Q. 10

Show that the four points with position vectors $\left(4\hat{i}+8\hat{j}+12\hat{k}\right),\left(2\hat{i}+4\hat{j}+6\hat{k}\right),$ $\left(3\hat{i}+5\hat{j}+4\hat{k}\right)$ **and** $\left(5\hat{i}+8\hat{j}+5\hat{k}\right)$ **are coplanar.**

Answer :

Given :

Let A, B, C & D be four points with position vectors $\bar{a}, \bar{b}, \bar{c}$ & $\bar{d}$.

Therefore,

$\bar{a} = 4\hat{\imath} + 8\hat{\jmath} + 12\hat{k}$

$\bar{b} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\bar{c} = 3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

$\bar{d} = 5\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = 4\hat{\imath} + 8\hat{\jmath} + 12\hat{k}$

$\bar{b} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\bar{c} = 3\hat{\imath} + 5\hat{\jmath} + 4\hat{k}$

$\bar{d} = 5\hat{\imath} + 8\hat{\jmath} + 5\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (4-2)\hat{\imath} + (8-4)\hat{\jmath} + (12-6)\hat{k}$

$\therefore\ \overline{BA} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (4-3)\hat{\imath} + (8-5)\hat{\jmath} + (12-4)\hat{k}$

$\therefore\ \overline{CA} = \hat{\imath} + 3\hat{\jmath} + 8\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (4-5)\hat{\imath} + (8-8)\hat{\jmath} + (12-5)\hat{k}$

$\therefore\ \overline{DA} = -\hat{\imath} + 0\hat{\jmath} + 7\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = 2\hat{\imath} + 4\hat{\jmath} + 6\hat{k}$

$\overline{CA} = \hat{\imath} + 3\hat{\jmath} + 8\hat{k}$

$\overline{DA} = -\hat{\imath} + 0\hat{\jmath} + 7\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} 2 & 4 & 6 \\ 1 & 3 & 8 \\ -1 & 0 & 7 \end{vmatrix}$$

$= 2(3 \times 7 - 0 \times 8) - 4(1 \times 7 - (-1) \times 8) + 6(1 \times 0 - (-1) \times 3)$

= 2(21) – 4(15) + 6(3)

= 42 – 60 + 18

= 0

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Q. 11

Show that the four points with position vectors $(6\hat{i}-7\hat{j}), (16\hat{i}-19\hat{j}-4\hat{k}), (3\hat{j}-6\hat{k})$ **and** $(2\hat{i}-5\hat{j}+10\hat{k})$ **are coplanar.**

Answer :

Given :

Let A, B, C & D be four points with position vectors $\bar{a}, \bar{b}, \bar{c}$ & $\bar{d}$.

Therefore,

$\bar{a} = 6\hat{\imath} - 7\hat{\jmath}$

$\bar{b} = 16\hat{\imath} - 19\hat{\jmath} - 4\hat{k}$

$\bar{c} = 3\hat{\jmath} - 6\hat{k}$

$\bar{d} = 2\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$

To Prove : Points A, B, C & D are coplanar.

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

2) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$$\bar{a} = 6\hat{\imath} - 7\hat{\jmath}$$

$$\bar{b} = 16\hat{\imath} - 19\hat{\jmath} - 4\hat{k}$$

$$\bar{c} = 3\hat{\jmath} - 6\hat{k}$$

$$\bar{d} = 2\hat{\imath} - 5\hat{\jmath} + 10\hat{k}$$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$$\overline{BA} = \bar{a} - \bar{b}$$

$$= (6 - 16)\hat{\imath} + (-7 + 19)\hat{\jmath} + (0 + 4)\hat{k}$$

$\therefore\ \overline{BA} = -10\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (6-0)\hat{\imath} + (-7-3)\hat{\jmath} + (0+6)\hat{k}$

$\therefore\ \overline{CA} = 6\hat{\imath} - 10\hat{\jmath} + 6\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (6-2)\hat{\imath} + (-7+5)\hat{\jmath} + (0-10)\hat{k}$

$\therefore\ \overline{DA} = 4\hat{\imath} - 2\hat{\jmath} - 10\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -10\hat{\imath} + 12\hat{\jmath} + 4\hat{k}$

$\overline{CA} = 6\hat{\imath} - 10\hat{\jmath} + 6\hat{k}$

$\overline{DA} = 4\hat{\imath} - 2\hat{\jmath} - 10\hat{k}$

$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = \begin{vmatrix} -10 & 12 & 4 \\ 6 & -10 & 6 \\ 4 & -2 & -10 \end{vmatrix}$

$= -10\big((-10)\times(-10) - (-2)\times 6\big) - 12(6\times(-10) - 4\times 6) + 4(6\times(-2) - (-10)\times 4)$

= -10(112) – 12(-84) + 4(28)

= -1120 + 1008 + 112

= 0

$\therefore [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}\ \&\ \overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$

Q. 12

Find the value of λ for which the four points with position vectors $(\hat{i}+2\hat{j}+3\hat{k})$, $(3\hat{i}-\hat{j}+2\hat{k}), (-2\hat{i}+\lambda\hat{j}+\hat{k})$ **and** $(6\hat{i}-4\hat{j}+2\hat{k})$ **are coplanar.**

Ans. λ = 3

Answer :

Given :

Let, A, B, C & D be four points with given position vectors

$\bar{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{c} = -2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\bar{d} = 6\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

To Find : value of λ

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = 1\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\bar{b} = 3\hat{\imath} - \hat{\jmath} + 2\hat{k}$

$\bar{c} = -2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\bar{d} = 6\hat{\imath} - 4\hat{\jmath} + 2\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (1-3)\hat{\imath} + (2+1)\hat{\jmath} + (3-2)\hat{k}$

$\therefore\ \overline{BA} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (1+2)\hat{\imath} + (2-\lambda)\hat{\jmath} + (3-1)\hat{k}$

$\therefore\ \overline{CA} = 3\hat{\imath} + (2-\lambda)\hat{\jmath} + 2\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (1-6)\hat{\imath} + (2+4)\hat{\jmath} + (3-2)\hat{k}$

$\therefore\ \overline{DA} = -5\hat{\imath} + 6\hat{\jmath} + \hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -2\hat{\imath} + 3\hat{\jmath} + \hat{k}$

$\overline{CA} = 3\hat{\imath} + (2-\lambda)\hat{\jmath} + 2\hat{k}$

$\overline{DA} = -5\hat{\imath} + 6\hat{\jmath} + \hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} -2 & 3 & 1 \\ 3 & (2-\lambda) & 2 \\ -5 & 6 & 1 \end{vmatrix}$$

$$= -2\big((2-\lambda)\times 1 - 2\times 6\big) - 3(3\times 1 - 2\times(-5)) + 1(6\times 3 - (2-\lambda)\times(-5))$$

= -2(- λ – 10) – 3(13) + 1(28 – 5λ)

= 2λ + 20 – 39 + 28 – 5λ

= 9 – 3λ

$\therefore [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 9 - 3\lambda$ ....eq(4)

Four points A, B, C & D are coplanar if and only if

$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$ ....eq(5)

From eq(4) and eq(5)

9 – 3λ = 0

3λ = 9

λ = 3

Q. 13

Find the value of λ for which the four points with position vectors $\left(-\hat{j}+\hat{k}\right)$, $\left(2\hat{i}-\hat{j}-\hat{k}\right)$, $\left(\hat{i}+\lambda\hat{j}+\hat{k}\right)$ **and** $\left(3\hat{j}+3\hat{k}\right)$ **are coplanar.**

Answer :

Given :

Let, A, B, C & D be four points with given position vectors

$\bar{a} = -\hat{j} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{j} - \hat{k}$

$\bar{c} = \hat{\imath} + \lambda\hat{j} + \hat{k}$

$\bar{d} = 3\hat{j} + 3\hat{k}$

To Find : value of λ

Formulae :

1) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

then vector $\overline{AB}$ is given by,

$\overline{AB} = \bar{b} - \bar{a}$

$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$

2) Scalar Triple Product:

If

$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$

$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$

$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

3) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given position vectors,

$\bar{a} = -\hat{\jmath} + \hat{k}$

$\bar{b} = 2\hat{\imath} - \hat{\jmath} - \hat{k}$

$\bar{c} = \hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\bar{d} = 3\hat{\jmath} + 3\hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (0 - 2)\hat{\imath} + (-1 + 1)\hat{\jmath} + (1 + 1)\hat{k}$

$\therefore\ \overline{BA} = -2\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (0-1)\hat{\imath} + (-1-\lambda)\hat{\jmath} + (1-1)\hat{k}$

$\therefore\ \overline{CA} = -\hat{\imath} + (-1-\lambda)\hat{\jmath} + 0\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (0-0)\hat{\imath} + (-1-3)\hat{\jmath} + (1-3)\hat{k}$

$\therefore\ \overline{DA} = 0\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -2\hat{\imath} + 0\hat{\jmath} + 2\hat{k}$

$\overline{CA} = -\hat{\imath} + (-1-\lambda)\hat{\jmath} + 0\hat{k}$

$\overline{DA} = 0\hat{\imath} - 4\hat{\jmath} - 2\hat{k}$

$$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = \begin{vmatrix} -2 & 0 & 2 \\ -1 & (-1-\lambda) & 0 \\ 0 & -4 & -2 \end{vmatrix}$$

$$= -2\big((-1-\lambda)\times(-2) - (-4)\times 0\big) - 0\big((-1)\times(-2) - 0\times 0\big) + 2\big((-1)\times(-4) - (-1-\lambda)\times 0\big)$$

= -2(2 + 2λ) – 0 + 2(4)

= - 4 - 4λ + 8

= 4 – 4λ

$\therefore\ [\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 4 - 4\lambda$ ....eq(4)

Four points A, B, C & D are coplanar if and only if

$[\overline{BA}\quad \overline{CA}\quad \overline{DA}] = 0$ ....eq(5)

From eq(4) and eq(5)

4 – 4λ = 0

4λ = 4

λ = 1

Q. 14

Using vector method, show that the points A(4, 5, 1), B(0, -1, -1), C(3, 9, 4) and D(-4, 4, 4) are coplanar.

Answer :

Given Points :

A ≡ (4, 5, 1)

B ≡ (0, -1, -1)

C ≡ (3, 9, 4)

D ≡ (-4, 4, 4)

To Prove : Points A, B, C & D are coplanar.

Formulae :

4) Position Vectors :

If A is a point with co-ordinates (a_1, a_2, a_3)

then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

5) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

6) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

7) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given points,

A ≡ (4, 5, 1)

B ≡ (0, -1, -1)

C ≡ (3, 9, 4)

D ≡ (-4, 4, 4)

Position vectors of above points are,

$$\bar{a} = 4\hat{\imath} + 5\hat{\jmath} + \hat{k}$$

$$\bar{b} = 0\hat{\imath} - \hat{\jmath} - \hat{k}$$

$$\bar{c} = 3\hat{\imath} + 9\hat{\jmath} + 4\hat{k}$$

$$\bar{d} = -4\hat{\imath} + 4\hat{\jmath} + 4\hat{k}$$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$$\overline{BA} = \bar{a} - \bar{b}$$

$$= (4 - 0)\hat{\imath} + (5 + 1)\hat{\jmath} + (1 + 1)\hat{k}$$

$\therefore\ \overline{BA} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$ ....eq(1)

$$\overline{CA} = \bar{a} - \bar{c}$$

$$= (4 - 3)\hat{\imath} + (5 - 9)\hat{\jmath} + (1 - 4)\hat{k}$$

$\therefore\ \overline{CA} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ ....eq(2)

$$\overline{DA} = \bar{a} - \bar{d}$$

$$= (4 + 4)\hat{\imath} + (5 - 4)\hat{\jmath} + (1 - 4)\hat{k}$$

$\therefore\ \overline{DA} = 8\hat{\imath} + 1\hat{\jmath} - 3\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = 4\hat{\imath} + 6\hat{\jmath} + 2\hat{k}$

$\overline{CA} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$

$\overline{DA} = 8\hat{\imath} + 1\hat{\jmath} - 3\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} 4 & 6 & 2 \\ 1 & -4 & -3 \\ 8 & 1 & -3 \end{vmatrix}$$

$$= 4((-4) \times (-3) - 1 \times (-3)) - 6(1 \times (-3) - (-3) \times 8) + 2(1 \times 1 - (-4) \times 8)$$

= 4(15) – 6(21) + 2(33)

= 60 – 126 + 66

= 0

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Hence, vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are coplanar.

Therefore, points A, B, C & D are coplanar.

Note : Four points A, B, C & D are coplanar if and only if $[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$

Q. 15

Find the value of λ for which the points A(3, 2, 1), B(4, λ, 5), C(4, 2, -2) and D(6, 5, -1) are coplanar.

Ans. λ = 5

Answer :

Given :

Points A, B, C & D are coplanar where,

A ≡ (3, 2, 1)

B ≡ (4, λ, 5)

C ≡ (4, 2, -2)

D ≡ (6, 5, -1)

To Find : value of λ

Formulae :

1) Position Vectors :

If A is a point with co-ordinates (a_1, a_2, a_3)

then its position vector is given by,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

2) Vectors :

If A & B are two points with position vectors $\bar{a}$ & $\bar{b}$,

Where,

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

then vector $\overline{AB}$ is given by,

$$\overline{AB} = \bar{b} - \bar{a}$$

$$(b_1 - a_1)\hat{\imath} + (b_2 - a_2)\hat{\jmath} + (b_3 - a_3)\hat{k}$$

3) Scalar Triple Product:

If

$$\bar{a} = a_1\hat{\imath} + a_2\hat{\jmath} + a_3\hat{k}$$

$$\bar{b} = b_1\hat{\imath} + b_2\hat{\jmath} + b_3\hat{k}$$

$$\bar{c} = c_1\hat{\imath} + c_2\hat{\jmath} + c_3\hat{k}$$

Then,

$$[\bar{a} \quad \bar{b} \quad \bar{c}] = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$$

4) Determinant :

$$\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = a_1(b_2.c_3 - c_2.b_3) - a_2(b_1.c_3 - c_1.b_3) + a_3(b_1.c_2 - c_1.b_2)$$

Answer :

For given points,

A ≡ (3, 2, 1)

B ≡ (4, λ, 5)

≡ (4, 2, -2)

D ≡ (6, 5, -1)

Position vectors of above points are,

$\bar{a} = 3\hat{i} + 2\hat{j} + \hat{k}$

$\bar{b} = 4\hat{i} + \lambda\hat{j} + 5\hat{k}$

$\bar{c} = 4\hat{i} + 2\hat{j} - 2\hat{k}$

$\bar{d} = 6\hat{i} + 5\hat{j} - \hat{k}$

Vectors $\overline{BA}, \overline{CA}$ & $\overline{DA}$ are given by,

$\overline{BA} = \bar{a} - \bar{b}$

$= (3-4)\hat{i} + (2-\lambda)\hat{j} + (1-5)\hat{k}$

$\therefore\ \overline{BA} = -\hat{i} + (2-\lambda)\hat{j} - 4\hat{k}$ ....eq(1)

$\overline{CA} = \bar{a} - \bar{c}$

$= (3-4)\hat{i} + (2-2)\hat{j} + (1+2)\hat{k}$

$\therefore\ \overline{CA} = -\hat{i} + 0\hat{j} + 3\hat{k}$ ....eq(2)

$\overline{DA} = \bar{a} - \bar{d}$

$= (3-6)\hat{i} + (2-5)\hat{j} + (1+1)\hat{k}$

$\therefore\ \overline{DA} = -3\hat{i} - 3\hat{j} + 2\hat{k}$ ....eq(3)

Now, for vectors

$\overline{BA} = -\hat{i} + (2-\lambda)\hat{j} - 4\hat{k}$

$\overline{CA} = -\hat{i} + 0\hat{j} + 3\hat{k}$

$\overline{DA} = -3\hat{i} - 3\hat{j} + 2\hat{k}$

$$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = \begin{vmatrix} -1 & (2-\lambda) & -4 \\ -1 & 0 & 3 \\ -3 & -3 & 2 \end{vmatrix}$$

$$= -1\big(0\times 2 - 3\times(-3)\big) - (2-\lambda)(2\times(-1) - (-3)\times 3) - 4((-1)\times(-3) - (-3)\times 0)$$

= - 1(9) – (2 - λ).(7) – 4(3)

= - 9 – 14 + 7λ – 12

= 7λ – 35

$\therefore [\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 7\lambda - 35$ .... eq(4)

But points A, B, C & D are coplanar if and only if

$[\overline{BA} \quad \overline{CA} \quad \overline{DA}] = 0$ .... eq(5)

From eq(4) and eq(5)

7λ – 35 = 0

$\therefore 7\lambda = 35$

Exercise 25B

Q. 1

If $\vec{a} = x\hat{i} + 2\hat{j} - z\hat{k}$ **and** $\vec{b} = 3\hat{i} - y\hat{j} + \hat{k}$ **are two equal vectors the x + y + z = ?**

Answer :

$\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}, \vec{b} = 2\hat{i} + \hat{j} - \hat{k}, \vec{c} = \hat{j} + \hat{k}$

$\vec{a} = x\hat{\imath} + 2\hat{\jmath} - z\hat{k}$

$\vec{b} = 3\hat{\imath} - y\hat{\jmath} + \hat{k}$

Since, these two vectors are equal, therefore comparing these two vectors we get,

x = 3 , - y = 2 , - z = 1

⇒x = 3,y = - 2,z = - 1

∴x + y + z = 3 + (- 2) + (- 1) = 3 - 2 - 1 = 0

Ans:x + y + z = 0

Q. 2

Write a unit vector in the direction of the sum of the vectors $\vec{a} = \left(2\hat{i} + 2\hat{j} - 5\hat{k}\right)$ **and** $\vec{b} = \left(2\hat{i} + \hat{j} - 7\hat{k}\right).$

Answer :

Let $\vec{s}$ be the sum of the vectors $\vec{a}$ and $\vec{b}$

$\Rightarrow \vec{s} = \vec{a} + \vec{b}$

$\Rightarrow \vec{s} = 2\hat{\imath} + 2\hat{\jmath} - 5\hat{k} + 2\hat{\imath} + \hat{\jmath} - 7\hat{k}$

$\Rightarrow \vec{s} = 4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}$

$|\vec{s}| = (4^2 + 3^2 + (-12)^2)^{1/2}$

$\Rightarrow |\vec{s}| = (16 + 9 + 144)^{1/2} = (169)^{1/2} = 13$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}}{13}$$

Ans: $\hat{s} = \frac{4\hat{\imath} + 3\hat{\jmath} - 12\hat{k}}{13}$

Q. 3

Write the value of λ so that the vectors $\vec{a} = \left(2\hat{i} + \lambda\hat{j} + \hat{k}\right)$ **and** $\vec{b} = \left(\hat{i} - 2\hat{j} + 3\hat{k}\right)$ **are perpendicular to each other.**

Answer :

$\vec{a} = 2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}$

$\vec{b} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$

Since these two vectors are perpendicular the dot product of these two vectors is zero.

i.e.: $\vec{a}.\vec{b} = 0$

$\Rightarrow (2\hat{\imath} + \lambda\hat{\jmath} + \hat{k}).(\hat{\imath} - 2\hat{\jmath} + 3\hat{k}) = 0$

$\Rightarrow 2 + \lambda \times (-2) + 3 = 0$

$\Rightarrow 5 = 2\lambda$

$\Rightarrow \lambda = 5/2$

Ans: $\lambda = 5/2$

Q. 4

Find the value of p for which the vectors $\vec{a} = \left(3\hat{i} + 2\hat{j} + 9\hat{k}\right)$ **and** $\vec{b} = \left(\hat{i} - 2p\hat{j} + 3\hat{k}\right)$ **are parallel.**

Answer :

$\vec{a} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$

$\vec{b} = \hat{\imath} - 2p\hat{\jmath} + 3\hat{k}$

Since these two vectors are parallel

$\therefore \frac{3}{1} = \frac{2}{-2p} = \frac{9}{3}$

$\Rightarrow \frac{3}{1} = \frac{1}{-p}$

$\Rightarrow p = \frac{-1}{3}$

Ans: $p = \frac{-1}{3}$

Q. 5

Find the value of λ when the projection of $\vec{a} = (\lambda\hat{i} + \hat{j} + 4\hat{k})$ on $\vec{b} = (2\hat{i} + 6\hat{j} + 3\hat{k})$ is 4 units.

Answer :

$\vec{a} = \lambda\hat{i} + \hat{j} + 4\hat{k}$

$\vec{b} = 2\hat{i} + 6\hat{j} + 3\hat{k}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$

$\Rightarrow |\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$\hat{b} = \frac{b}{|\vec{b}|} = \frac{2\hat{i} + 6\hat{j} + 3\hat{k}}{7}$

Now it is given that: $\vec{a}.\hat{b} = 4$

$\Rightarrow (\lambda\hat{i} + \hat{j} + 4\hat{k}).\left(\frac{2\hat{i} + 6\hat{j} + 3\hat{k}}{7}\right) = 4$

$\Rightarrow 2\lambda + 6 + (3\times4) = 28$

$\Rightarrow \lambda = (28 - 12 - 6)/2$

$\Rightarrow \lambda = 10/2 = 5$

Ans: $\lambda = 5$

Q. 6

If $\vec{a}$ and $\vec{b}$ are perpendicular vectors such that $|\vec{a}+\vec{b}|=13$ and $|\vec{a}|=5,$ find the value of $|\vec{b}|.$

Answer :

Since a and b vectors are perpendicular .

$\Rightarrow \theta = \frac{\pi}{2}$

Now,

$|\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2|\vec{a}||\vec{b}|\cos\theta$

$\Rightarrow 13^2 = 5^2 + |\vec{b}|^2 + 0$ ...($\cos\theta = \cos\frac{\pi}{2} = 0$)

$\Rightarrow |\vec{b}|^2 = 169 - 25 = 144$

$\Rightarrow |\vec{b}| = 12$

Ans: $|\vec{b}| = 12$

Q. 7

If $\vec{a}$ is a unit vector such that $(\vec{x}-\vec{a})\cdot(\vec{x}+\vec{a})=15,$ find $|\vec{x}|.$

Answer :

$(\vec{x} - \vec{a})(\vec{x} + \vec{a}) = 15$

$\Rightarrow |\vec{x}|^2 - |\vec{a}|^2 = 15$

$\Rightarrow |\vec{x}|^2 = |\vec{a}|^2 + 15$

Now , a is a unit vector,

$\Rightarrow |\vec{a}| = 1$

$\Rightarrow |\vec{x}|^2 = 1^2 + 15$

$\Rightarrow |\vec{x}|^2 = 16$

$\Rightarrow |\vec{x}| = 4$

Ans: $|\vec{x}| = 4$

Q. 8

Find the sum of the vectors $\vec{a}=\left(\hat{i}-3\hat{k}\right),\ \vec{b}=\left(2\hat{j}-\hat{k}\right)$ **and** $\vec{c}=\left(2\hat{i}-3\hat{j}+2\hat{k}\right).$

Answer :

$\vec{a} = \hat{\imath} - 3\hat{k}$

$\vec{b} = 2\hat{\jmath} - \hat{k}$

$\vec{c} = 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

Now ,

$\vec{a} + \vec{b} + \vec{c} = \hat{\imath} - 3\hat{\jmath} + 2\hat{\jmath} - \hat{k} + 2\hat{\imath} - 3\hat{\jmath} + 2\hat{k}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

Ans: $\vec{a} + \vec{b} + \vec{c} = 3\hat{\imath} - \hat{\jmath} - 2\hat{k}$

Q. 9

Find the sum of the vectors $\vec{a}=\left(\hat{i}-2\hat{j}\right),\ \vec{b}=\left(2\hat{i}-3\hat{j}\right)$ **and** $\vec{c}=\left(2\hat{i}+3\hat{k}\right).$

Answer :

$\vec{a} = \hat{\imath} - 2\hat{\jmath}$

$\vec{b} = 2\hat{\imath} - 3\hat{\jmath}$

$\vec{c} = 2\hat{\imath} + 3\hat{k}$

Now ,

$\vec{a} + \vec{b} + \vec{c} = \hat{\imath} - 2\hat{\jmath} + 2\hat{\imath} - 3\hat{\jmath} + 2\hat{\imath} + 3\hat{k}$

$\Rightarrow \vec{a} + \vec{b} + \vec{c} = 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

Ans: $\vec{a} + \vec{b} + \vec{c} = 5\hat{\imath} - 5\hat{\jmath} + 3\hat{k}$

Q. 10

Write the projection of the vector $\left(\hat{i}+\hat{j}+\hat{k}\right)$ **along the vector** $\hat{j}.$

Answer :

projection of a on b is given by: $\vec{a}.\hat{b}$

∴ the projection of the vector $\left(\hat{i}+\hat{j}+\hat{k}\right)$ along the vector $\hat{j}$. is :

$(\hat{\imath} + \hat{\jmath} + \hat{k}).\hat{\jmath} = 0 + 1 + 0 = 1$

Ans: the projection of the vector $\left(\hat{i}+\hat{j}+\hat{k}\right)$ along the vector $\hat{j}$. is:1

Q. 11

Write the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ **on the vector** $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Answer :

$\vec{a} = 7\hat{\imath} + \hat{\jmath} - 4\hat{k}$

$\vec{b} = 2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (2^2 + 6^2 + 3^2)^{1/2}$

$\Rightarrow |\vec{b}| = (4 + 36 + 9)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}$$

$$\vec{a}.\hat{b} = (7\hat{\imath} + \hat{\jmath} - 4\hat{k}).\left(\frac{2\hat{\imath} + 6\hat{\jmath} + 3\hat{k}}{7}\right) = \frac{(7\times 2) + (1\times 6) - (4\times 3)}{7}$$
$$= \frac{14 + 6 - 12}{7} = \frac{8}{7}$$

Ans: the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ on the vector $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Q. 12

Find $\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$ **when** $\vec{a}=\left(2\hat{i}+\hat{j}+3\hat{k}\right)$, $\vec{b}=\left(-\hat{i}+2\hat{j}+\hat{k}\right)$ **and** $\vec{c}=\left(3\hat{i}+\hat{j}+2\hat{k}\right)$.

Answer :

$\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$

$\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\vec{c} = 3\hat{\imath} + \hat{\jmath} + 2\hat{k}$

$$\vec{b} \times \vec{c} = \left(-\hat{\imath} + 2\hat{\jmath} + \hat{k}\right) \times \left(3\hat{\imath} + \hat{\jmath} + 2\hat{k}\right) = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 2 & 1 \\ 3 & 1 & 2 \end{bmatrix} = \hat{\imath}(4-1) - \hat{\jmath}(-2-3) + \hat{k}(-1-6) = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$$

$\therefore \vec{b} \times \vec{c} = 3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}$

$\therefore \vec{a}.\left(\vec{b} \times \vec{c}\right) = \left(2\hat{\imath} + \hat{\jmath} + 3\hat{k}\right).\left(3\hat{\imath} + 5\hat{\jmath} - 7\hat{k}\right)$ = (2×3) + (1×5) + (3× - 7)

= 6 + 5 - 21 = - 10

Ans: - 10

Q. 13

Find a vector in the direction of $\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ which has magnitude 21 units.

Answer :

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$

$|\vec{a}| = (2^2 + (-3)^2 + 6^2)^{1/2}$

$\Rightarrow |\vec{a}| = (4 + 9 + 36)^{1/2} = (49)^{1/2} = 7$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}}{7}$$

a vector in the direction of $\left(2\hat{i} - 3\hat{j} + 6\hat{k}\right)$ which has magnitude 21 units.

$= 21\hat{a} = 21 \times \frac{2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}}{7} = 3\left(2\hat{\imath} - 3\hat{\jmath} + 6\hat{k}\right) = 6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}$

Ans: $6\hat{\imath} - 9\hat{\jmath} + 18\hat{k}$

Q. 14

If $\vec{a} = \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right), \vec{b} = \left(-\hat{i} + 2\hat{j} + \hat{k}\right)$ **and** $\vec{c} = \left(3\hat{i} + \hat{j}\right)$ **are such that** $\left(\vec{a} + \lambda\vec{b}\right)$ **is perpendicular to** $\vec{c}$ **then find the value of λ.**

Answer :

$\vec{a} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k}$

$\vec{b} = -\hat{\imath} + 2\hat{\jmath} + \hat{k}$

$\vec{c} = 3\hat{\imath} + \hat{\jmath}$

$\vec{a} + \lambda\vec{b} = 2\hat{\imath} + 2\hat{\jmath} + 3\hat{k} + \lambda(-\hat{\imath} + 2\hat{\jmath} + \hat{k})$

$\Rightarrow \vec{a} + \lambda\vec{b} = (2 - \lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}$

Since $\vec{a} + \lambda\vec{b}$ *is perpendicular to* $\vec{c}$

$\Rightarrow \overrightarrow{(a + \lambda\vec{b})}.\vec{c} = 0$

$\Rightarrow \left((2 - \lambda)\hat{\imath} + (2 + 2\lambda)\hat{\jmath} + (3 + \lambda)\hat{k}\right).(3\hat{\imath} + \hat{\jmath}) = 0$

$\Rightarrow (2 - \lambda)\times 3 + (2 + 2\lambda)\times 1 = 0$

$\Rightarrow 6 + 2 - 3\lambda + 2\lambda = 0$

$\Rightarrow \lambda = 8$

Ans: $\lambda = 8$

Q. 15

Write the vector of magnitude 15 units in the direction of vector $\left(\hat{i} - 2\hat{j} + 2\hat{k}\right)$.

Answer :

$\vec{a} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$|\vec{a}| = (1^2 + (-2)^2 + 2^2)^{1/2}$

$\Rightarrow |\vec{a}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{a} = \frac{\vec{a}}{|\vec{a}|} = \frac{\hat{\imath} - 2\hat{\jmath} + 2\hat{k}}{3}$$

a vector in the direction of $\left(\hat{i}-2\hat{j}+2\hat{k}\right).$ which has magnitude 15 units.

= $15\hat{a} = 15 \times \frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3} = 5(\hat{\imath}-2\hat{\jmath}+2\hat{k}) = 5\hat{\imath}-10\hat{\jmath}+10\hat{k}.$

Ans: $5\hat{\imath}-10\hat{\jmath}+10\hat{k}.$

Q. 16

If $\vec{a}=\left(\hat{i}+\hat{j}+\hat{k}\right),\vec{b}=\left(4\hat{i}-2\hat{j}+3\hat{k}\right)$ **and** $\vec{c}=\left(\hat{i}-2\hat{j}+\hat{k}\right),$ **find a vector of magnitude 6 units which is parallel to the vector** $\left(2\vec{a}-\vec{b}+3\vec{c}\right).$

Answer :

$\vec{a} = \hat{\imath}+\hat{\jmath}+\hat{k}$

$\vec{b} = 4\hat{\imath}-2\hat{\jmath}+3\hat{k}$

$\vec{c} = \hat{\imath}-2\hat{\jmath}+\hat{k}$

$\therefore (2\vec{a}-\vec{b}+3\vec{c}) = 2(\hat{\imath}+\hat{\jmath}+\hat{k})-(4\hat{\imath}-2\hat{\jmath}+3\hat{k})+3(\hat{\imath}-2\hat{\jmath}+\hat{k})$

$\Rightarrow (2\vec{a}-\vec{b}+3\vec{c}) = \hat{\imath}-2\hat{\jmath}+2\hat{k}$

LET, $(2\vec{a}-\vec{b}+3\vec{c}) = \vec{s}$

$\vec{s} = \hat{\imath}-2\hat{\jmath}+2\hat{k}$

$|\vec{s}| = (1^2 + (-2)^2 + 2^2)^{1/2}$

$\Rightarrow |\vec{s}| = (1 + 4 + 4)^{1/2} = (9)^{1/2} = 3$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{s} = \frac{\vec{s}}{|\vec{s}|} = \frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3}$$

a vector of magnitude 6 units which is parallel to the vector $\left(2\vec{a}-\vec{b}+3\vec{c}\right).$ is:

$$6\hat{s} = 6 \times \frac{\hat{\imath}-2\hat{\jmath}+2\hat{k}}{3} = 2(\hat{\imath}-2\hat{\jmath}+2\hat{k}) = 2\hat{\imath}-4\hat{\jmath}+4\hat{k}.$$

Ans: $2\hat{\imath}-4\hat{\jmath}+4\hat{k}$

Q. 17

Write the projection of the vector $\left(\hat{i}-\hat{j}\right)$ on the vector $\left(\hat{i}+\hat{j}\right)$.

Answer :

$\vec{a} = \hat{\imath} - \hat{\jmath}$

$\vec{b} = \hat{\imath} + \hat{\jmath}$

projection of a on b is given by: $\vec{a}.\hat{b}$

$|\vec{b}| = (1^2 + 1^2 + 0^2)^{1/2}$

$\Rightarrow |\vec{b}| = (1 + 1)^{1/2} = (2)^{1/2}$

a unit vector in the direction of the sum of the vectors is given by:

$$\hat{b} = \frac{\vec{b}}{|\vec{b}|} = \frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}$$

$$\vec{a}.\hat{b} = (\hat{\imath} - \hat{\jmath}).\left(\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}\right) = \frac{(1 \times 1) + (-1 \times 1)}{\sqrt{2}} = \frac{0}{\sqrt{2}} = 0$$

Ans: the projection of the vector $\left(7\hat{i}+\hat{j}-4\hat{k}\right)$ on the vector $\left(2\hat{i}+6\hat{j}+3\hat{k}\right)$.

Q. 18

Write the angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes $\sqrt{3}$ and 2 respectively having $\vec{a}\cdot\vec{b} = \sqrt{6}$.

Answer :

$|\vec{a}| = \sqrt{3}$

$|\vec{b}| = 2$

Since, $\vec{a}.\vec{b} = |\vec{a}||\vec{b}|cos\theta$

Substituting the given values we get:

$\Rightarrow \sqrt{6} = \sqrt{3} \times 2 \times cos\theta$

$\Rightarrow cos\theta = \frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \theta = \cos^{-1}\frac{1}{\sqrt{2}}$

$\Rightarrow \theta = 45^\circ = \frac{\pi}{4}$

Ans: $\theta = 45^\circ = \frac{\pi}{4}$

Q. 19

If $\vec{a} = (\hat{i} - 7\hat{j} + 7\hat{k})$ **and** $\vec{b} = (3\hat{i} - 2\hat{j} + 2\hat{k})$ **then find** $|\vec{a} \times \vec{b}|$.

Answer :

$\vec{a} = \hat{\imath} - 7\hat{\jmath} + 7\hat{k}$

$\vec{b} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$$\vec{a} \times \vec{b} = (\hat{\imath} - 7\hat{\jmath} + 7\hat{k}) \times (3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}) = \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & -7 & 7 \\ 3 & -2 & 2 \end{bmatrix} = \hat{\imath}(-14 - (-14)) - \hat{\jmath}(2 - 21) + \hat{k}(-2 - (-21))$$

$$= 0\hat{\imath} + 19\hat{\jmath} + 19\hat{k}$$

$\therefore \vec{a} \times \vec{b} = 0\hat{\imath} + 19\hat{\jmath} + 19\hat{k}$

$\therefore |\vec{a} \times \vec{b}| = (0^2 + 19^2 + 19^2)^{1/2} = (2 \times 19^2)^{1/2} = 19\sqrt{2}$

Ans: $\therefore |\vec{a} \times \vec{b}| = 19\sqrt{2}$

Q. 20

Find the angle between two vectors $\vec{a}$ **and** $\vec{b}$ **with magnitudes 1 and 2 respectively, when** $|\vec{a} \times \vec{b}| = \sqrt{3}$.

Answer :

$|\vec{a}| = 1$

$|\vec{b}| = 2$

Since, $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|sin\theta$

Substituting the given values we get:

$\Rightarrow \sqrt{3} = 1 \times 2 \times sin\theta$

$\Rightarrow sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \theta = \sin^{-1}\frac{\sqrt{3}}{2}$

$\Rightarrow \theta = 60^{\circ} = \frac{\pi}{3}$

Ans: $\theta = 60^{\circ} = \frac{\pi}{3}$

Q. 21

What conclusion can you draw about vectors $\vec{a}$ **and** $\vec{b}$ **when** $\vec{a} \times \vec{b} = \vec{0}$ **and** $\vec{a} \cdot \vec{b} = 0$?

Answer :

It is given that:

$\vec{a} \times \vec{b} = \vec{0}$ *and* $\vec{a}.\vec{b} = \vec{0}$

$\Rightarrow |\vec{a}||\vec{b}|sin\theta = |\vec{a}||\vec{b}|cos\theta = \vec{0}$

Since sinθ and cosθ cannot be 0 simultaneously $\therefore |\vec{a}| = |\vec{b}| = 0$

Conclusion: when $\vec{a} \times \vec{b} = \vec{0}$ *and* $\vec{a}.\vec{b} = \vec{0}$

Then $|\vec{a}| = |\vec{b}| = 0$

Q. 22

Find the value of λ when the vectors $\vec{a} = \left(\hat{i} + \lambda\hat{j} + 3\hat{k}\right)$ **and** $\vec{b} = \left(3\hat{i} + 2\hat{j} + 9\hat{k}\right)$ **are parallel.**

Answer :

$\vec{a} = \hat{\imath} + \lambda\hat{\jmath} + 3\hat{k}$

$\vec{b} = 3\hat{\imath} + 2\hat{\jmath} + 9\hat{k}$

It is given that $\vec{a} \parallel \vec{b}$

$\Rightarrow \frac{1}{3} = \frac{\lambda}{2} = \frac{3}{9}$

$\Rightarrow \frac{1}{3} = \frac{\lambda}{2}$

$\Rightarrow \lambda = 2 \times \frac{1}{3} = \frac{2}{3}$

Ans: λ = 2/3

Q. 23

Write the value of

$$\hat{i} \cdot \left(\hat{j} \times \hat{k}\right) + \hat{j} \cdot \left(\hat{i} \times \hat{k}\right) + \hat{k} \cdot \left(\hat{i} \times \hat{j}\right).$$

Answer :

We know that:

$\hat{\imath} \times \hat{\jmath} = \hat{k}, \hat{\jmath} \times \hat{k} = \hat{\imath}, \hat{k} \times \hat{\imath} = \hat{\jmath},$

$\hat{\jmath} \times \hat{\imath} = -\hat{k}, \hat{k} \times \hat{\jmath} = -\hat{\imath}, \hat{\imath} \times \hat{k} = -\hat{\jmath}$

$\hat{\imath}.\hat{\imath} = \hat{\jmath}.\hat{\jmath} = \hat{k}.\hat{k} = 1$

$\therefore \hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}.(\hat{\imath} \times \hat{\jmath}) = \hat{\imath}.\hat{\imath} + \hat{\jmath}.(-\hat{\jmath}) + \hat{k}.\hat{k} = 1 - 1 + 1 = 1$

Ans: $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}.(\hat{\imath} \times \hat{\jmath}) = 1$

Q. 24

Find the volume of the parallelepiped whose edges are represented by the vectors $\vec{a} = \left(2\hat{i} - 3\hat{j} + 4\hat{k}\right), \vec{b} = \left(\hat{i} + 2\hat{j} - \hat{k}\right)$ **and** $\vec{c} = \left(3\hat{i} - 2\hat{j} + 2\hat{k}\right).$

Answer :

Scalar triple product geometrically represents the volume of the parallelepiped whose three coterminous edges are represented by $\vec{a}, \vec{b}, \vec{c}$ i.e. V = $[\vec{a}\vec{b}\vec{c}]$

$\vec{a} = 2\hat{\imath} - 3\hat{\jmath} + 4\hat{k}$

$\vec{b} = \hat{\imath} + 2\hat{\jmath} - \hat{k}$

$\vec{c} = 3\hat{\imath} - 2\hat{\jmath} + 2\hat{k}$

$\therefore$ V = $[\vec{a}\vec{b}\vec{c}] = \begin{bmatrix} 2 & -3 & 4 \\ 1 & 2 & -1 \\ 3 & -2 & 2 \end{bmatrix}$ = 2(4 - 2) - (- 3)(2 - (- 3)) + 4(- 2 - 6) = 4 + 15 - 32 = | - 13| = 13 cubic units.

Ans:13 cubic units.

Q. 25

If $\vec{a} = \left(-2\hat{i} - 2\hat{j} + 4\hat{k}\right), \vec{b} = \left(-2\hat{j} + 4\hat{j} - 2\hat{k}\right)$ **and** $\vec{c} = \left(4\hat{i} - 2\hat{j} - 2\hat{k}\right)$ **then prove that** $\vec{a}, \vec{b}$ **and** $\vec{c}$ **are coplanar.**

Answer :

$\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

If $\vec{a}, \vec{b}, \vec{c}$ are coplanar then $[\vec{a}\vec{b}\vec{c}] = 0$

L.H.S = $\begin{bmatrix} -2 & -2 & 4 \\ -2 & 4 & -2 \\ 4 & -2 & -2 \end{bmatrix}$ = - 2(- 8 - 4) + 2(4 + 8) + 4(4 - 16) = 24 + 24 - 48 = 0 = R.H.S

$\therefore$L.H.S = R.H.S

Hence proved that the vectors $\vec{a} = -2\hat{\imath} - 2\hat{\jmath} + 4\hat{k}$

$\vec{b} = -2\hat{\imath} + 4\hat{\jmath} - 2\hat{k}$

$\vec{c} = 4\hat{\imath} - 2\hat{\jmath} - 2\hat{k}$

Are coplanar.

Q. 26

If $\vec{a} = \left(2\hat{i} + 6\hat{j} + 27\hat{k}\right)$ **and** $\vec{b} = \left(\hat{i} + \lambda\hat{j} + \mu\hat{k}\right)$ **are such that** $\vec{a} \times \vec{b} = \vec{0}$ **then find the values of λ and μ.**

Answer :

$\vec{a} = 2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}$

$\vec{b} = \hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}$

It is given that $\vec{a} \times \vec{b} = \vec{0}$

$\Rightarrow (2\hat{\imath} + 6\hat{\jmath} + 27\hat{k}) \times (\hat{\imath} + \lambda\hat{\jmath} + \mu\hat{k}) = 0$

$\Rightarrow \begin{bmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 6 & 27 \\ 1 & \lambda & \mu \end{bmatrix} = 0 = \hat{\imath}(6\mu - 27\lambda) - \hat{\jmath}(2\mu - 27) + \hat{k}(2\lambda - 6)$

$\Rightarrow 2\lambda - 6 = 0$

$\Rightarrow \lambda = 6/2 = 3$

$\Rightarrow 2\mu - 27 = 0$

$\Rightarrow \mu = 27/2$

Ans: $\lambda = 3$, $\mu = 27/2$

Q. 27

If θ is the angle between $\vec{a}$ and $\vec{b}$, and $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$ then what is the value of θ?

Answer :

It is given that:

$|\vec{a} \times \vec{b}| = |\vec{a}.\vec{b}|$

$\Rightarrow |\vec{a}||\vec{b}| sin\theta = |\vec{a}||\vec{b}| cos\theta$

$\Rightarrow \sin\theta = \cos\theta$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \theta = \tan^{-1} 1 = \frac{\pi}{4}$

Ans: $\theta = \frac{\pi}{4}$

Q. 28

When does $|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$ hold?

Answer :

When the two vectors are parallel or collinear, they can be added in a scalar way because the angle between them is zero degrees,they are I the same or opposite direction.

Therefore when two vectors $\vec{a}$ *and* $\vec{b}$ are either parallel or collinear then

$|\vec{a} + \vec{b}| = |\vec{a}| + |\vec{b}|$