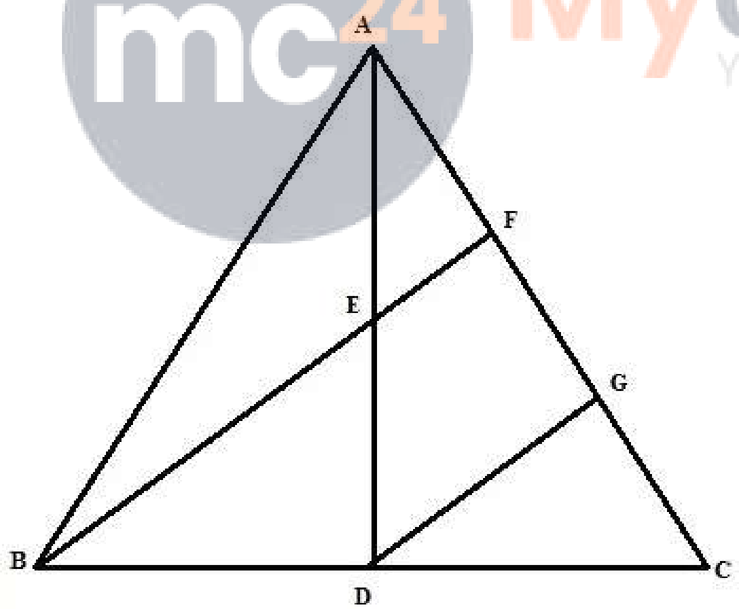
Again CE||AB

Therefore PQ||AB

Hence proved

Solution 13:

The required figure is shown below



For help we draw a line DG||BF

Now from triangle ADG, DG||BF and E is the midpoint of AD

Therefore F is the midpoint of AG,ie AF=GF .....(1)

From triangle BCF, DG||BF and D is the midpoint of BC

Therefore G is the midpoint of CF,ie GF=CF ...(2)

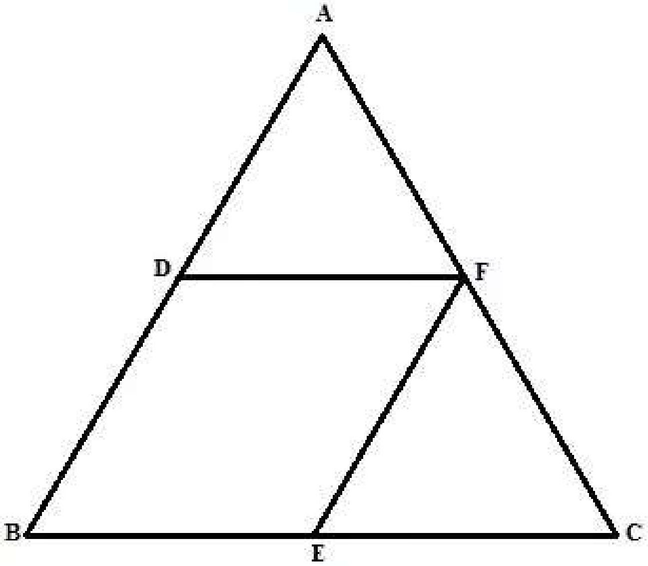
AC=AF+GF+CF

AC=3AF(From (1) and (2))

Hence proved

Solution 14:

The required figure is shown below



(i)Since F is the midpoint and EF||AB.
Therefore E is the midpoint of BC

So $BE = \frac{1}{2}BC$ and $EF = \frac{1}{2}AB$ .....(1)

Since D and F are the midpoint of AB and AC
Therefore DE||BC

SO $DF = \frac{1}{2}BC$ and $DB = \frac{1}{2}AB$ .....(2)

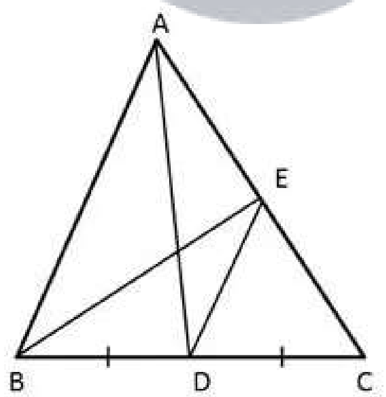
From (1),(2) we get
BE=DF and BD=EF
Hence BDEF is a parallelogram.
(ii)Since

$AB = 2EF$

$= 2\times4.8$

$= 9.6cm$

Solution 15:



In ΔABC,
AD is the median of BC
⇒ D is the mid - point of BC.
Given that DE P BA
By the Converse of the Mid - point theorem,
⇒ DE bisects AC
⇒ E is the mid - point of AC
⇒ BE is the median of AC,
that is BE is also a median.

Solution 16:

Construction : Draw $DY \parallel BQ$

In ΔBCQ and ΔDCY,

$\angle BCQ = \angle DCY$ (Common)

$\angle BQC = \angle DYC$ (Corresponding angles)

So, $\Delta BCQ \sim \Delta DCY$ (AA Similarity criterion)

$\Rightarrow \frac{BQ}{DY} = \frac{BC}{DC} = \frac{CQ}{CY}$ (Corresponding sides are proportional)

$\Rightarrow \frac{BQ}{DY} = \frac{2CD}{CD}$ (D is the mid-point of BC)

$\Rightarrow \frac{BQ}{DY} = 2 \ldots.(i)$

Similarly, $\Delta AEQ \sim \Delta ADY$

$\Rightarrow \frac{EQ}{DY} = \frac{AE}{ED} = \frac{1}{2}$ (E is the mid-point of AD)

that is $\frac{EQ}{DY} = \frac{1}{2} \ldots\ldots(ii)$

Dividing (i) by (ii), we get

$\Rightarrow \frac{BQ}{EQ} = 4$

$\Rightarrow BE + EQ = 4EQ$

$\Rightarrow BE = 3EQ$

$\Rightarrow \frac{BE}{EQ} = \frac{3}{1}$

Solution 17:

In ΔEDF,

M is the mid-point of AB and N is the mid-point of DE.

$\Rightarrow MN = \frac{1}{2}EF$ (Mid-point theorem)

$\Rightarrow EF = 2MN \ldots\ldots(i)$

In ΔABC,

M is the mid-point of AB and N is the mid-point of BC.

$\Rightarrow MN = \frac{1}{2}AC$ (Mid-point theorem)

$\Rightarrow AC = 2MN \ldots\ldots(ii)$

From (i) and (ii), we get

$\Rightarrow EF = AC$