$$\frac{-1}{4} - B = 0$$

$$B = -\frac{1}{4}$$

From equation(1),we get,

$$\frac{t}{(1-t)(1+t)(1-t)} = \frac{-1}{4} \times \frac{1}{1+t} - \frac{1}{4} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{(1-t)^2}$$

$$\int \frac{t}{(1-t)(1+t)(1-t)} dt = \frac{-1}{4} \int \frac{1}{1+t} dt - \frac{1}{4} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{(1-t)^2} dt$$

$$= \frac{-1}{4} \int \frac{1}{1+t} dt - \frac{1}{4} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{(1-t)^2} dt$$

$$= -\frac{1}{4} log|1+t| - \frac{1}{4} log|1-t| - \frac{1}{2} \times \frac{1}{1-t} + c$$

$$= -\frac{1}{4} log|1+sinx| - \frac{1}{4} log|1-sinx| - \frac{1}{2} \times \frac{1}{1-sinx} + c$$

35. Question

Evaluate:

$$\int \frac{(5x+8)}{x^2(3x+8)} dx$$

Answer

Let $I = \int \frac{5x+8}{x^2(3x+8)} dx$

Now putting, $\frac{5x+8}{x^2(3x+8)} = \frac{A}{(3x+8)} + \frac{Bx+C}{x^2}$ ……..(1)

$Ax^2+(Bx + C)(3x+8) = 5x+8$

Putting 3x+8=0,

$$x = -\frac{8}{3}$$

$$A\left(\frac{64}{9}\right) + B(0) = 5\left(-\frac{8}{3}\right) + 8$$

$$A\left(\frac{64}{9}\right) = \frac{-40+24}{3}$$

$$A\left(\frac{64}{9}\right) = \frac{-16}{3}$$

$$A = \frac{-3}{4}$$

By equating the coefficient of x^2 and constant term,

A+3B=0

$$\frac{-3}{4} + 3B = 0$$

$$3B = \frac{3}{4}$$

$B = \frac{1}{4}$

8C=8

C=1

From equation (1), we get,

$$\int \frac{5x+8}{x^2(3x+8)}dx = \frac{-3}{4} \times \int \frac{1}{(3x+8)}dx + \frac{1}{4} \times \int \frac{x+1}{x^2}dx$$

$$= \frac{-3}{4} \times \frac{\log(3x+8)}{3} + \frac{1}{4}\int \frac{x}{x^2}dx + \int \frac{1}{x^2}dx$$

$$= -\frac{1}{4}\log|3x+8| + \frac{1}{4}logx - \frac{1}{x} + c$$

Putting x+2=0,

X=-2

$A(-4)^2+B(0)+C(0)=-6+1=-5$

$A = \frac{-5}{16}$

54. Question

$\int \frac{dx}{(\sin x + \sin 2x)}$

Answer

let $I = \int \frac{dx}{(sinx+sin2x)} = \int \frac{dx}{(sinx+2sinxcosx)}$

Put t=cosx

dt=-sinxdx

$\frac{-dt}{sinx} = dx$

$$I = \int \frac{-dt}{sin^2 x\,(1+2t)} = \int \frac{dt}{(1-cos^2 x)(1+2t)} = \int \frac{dt}{(1-t^2)(1+2t)}$$

Putting $\frac{t}{(1-t)(1+t)(1+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{1+2t} \dots \dots (1)$

$A(1+t)(1+2t)+B(1-t)(1+2t)+C(1-t^2)=1$

Putting 1+t=0

t=-1

A(0)+B(2)(1-2)+C(0)=1

$B = -\frac{1}{2}$

Putting 1-t=0

t=1

A(2)(3)+B(0)+C(0)=1

$A = \frac{1}{6}$

Putting 1+2t=0

$$t = -\frac{1}{2}$$

$$A(0) + B(0) + C\left(1 - \frac{1}{4}\right) = 1$$

$$C = \frac{4}{3}$$

From equation(1),we get,

$$\frac{1}{(1-t)(1+t)(1+2t)} = \frac{1}{6} \times \frac{1}{1-t} - \frac{1}{2} \times \frac{1}{1+t} + \frac{4}{3} \times \frac{1}{1+2t}$$

$$\int \frac{1}{(1-t)(1+t)(1+2t)} dt = \frac{1}{6}\int \frac{1}{1-t} dt - \frac{1}{2}\int \frac{1}{1+t} dt + \frac{4}{3}\int \frac{1}{1+2t} dt$$

$$= \frac{1}{6} log|1-t| - \frac{1}{2} log|1+t| + \frac{2}{3} log|1+2t| + c$$

$$= \frac{1}{6} log|1-cosx| - \frac{1}{2} log|1+cosx| + \frac{2}{3} log|1+2cosx| + c$$

36. Question

Evaluate:

$$\int \frac{(5x^2 - 18x + 17)}{(x-1)^2 (2x-3)} dx$$

Answer

Let $I = \int \frac{5x^2 18x+17}{(x-1)^2(2x-3)} dx$

Now putting, $\frac{5x^2 18x+17}{(x-1)^2(2x-3)} = \frac{A}{(2x-3)} + \frac{B}{x-1} + \frac{C}{(x-1)^2} \ldots. (1)$

$A(x-1)^2+B(2x-3)(x-1)+C(2x-3) = 5x^2-18x+17$

Putting x-1=0,

X=1

A(0)+B(0)+C(2-3)=5-18+17

C(-1)=4

Putting 2x-3=0,

$$x = \frac{3}{2}$$

$$A\left(\frac{3}{2} - 1\right)^2 + B(0) + C(0) = 5\left(\frac{3}{2}\right)^2 - 18\left(\frac{3}{2}\right) + 17$$

$$A\left(\frac{1}{4}\right) + 0 = 5 \times \frac{9}{4} - 27 + 17$$

$$A\left(\frac{1}{4}\right) = \frac{45}{4} - 10 = \frac{5}{4}$$

A=5

By equating the coefficient of x^2, we get ,

A+2B=5

5+2B=5

2B=0

B=0

From equation (1), we get,

$$\frac{5x^2 18x+17}{(x-1)^2(2x-3)} = 5\times\frac{1}{(2x-3)} + 0 - 4\times\frac{1}{(x-1)^2}$$

$$\int \frac{5x^2 18x+17}{(x-1)^2(2x-3)}dx = \frac{5}{2}\log(2x-3) + \frac{4}{x-1} + c$$

37. Question

Evaluate:

$$\int \frac{8}{(x+2)(x^2+4)}dx$$

Answer

Let $I = \int \frac{8}{(x+2)(x^2+4)}dx$

Now putting, $\frac{8}{(x+2)(x^2+4)} = \frac{A}{x+2} + \frac{Bx+C}{(x^2+4)} \dots\dots (1)$

$A(x^2+4)+(Bx+C)(x+2)= 8$

Putting x+2=0,

X=-2

A(4+4)+0=8

A=1

By equating the coefficient of x^2 and constant term, A+B=0

1+B=0

B=-1

4A+2C=8

4×1+2C=8

2C=4

C=2

From equation (1),we get,

$$\frac{8}{(x+2)(x^2+4)} = \frac{1}{x+2} + \frac{-x+2}{(x^2+4)}$$

$$\int \frac{8}{(x+2)(x^2+4)}dx = \int \frac{1}{x+2}dx - \int \frac{x}{(x^2+4)}dx + 2\int \frac{1}{(x^2+4)}dx$$

$$= log|x+2| - \frac{1}{2}\log(x^2+4) + 2\times\frac{1}{2}\times tan^{-1}\frac{x}{2} + c$$

$$= log|x+2| - \frac{1}{2}\log|x^2+4| + tan^{-1}\frac{x}{2} + c$$

55. Question

$\int \frac{x^2}{(x^4 - x^2 - 12)} dx$

Answer

Let $I = \int \frac{x^2}{(x^4 - x^2 - 12)} dx$

Putting $\frac{x^2}{(x^4 - x^2 - 12)} = \frac{t}{t^2 - t - 12} = \frac{t}{(t-4)(t+3)} = \frac{A}{t-4} + \frac{B}{t+3} \dots\dots\dots (1)$

Where $t=x^2$

A(t+3)+B(t-4)=t

Now put t+3=0

t=-3

A(0)+B(-7)=-3

$B = \frac{3}{7}$

Now put t-4=0

t=4

A(4+3)+B(0)=4

$A = \frac{4}{7}$

From equation(1)

$$\frac{t}{(t-4)(t+3)} = \frac{4}{7} \times \frac{1}{t-4} + \frac{3}{7} \times \frac{1}{t+3}$$

$$\frac{x^2}{(x^2-4)(x^2+3)} = \frac{4}{7} \times \frac{1}{x^2 - 2^2} + \frac{3}{7} \times \frac{1}{x^2 + (\sqrt{3})^2}$$

$$\int \frac{x^2}{(x^2-4)(x^2+3)} dx = \frac{4}{7} \int \frac{1}{x^2 - 2^2} dx + \frac{3}{7} \int \frac{1}{x^2 + \left(\sqrt{3}\right)^2} dx$$

$$= \frac{4}{7} \times \frac{1}{2} \times \frac{1}{2} log \left|\frac{x-2}{x+2}\right| + \frac{3}{7} \times \frac{1}{\sqrt{3}} tan^{-1} \frac{x}{\sqrt{3}} + c$$

$$= \frac{1}{7} log \left|\frac{x-2}{x+2}\right| + \frac{\sqrt{3}}{7} tan^{-1} \frac{x}{\sqrt{3}} + c$$

56. Question

$\int \frac{x^4}{(x^2+1)(x^2+9)(x^2+16)} dx$

Answer

Let $I = \int \frac{x^4}{(x^2+1)(x^2+9)(x^2+16)} dx$

Putting $\frac{(x^2)^2}{(x^2+1)(x^2+9)(x^2+16)} = \frac{t^2}{(t+1)(t+9)(t+16)} = \frac{A}{t+1} + \frac{B}{t+9} + \frac{C}{t+16} \dots\dots\dots (1)$

Where $t=x^2$

t^2=A(t+9)(t+16)+B(t+1)(t+16)+C(t+1)(t+9)

Now put t+1=0

t=-1

A(8)(15)+B(0)+C(0)=1

$$A = \frac{1}{120}$$

Now put t+9=0

t=-9

A(-9+9)(-9+16)+B(-9+1)(-9+16)+C(-9+1)(-9+9)=(-9)^2

A(0)+B(-56)+C(0)=81

$$B = -\frac{81}{56}$$

Now put t+16=0

t=-16

A(0)+B(0)+C(-15)(-7)=(-16)^2

A(0)+B(0)+C(105)=256

$$C = \frac{256}{105}$$

From equation(1)

$$\frac{t^2}{(t+1)(t+9)(t+16)} = \frac{A}{t+1} + \frac{B}{t+9} + \frac{C}{t+16}$$

$$\int \frac{t^2}{(t+1)(t+9)(t+16)} dt = \int \left[\frac{\frac{1}{120}}{t+1} - \frac{\frac{81}{56}}{t+9} + \frac{\frac{256}{105}}{t+16}\right] dt$$

$$= \frac{1}{120}\int \frac{1}{t+1} dt - \frac{81}{56}\int \frac{1}{t+9} dt + \frac{256}{105}\int \frac{1}{t+16} dt$$

$$= \frac{1}{120}\int \frac{1}{x^2+1} dx - \frac{81}{56}\int \frac{1}{x^2+9} dx + \frac{256}{105}\int \frac{1}{x^2+16} dx$$

$$= \frac{1}{120}\int \frac{1}{x^2+1} dx - \frac{81}{56}\int \frac{1}{x^2+(3)^2} dx + \frac{256}{105}\int \frac{1}{x^2+(4)^2} dx$$

$$= \frac{1}{120} tan^{-1} x - \frac{81}{56} \times \frac{1}{3} tan^{-1}\left(\frac{x}{3}\right) + \frac{256}{105} \times \frac{1}{4} tan^{-1}\left(\frac{x}{4}\right) + c$$

$$= \frac{1}{120} tan^{-1} x - \frac{27}{56} tan^{-1}\left(\frac{x}{3}\right) + \frac{64}{105} tan^{-1}\left(\frac{x}{4}\right) + c$$

38. Question

Evaluate:

$$\int \frac{(3x+5)}{(x^3 - x^2 + x - 1)} dx$$

Answer

Let $I = \int \frac{3x+5}{(x^3-x^2+x-1)} dx$

Now putting, $\frac{3x+5}{(x^3-x^2+x-1)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+1)} \ldots\ldots (1)$

A(x^2+1)+(Bx+C)(x-1)=3x+5

Putting x-1=0,

X=1

A(2)+B(0)=3+5=8

A=4

By equating the coefficient of x^2 and constant term, A+B=0

4+B=0

B=-4

A-C=5

4-C=5

C=-1

From equation (1),we get,

$$\frac{3x+5}{(x-1)(x^2+1)} = \frac{4}{x-1} + \frac{-4x-1}{(x^2+1)}$$

$$\int \frac{3x+5}{(x-1)(x^2+1)} dx = 4\int \frac{1}{x-1} dx - 4\int \frac{1}{(x^2+1)} dx - \int \frac{1}{(x^2+1)} dx$$

$$= 4\log(x-1) - \frac{4}{2}\log(x^2+1) - tan^{-1}x + c$$

$$= 4\log(x-1) - 2\log(x^2+1) - tan^{-1}x + c$$

57. Question

$\int \frac{\sin 2x}{(1-\cos 2x)(2-\cos 2x)} dx$

Answer

let $I = \int \frac{sin2x}{(1-cos2x)(2-cos2x)} dx$

Put t=cos2x

dt=-2sin2xdx

$$I = \int \frac{-dt/2}{(1-t)(2-t)} = \frac{1}{2}\int \frac{dt}{(t-2)(1-t)}$$

Putting $\frac{1}{(t-2)(1-t)} = \frac{A}{t-2} + \frac{B}{1-t} \dots\dots(1)$

A(1-t)+B(t-2)=1

Putting 1-t=0

t=1

A(0)+B(1-2) =1

B=-1

Putting t-2=0

t=2

A(1-2)+B(0) =1

A=-1

From equation (1), we get,

$$\frac{1}{(t-2)(1-t)}=\frac{-1}{t-2}+\frac{-1}{1-t}$$

$$\int\frac{1}{(t-2)(1-t)}dt=\int\frac{1}{2-t}dt+\int\frac{1}{t-1}dt$$

$$=-log|2-t|+log|t-1|+c$$

$$=log|t-1|-log|2-t|+c$$

$$=log|cos2x-1|-log|2-cos2x|+c$$

39. Question

Evaluate:

$$\int\frac{2x}{(x^2+1)(x^2+3)}dx$$

Answer

Let $I=\int\frac{2x}{(x^2+1)(x^2+3)}dx$

Put $t=x^2$

$dt=2xdx$

Now putting, $\frac{1}{(t+1)(t+3)}=\frac{A}{t+1}+\frac{B}{t+3}\ldots\ldots(1)$

A(t+3) +B(t+1) = 1

Putting t+3=0,

X=-3

A(0) +B(-3+1)=1

$$B=-\frac{1}{2}$$

Putting t+1=0,

X=-1

A(-1+3)+B(0)=1

$$A=\frac{1}{2}$$

From equation(1),we get,

$$\frac{1}{(t+1)(t+3)}=\frac{1}{2}\times\frac{1}{t+1}-\frac{1}{2}\times\frac{1}{t+3}$$

$$\int\frac{1}{(t+1)(t+3)}dt=\frac{1}{2}\int\frac{1}{t+1}dt-\frac{1}{2}\int\frac{1}{t+3}dt$$

$$=\frac{1}{2}\log|t+1|-\frac{1}{2}\log|t+3|+c$$

$$\int\frac{2x}{(x^2+1)(x^2+3)}dx=\frac{1}{2}\log|x^2+1|-\frac{1}{2}\log|x^2+3|+c$$

58. Question

$$\int\frac{2}{(1-x)(1+x^2)}dx$$

Answer

Let $I = \int \frac{2}{(1-x)(1+x^2)} dx$

Put $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{x^2+1} \dots\dots\dots (1)$

A(1+x^2)+Bx(1-x)+C(1-x) =2

Put x=1

2=2A+0+0

A=1

Put x=0

2=A+C

C=2-A

C=2-1=1

Putting x=2

We have 2=5A-2B-C

2=5×1-2B-1

2B=2

B=1

$$\frac{2}{(1-x)(1+x^2)} = \frac{1}{1-x} + \frac{x}{1+x^2} + \frac{1}{1+x^2}$$

$$\int \frac{2}{(1-x)(1+x^2)} dx = \int \frac{1}{1-x} dx + \int \frac{x}{1+x^2} dx + \int \frac{1}{1+x^2} dx$$

$$-log|1-x| + \frac{1}{2} log|1+x^2| + tan^{-1}x + c$$

40. Question

Evaluate:

$\int \frac{x^2}{(x^4-1)} dx$

Answer

Let $I = \int \frac{x^2}{(x^4-1)} dx$

Put t=x^2

dt=2xdx

Now putting, $\frac{x^2}{(x^4-1)} = \frac{t}{(t-1)(t+1)} = \frac{A}{t-1} + \frac{B}{t+1} \dots\dots (1)$

A(t+1)+B(t-1) = t

Putting t+1=0,

t=-1

A(0)+B(-1-1)=-1

$B = \frac{1}{2}$

Putting t-1=0,

t=1

A(1+1)+B(0)=1

$A=\frac{1}{2}$

From equation(1),we get,

$$\frac{t}{(t-1)(t+1)}=\frac{1}{2}\times\frac{1}{t-1}+\frac{1}{2}\times\frac{1}{t+1}$$

$$\int\frac{x^2}{(x^4-1)}dt=\frac{1}{2}\int\frac{1}{x^2-1}dt+\frac{1}{2}\int\frac{1}{x^2+1}dt$$

$$=\frac{1}{2}\times\frac{1}{2}\log\left|\frac{x-1}{x+1}\right|+\frac{1}{2}tan^{-1}x+c$$

$$=\frac{1}{4}\log\left|\frac{x-1}{x+1}\right|+\frac{1}{2}tan^{-1}x+c$$

59. Question

$\int\frac{2x^2+1}{x^2(x^2+4)}dx$

Answer

Let $I=\int\frac{2x^2+1}{x^2(x^2+4)}dx$

Again let x^2=t

$$\frac{2t+1}{t(t+4)}=\frac{A}{t}+\frac{B}{(t+4)}\ldots\ldots(1)$$

2t+1=A(t+4)+B(t)

Putting t=-4

2(-4)+1=A(-4+4)+B(-4)

-8+1=0-4B

-7=-4B

$B=\frac{7}{4}$

Putting t=0

2(0)+1=A(0+4)+B(0)

1=4A

$A=\frac{1}{4}$

$$\frac{2t+1}{t(t+4)}=\frac{\frac{1}{4}}{t}+\frac{\frac{7}{4}}{(t+4)}$$

$$\int\frac{2t+1}{t(t+4)}dt=\int\frac{2x^2+1}{x^2(x^2+4)}dx=\frac{1}{4}\int\frac{1}{x^2}dx+\frac{7}{4}\int\frac{1}{(x^2+2^2)}dx$$

$$=\frac{1}{4}\times\frac{(-1)}{x}+\frac{7}{4}\times\frac{1}{2}tan^{-1}\left(\frac{x}{2}\right)+c$$

$I = \frac{-1}{4x} + \frac{7}{8} tan^{-1}\left(\frac{x}{2}\right) + c$

Exercise 15B

1. Question

Evaluate:

$$\int x^{-6} dx$$

Answer

$\int x^{-6} dx = \frac{x^{-6+1}}{-6+1} + c$

$\because \left\{\int x^n = \frac{x^{n+1}}{n+1} + c\right\}$

$= \frac{x^{-5}}{-5} + c$

$\int x^{-6} dx = -\frac{1}{5x^5} + c$

2. Question

Evaluate:

$$\int\left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx$$

Answer

$\int (\sqrt{x} + 1/\sqrt{x}) dx = \int \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right) dx$

$\left\{\int x^n = \frac{x^{n+1}}{n+1} + c\right\}$

$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx = \int \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + \frac{x^{-\frac{1}{2}+1}}{-\frac{1}{2}+1} dx$

$\int \left(\sqrt{x} + \frac{1}{\sqrt{x}}\right) dx = \int \frac{2}{3} x^{\frac{3}{2}} + 2\sqrt{x} + c$

3. Question

Evaluate:

$$\int \sin 3x \, dx$$

Answer

$\int si\,n\, 3x\, dx = \frac{-1}{3} cos\, 3x + c$

$\left\{\int \sin ax\, dx = \frac{-1}{a} \cos ax\right\}$

4. Question

Evaluate:

$$\int \frac{x^2}{\left(1+x^3\right)} dx$$

Answer

Let $x^3 + 1 = t$

$3x^2 dx = dt$

$$\frac{1}{3}\int \frac{dt}{t} = \frac{1}{3} ln\, t + c$$

$$\int \frac{x^2}{1+x^3} dx = \frac{1}{3} ln(x^3+1) + c$$

5. Question

Evaluate:

$$\int \frac{2\cos x}{3\sin^2 x} dx$$

Answer

Let sin x=t

cos x dx=dt

$$\int \frac{2\cos x}{3\sin^2 x} dx = \int \frac{2}{3t^2} dt = -\frac{2}{3t} + c$$

$$\int \frac{2\cos x}{3\sin^2 x} dx = \frac{-2}{3} cs\,c x + c$$

6. Question

Evaluate:

$$\int \frac{\left(3\sin\phi - 2\right)\cos\phi}{\left(5-\cos^2\phi - 4\sin\phi\right)} d\phi$$

Answer

$$\frac{(3\sin \emptyset - 6 + 4)\cos\emptyset}{(4+1-cos^2\emptyset - 4\sin\emptyset)} = \frac{3(\sin\emptyset - 2)\cos\emptyset + 4\cos\emptyset}{(\sin\emptyset - 2)^2}$$

$$= \frac{3\cos\emptyset}{(\sin\emptyset - 2)} + \frac{4\cos\emptyset}{(\sin\emptyset - 2)^2}$$

$$\int \left(\frac{3\cos\emptyset}{(\sin\emptyset - 2)} + \frac{4\cos\emptyset}{(\sin\emptyset - 2)^2}\right) d\,\emptyset$$

Let (sin ∅-2)= t

cos ∅ d∅=dt

$$\int \frac{3dt}{t} + \frac{4dt}{t^2} = 3\ln t - \frac{4}{t} + c$$

$$\int \frac{(3si\,n\phi - 2)co\,s\,\phi}{(5-cos^2\phi - 4si\,n\,\phi)} d\phi = 3l\,n|si\,n\,\phi - 2| - \frac{4}{(si\,n\phi - 2)} + c$$

7. Question

Evaluate:

$$\int \sin^2 x \, dx$$

Answer

$$\int sin^2 x \, dx = \int \frac{1}{2} - \frac{\cos 2x}{2} \, dx$$

{1-cos 2x=2 $\sin^2$ x}

$$\int sin^2 x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + c$$

$$\left\{ \int \cos ax \, dx = \frac{1}{a} \sin ax \right\}$$

8. Question

Evaluate:

$$\int \frac{(\log x)^2}{x} dx$$

Answer

Let log x = t

$$\frac{1}{x} dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$\int \frac{(log\, x)^2}{x} dx = \frac{(log\, x)^3}{3} + c$$

9. Question

Evaluate:

$$\int \frac{(x+1)(x+\log x)^2}{x} dx$$

Answer

$$\int \frac{(x+1)(x+logx)]^2}{x} = \int \left(1 + \frac{1}{x}\right)(x + logx)^2 dx$$

Let $x + logx = t$

$$\left(1 + \frac{1}{x}\right) dx = dt$$

$$\int t^2 dt = \frac{t^3}{3} + c$$

$$\int \frac{(x+1)(x+logx)^2}{x} = \frac{(x+logx)^3}{3} + c$$

10. Question

Evaluate:

$$\int \frac{\sin x}{(1+\cos x)} dx$$

Answer

Let 1+cosx=t

-sin x dx=dt

$$\int \frac{-dt}{t} = -\ln t + c$$

$$\int \frac{\sin x}{(1+\cos x)} dx = -\ln|1+\cos x| + c$$

11. Question

Evaluate:

$$\int \frac{(1+\tan x)}{(1-\tan x)} dx$$

Answer

$$\frac{1+\tan x}{1-\tan x} = \frac{\cos x + \sin x}{\cos x - \sin x}$$

$$\int \frac{\cos x + \sin x}{\cos x - \sin x} dx$$

Let cos x-sin x=t

-(sin x + cos x)dx=dt

$$\int \frac{-dt}{t} = -\ln t + c$$

$$\int \frac{1+\tan x}{1-\tan x} dx = -\ln|\cos x - \sin x| + c$$

12. Question

Evaluate:

$$\int \frac{(1-\cot x)}{(1+\cot x)} dx$$

Answer

$$\frac{1-\cot x}{1+\cot x} = \frac{\sin x - \cos x}{\sin x + \cos x}$$

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx$$

Let sin x + cos x=t

(cos x-sin x)dx=dt

$$\int \frac{\sin x - \cos x}{\sin x + \cos x} dx = \int \frac{-dt}{t} = -\ln|\sin x + \cos x| + c$$

$$\int \frac{1-\cot x}{1+\cot x} dx = -\ln|\sin x + \cos x| + c$$

13. Question

Evaluate:

$$\int \frac{(1+\cot x)}{(x+\log \sin x)} dx$$

Answer

Let (x + log (sin x))=t

(1+cot x) dx=dt

$$\int \frac{dt}{t} = ln\, t + c$$

$$\int \frac{(1+\cot x)}{(x+\log \sin x)} = \ln|x+\log(\sin x)| + c$$

14. Question

Evaluate:

$$\int \frac{(1-\sin 2x)}{\left(x+\cos^2 x\right)} dx$$

Answer

Let (x + $\cos^2$ x)=t

(1-sin 2x) dx=dt

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{1-\sin 2x}{x+\cos^2 x} = ln(|x+\cos^2 x|) + c$$

15. Question

Evaluate:

$$\int \frac{\sec^2(\log x)}{x} dx$$

Answer

Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\int \sec^2 t\, dt = \tan t + c$$

$$\int \frac{\sec^2(\log x)}{x} dx = \tan(\log x) + c$$

16. Question

Evaluate:

$$\int \frac{\sin\left(2 \tan^{-1} x\right)}{\left(1+x^2\right)} dx$$

Answer

Let $\tan^{-1} x = t$

$\frac{1}{1+x^2}dx = dt$

$\int \sin 2t = -\frac{\cos 2t}{2} + c$

$\int \frac{sin(2\,tan^{-1} x)}{(1+x^2)}dx \ = \frac{-1}{2}\cos(2\tan^{-1} x) + c$

17. Question

Evaluate:

$$\int \frac{\tan x \sec^2 x}{\left(1-\tan^2 x\right)}dx$$

Answer

Let 1-tan^2 x=t

-2 tan x. sec^2 x dx=dt

$\frac{-1}{2}\int \frac{dt}{t} = \frac{-1}{2}logt + c$

$\int \frac{\tan x \sec^2 x}{} dx = \frac{-1}{2}log\,|1-tan^2 x| + c$

18. Question

Evaluate:

$$\int \frac{\left(x^4+1\right)}{\left(x^2+1\right)}dx$$

Answer

$\frac{x^4+1}{x^2+1} = \frac{x^4-1+2}{x^2+1}$

$= x^2 - 1 + \frac{2}{x^2+1}$

$\int \left(x^2 - 1 + \frac{2}{x^2+1}\right)dx \ = \frac{x^3}{3} - x + 2tan^{-1} x + c$

19. Question

Evaluate:

$$\int \tan^{-1}\sqrt{\frac{1-\sin x}{1+\sin x}}dx$$

Answer

$\sin x = \cos\left(\frac{\pi}{2} - x\right)$

$$tan^{-1}\sqrt{\frac{1-cos\left(\frac{\pi}{2}-x\right)}{1+cos\left(\frac{\pi}{2}-x\right)}} = tan^{-1}\sqrt{\frac{2sin^2(\frac{\pi}{4}-\frac{x}{2})}{2cos^2(\frac{\pi}{4}-\frac{x}{2})}}$$

$$= tan^{-1}\left(\left(tan\left(\frac{\pi}{4}-\frac{x}{2}\right)\right)\right)$$

$$\int\left(\frac{\pi}{4}-\frac{x}{2}\right)dx = \frac{\pi}{4}x-\frac{x^2}{4}+c$$

20. Question

Evaluate:

$$\int \log\left(1+x^2\right)dx$$

Answer

Using Integration by Parts

$$\int u_{II}v_{I}dx = u\int vdx - \int u'\int vdx\,dx + c$$

Here 1 is the first function and $\log(x^2+1)$ is second function

$$\int log(1+x^2)\,dx = (log(1+x^2))x - \int\frac{2x}{1+x^2}xdx$$

$$= (log(1+x^2))x - 2\int\frac{x^2+1-1}{x^2+1}dx$$

$$= (log(1+x\text{^}2))x - 2x + 2\ \tan^{-1} x + c$$

21. Question

Evaluate:

$$\int \cos x \cos 3x\, dx$$

Answer

$\frac{1}{2}\int 2\cos x\cos 3x\,dx$

{2 cos A cos B=cos(A+B)+cos(A-B)}

$$\frac{1}{2}\int(\cos 4x+\cos 2x)\,dx = \frac{\sin 4x}{8}+\frac{\sin 2x}{4}+c$$

22. Question

Evaluate:Evaluate $\int \sin 3x \sin x\, dx$

Answer

$\frac{1}{2}\int 2\sin 3x\sin x\,dx$

{ 2 sin A sin B=cos(A-B)-cos(A+B) }

$$\frac{1}{2}\int(cos\,2x - cos\,4x)dx = \frac{sin\,2x}{4}-\frac{sin\,4x}{8}+c$$

23. Question

Evaluate:

$$\int \frac{xe^x}{(x+1)^2} dx$$

Answer

$$\frac{e^x(x+1-1)}{(x+1)^2} = e^x\left(\frac{1}{x+1} - \frac{1}{(x+1)^2}\right)$$

$$\{\int (e^x(f(x) + f'(x))dx = e^x f(x) + c\}$$

$$\int \frac{xe^x}{(x+1)^2} dx = \frac{e^x}{x+1} + c$$

24. Question

Evaluate:

$$\int e^x \{\tan x - \log \cos x\} dx$$

Answer

$$\int (e^x(f(x) + f'(x))dx = e^x f(x) + c$$

Here f(x)=-log cos x

$\int e^x$ (tan x- log cos x)dx=-e^x (log cos x)+c

25. Question

Evaluate:

$$\int \frac{dx}{(1 - \sin x)}$$

Answer

Multiplying Num^r and Den^r with (1+sinx)

$$\int \frac{1 + \sin x}{\cos^2 x} dx = \int \sec^2 x + \sec x \tan x \, dx$$

= tan x + sec x + c

26. Question

Evaluate:

$$\int c \cos x^2 dx$$

Answer

Let x^2 =t

2xdx=dt

$$\frac{1}{2}\int \cos t \, dt = \frac{1}{2}\sin t + c$$

$$\int x \cos x^2 \, dx = \frac{1}{2}\sin x^2 + c$$

27. Question

Evaluate:

$$\int \frac{\cot x}{\sqrt{\sin x}} dx$$

Answer

$$\frac{\cot x}{\sqrt{\sin x}} = \frac{\cos x}{(\sin x)^{3/2}}$$

Let sin x=t

cos x dx=dt

$$\int \frac{dt}{t^{3/2}} = \frac{-2}{\sqrt{t}} + c$$

$$\int \frac{\cot x}{\sqrt{\sin x}} dx = \frac{-2}{\sqrt{\sin x}} + c$$

28. Question

Evaluate:

$$\int \frac{\sec^2 x}{\operatorname{cosec}^2 x} dx$$

Answer

$$\frac{sec^2 x}{cosec^2 x} = tan^2 x$$

$$\int tan^2 x\, dx = \int (sec^2 x - 1)\, dx$$

$$\int \frac{sec^2 x}{cosec^2 x} dx = \tan x - x + c$$

29. Question

Evaluate:

$$\int \sin^{-1}(\cos x) dx$$

Answer

$$\int sin^{-1}(cos\, x)\, dx = \int \left(\frac{\pi}{2} - \cos^{-1}(cosx)\right) dx$$

$$\int (\frac{\pi}{2} - x)\, dx = \frac{\pi}{2}x - \frac{x^2}{2} + c$$

30. Question

Evaluate:

$$\int \frac{dx}{\left(\sqrt{x+2} + \sqrt{x+1}\right)}$$

Answer

On rationalizing

$$\int \frac{dx}{(\sqrt{x+2} + \sqrt{x+1})} = \int \frac{\sqrt{x+2} - \sqrt{x+1}}{(\sqrt{x+2} + \sqrt{x+1})\sqrt{x+2} - \sqrt{x+1}} dx$$

$$= \int \frac{\sqrt{x+2} - \sqrt{x+1}}{(x+2-x-1)} dx$$

$$\int \frac{\sqrt{x+2} - \sqrt{x+1}}{1} dx = \frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{\frac{3}{2}} + c$$

31. Question

Evaluate:

$$\int 2^x \, dx$$

Answer

We know that,

$$\int a^x dx = \frac{a^x}{\ln a} + c$$

$$\int 2^x dx = \frac{2^x}{\ln 2} + c$$

.

32. Question

Evaluate:

$$\int \frac{(1 + \tan x)}{(x + \log \sec x)} dx$$

Answer

Let (x + log (sec x))=t

(1+tan x) dx=dt

$$\int \frac{dt}{t} = \ln t + c$$

$$\int \frac{(1 + \tan x)}{(x + \log \sec x)} = \ln|x + \log(\sec x)| + c$$

33. Question

Evaluate:

$$\int \frac{\sec^2(\log x)}{x} dx$$

Answer

Let $\log x = t$

$$\frac{1}{x} dx = dt$$

$$\int \sec^2 t dt = \tan t + c$$

$$\int \frac{\sec^2(\log x)}{x} dx = \tan(\log x) + c$$

34. Question

Evaluate:

$$\int (2x+1)\left(\sqrt{x^2+x+1}\right)dx$$

Answer

Let $x^2+x+1=t$

$(2x+1)dx=dt$

$$\int \sqrt{t}dt = \frac{2}{3}t^{3/2} + c = \frac{2}{3}(x^2+x+1)^{3/2} + c$$

35. Question

Evaluate:

$$\int \frac{dx}{\sqrt{9x^2+16}}$$

Answer

We know that,

$$\int \frac{dx}{\sqrt{(ax)^2+b^2}} = \frac{1}{a}\log\left|ax+\sqrt{(ax)^2+b^2}\right| + c$$

$$\int \frac{dx}{\sqrt{(3x)^2+4^2}} = \frac{1}{3}\log\left|3x+\sqrt{9x^2+16}\right| + c$$

36. Question

Evaluate:

$$\int \frac{dx}{\sqrt{4-9x^2}}$$

Answer

We know that,

$$\int \frac{dx}{\sqrt{b^2-(ax)^2}} = \frac{1}{a}\sin^{-1}\frac{ax}{b} + c$$

$$\int \frac{dx}{\sqrt{2^2-(3x)^2}} = \frac{1}{3}\sin^{-1}\frac{3x}{2} + c$$

37. Question

Evaluate:

$$\int \frac{dx}{\sqrt{4x^2-25}}$$

Answer

We know that,

$$\int \frac{dx}{\sqrt{(ax)^2-b^2}} = \frac{1}{a}\log\left|ax+\sqrt{(ax)^2-b^2}\right| + c$$

$$\int \frac{dx}{\sqrt{(2x)^2-5^2}} = \frac{1}{2}\log\left|2x+\sqrt{4x^2-25}\right| + c$$

38. Question

Evaluate:

$$\int \sqrt{4-x^2}\,dx$$

Answer

We know that,

$$\int \sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}+c$$

$$\int \sqrt{2^2-x^2}\,dx = \frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}+c$$

39. Question

Evaluate:

$$\int \sqrt{9+x^2}\,dx$$

Answer

We know that,

$$\int \sqrt{a^2+x^2}\,dx = \frac{x}{2}\sqrt{a^2+x^2}+\frac{a^2}{2}\log\left|x+\sqrt{a^2+x^2}\right|+c$$

$$\int \sqrt{3^2+x^2}\,dx = \frac{x}{2}\sqrt{9+x^2}+\frac{9}{2}\log\left|x+\sqrt{9+x^2}\right|+c$$

40. Question

Evaluate:

$$\int \sqrt{x^2-16}\,dx$$

Answer

We know that,

$$\int \sqrt{x^2-a^2}\,dx = \frac{x}{2}\sqrt{x^2-a^2}-\frac{a^2}{2}\log\left|x+\sqrt{x^2-a^2}\right|+c$$

$$\int \sqrt{x^2-4^2}\,dx = \frac{x}{2}\sqrt{x^2-16}-8\log\left|x+\sqrt{x^2-16}\right|+c$$

Objective Questions I

1. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(9+x^2\right)} = ?$$

A. $\tan^{-1}\frac{x}{3}+C$

B. $\frac{1}{3}\tan^{-1}\frac{x}{3}+C$

C. $3\tan^{-1}\frac{x}{3}+C$

D. none of these

Answer

$$= \int \frac{dx}{x^2 + 3^2}$$

We know, $\int \frac{dx}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{3}\tan^{-1}\frac{x}{3} + c$$

2. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4+16x^2\right)} = ?$$

A. $\frac{1}{32}\tan^{-1}4x + C$

B. $\frac{1}{16}\tan^{-1}\frac{x}{2} + C$

C. $\frac{1}{8}\tan^{-1}2x + C$

D. $\frac{1}{4}\tan^{-1}\frac{x}{2} + C$

Answer

$$= \int \frac{dx}{(4x)^2 + 2^2}$$

4x=t

4dx=dt

$$dx = \frac{dt}{4}$$

$$= \frac{1}{4}\int \frac{dt}{t^2 + 2^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{1}{8}\tan^{-1}\frac{t}{2} + c$$

put t=4x

$$= \frac{1}{8}\tan^{-1}\frac{4x}{2} + c$$

$$= \frac{1}{8}\tan^{-1}2x + c$$

3. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(9+4x^2\right)} dx = ?$$

A. $\frac{1}{2}\tan^{-1}\frac{2x}{3}+C$

B. $\frac{1}{6}\tan^{-1}\frac{2x}{3}+C$

C. $\frac{1}{6}\tan^{-1}\frac{3x}{2}+C$

D. none of these

Answer

$$\int \frac{dx}{(2x)^2+3^2}$$

2x=t

2dx=dt

$$dx=\frac{dt}{2}$$

$$=\frac{1}{2}\int \frac{dt}{t^2+3^2}$$

We know, $\int \frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=\frac{1}{6}\tan^{-1}\frac{t}{3}+c$$

put t=2x

$$=\frac{1}{6}\tan^{-1}\frac{2x}{3}+c$$

4. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin x}{\left(1+\cos^2 x\right)} dx = ?$$

A. $-\tan^{-1}(\cos x)+C$

B. $\cot^{-1}(\cos x)+C$

C. $-\cot^{-1}(\cos x)+C$

D. $\tan^{-1}(\cos x)+C$

Answer

$$\int \frac{\sin x}{(\cos x)^2+1^2}\,dx$$

cos x=t

-sin x dx=dt

$$= -\int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= -\tan^{-1}\frac{t}{1} + c$$

put t=cos x

$= -\tan^{-1}(\cos x)+c$

5. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\cos x}{(1+\sin^2 x)} dx = ?$$

A. $-\tan^{-1}(\sin x)+C$

B. $\tan^{-1}(\cos x)+C$

C. $\tan^{-1}(\sin x)+C$

D. $-\tan^{-1}(\cos x)+C$

Answer

$$\int \frac{\cos x}{(\sin x)^2 + 1^2} dx$$

sin x=t

cos x dx=dt

$$= \int \frac{dt}{t^2 + 1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \tan^{-1}\frac{t}{1} + c$$

put t=sin x

$= \tan^{-1}(\sin x)+c$

6. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{e^x}{(e^{2x}+1)} dx = ?$$

A. $\cot^{-1}(e^x)+C$

B. $\tan^{-1}(e^x)+C$

C. $2\tan^{-1}\left(e^{x}\right)+C$

D. none of these

Answer

$$=\int \frac{e^x}{(e^x)^2+1^2}\,dx$$

$e^x = t$

$e^x\,dx = dt$

$$=\int \frac{dt}{t^2+1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=\tan^{-1}\frac{t}{1}+c$$

put $t=e^x$

$\tan^{-1} e^x + c$

7. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{3x^5}{\left(1+x^{12}\right)}dx = ?$$

A. $\tan^{-1}x^6 + C$

B. $\frac{1}{4}\tan^{-1}x^6 + C$

C. $\frac{1}{2}\tan^{-1}x^6 + C$

D. none of these

Answer

$$=\int \frac{3x^5}{(x^6)^2+1^2}\,dx$$

Let $x^6 = t$

$6x^5\,dx = dt$

$$3x^5\,dx = \frac{dt}{2}$$

$$=\frac{1}{2}\int \frac{dt}{t^2+1^2}$$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=\frac{1}{2}\tan^{-1}\frac{t}{1}+c$$

put $t=x^6$

$= \frac{1}{2}\tan^{-1}\frac{x^6}{1} + c$

$= \frac{1}{2}\tan^{-1} x^6 + c$

8. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{2x^3}{\left(4+x^8\right)} dx = ?$

A. $\frac{1}{2}\tan^{-1}\frac{x^4}{2} + C$

B. $\frac{1}{4}\tan^{-1}\frac{x^4}{2} + C$

C. $\frac{1}{2}\tan^{-1} x^4 + C$

D. none of these

Answer

$= \int \frac{2x^3}{(x^4)^2 + 2^2} dx$

Let x^4=t

$4x^3$ dx=dt

$2x^3\ dx = \frac{dt}{2}$

$= \frac{1}{2}\int \frac{dt}{t^2 + 2^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{1}{4}\tan^{-1}\frac{t}{2} + c$

put $t=x^4$

$= \frac{1}{4}\tan^{-1}\frac{x^4}{2} + c$

9. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\left(x^2+4x+8\right)} = ?$

A. $\frac{1}{2}\tan^{-1}\left(\frac{x+1}{2}\right) + C$

B. $\frac{1}{2}\tan^{-1}\left(\frac{x+2}{2}\right)+C$

C. $\frac{1}{2}\tan^{-1}(x+2)+C$

D. $\tan^{-1}\left(\frac{x+2}{2}\right)+C$

Answer

$= \int \frac{dx}{x^2+4x+8}$

Completing the square

$x^2 + 4x + 8 = x^2 + 4x + 8\ (+4-4)$

$= x^2 + 4x + 4 + 4$

$= (x+2)^2 + 2^2$

$= \int \frac{dx}{(x+2)^2+2^2}$

Let x+2=t

dx=dt

$= \int \frac{dt}{t^2+2^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{1}{2}\tan^{-1}\frac{t}{2} + c$

put t=x+2

$= \frac{1}{2}\tan^{-1}\frac{x+2}{2} + c$

10. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\left(2x^2+x+3\right)} = ?$

A. $\frac{1}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$

B. $\frac{1}{\sqrt{23}}\tan^{-1}\left(\frac{x+1}{\sqrt{23}}\right)+C$

C. $\frac{2}{\sqrt{23}}\tan^{-1}\left(\frac{4x+1}{\sqrt{23}}\right)+C$

D. none of these

Answer

$= \int \frac{dx}{2x^2 + x + 3}$

Completing the square

$\Rightarrow 2x^2 + x + 3 \;= 2x^2 + \frac{1}{2}x + \frac{3}{2})$

$= 2(x^2 + \frac{1}{2}x + \frac{3}{2} + \frac{1}{16} - \frac{1}{16})$

$= 2(\,(x + \frac{1}{4})^2 + \frac{23}{16})$

$= \frac{1}{2}\int \frac{dx}{(\,(x+\frac{1}{4})^2 + \frac{23}{16})}$

$Let\ x + \frac{1}{4} = t$

dx=dt

$= \int \frac{dt}{t^2 + \frac{\sqrt{23}}{4}^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{4}{2\sqrt{23}}\tan^{-1}\frac{t}{\frac{\sqrt{23}}{4}} + c$

$put\ t = x + \frac{1}{4}$

$= \frac{2}{\sqrt{23}}\tan^{-1}\frac{x+\frac{1}{4}}{\frac{\sqrt{23}}{4}} + c$

$= \frac{2}{\sqrt{23}}\tan^{-1}\frac{4x+1}{\sqrt{23}} + c$

11. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\left(e^x + e^{-x}\right)} = ?$

A. $\tan^{-1}\left(e^x\right) + C$

B. $\tan^{-1}\left(e^{-x}\right) + C$

C. $-\tan^{-1}\left(e^{-x}\right) + C$

D. none of these

Answer

$= \int \frac{1}{e^x + e^{-x}}\, dx$

$= \int \frac{e^x}{(e^x)^2 + 1^2}\, dx$

e^x =t e^x

e^x dx=dt

$= \int \frac{dt}{t^2 + 1^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \tan^{-1}\frac{t}{1} + c$

put t= e^x

= $\tan^{-1} e^x$ +c

12. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{x^2}{\left(9+4x^2\right)} = ?$

A. $\frac{x}{4} - \frac{1}{8}\tan^{-1}\frac{x}{3} + C$

B. $\frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{x}{3} + C$

C. $\frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{2x}{3} + C$

D. none of these

Answer

$\int \frac{x^2}{4x^2+9} = \frac{1}{4}\int \frac{4x^2+9-9}{4x^2+9}\, dx$

$= \frac{1}{4}\int 1 + \frac{1}{4}\int \frac{-9}{4x^2+9} dx$

$= \frac{x}{4} - \frac{9}{4}\int \frac{1}{(2x)^2+3^2} dx$

Let 2x=t

2 dx=dt

$= \frac{x}{4} - \frac{9}{8}\int \frac{1}{(t)^2+3^2} dx$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{x}{4} - \frac{9}{4.2.3}\tan^{-1}\frac{t}{3} + c$

put t=2x

$= \frac{x}{4} - \frac{3}{8}\tan^{-1}\frac{2x}{3} + c$

13. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\left(x^2-1\right)}{\left(x^2+4\right)} dx = ?$$

A. $x - 5\tan^{-1}\frac{x}{2} + C$

B. $x - \frac{5}{2}\tan^{-1}\frac{x}{2} + C$

C. $x - \frac{5}{2}\tan^{-1}\frac{5x}{2} + C$

D. none of these

Answer

$$\int \frac{x^2-1}{x^2+4} = \int \frac{x^2}{x^2+4} - \int \frac{1}{x^2+4}$$

$$= \int \frac{x^2}{x^2+4} - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= \int \frac{x^2+4-4}{x^2+4} - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= \int (1 - \frac{4}{x^2+4}) - \frac{1}{2}\tan^{-1}\frac{x}{2}$$

$$= x - 2\tan^{-1}\frac{x}{2} - \frac{1}{2}\tan^{-1}\frac{x}{2} + c$$

$$= x - \frac{5}{2}\tan^{-1}\frac{x}{2} + c$$

14. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(4+9x^2\right)} = ?$$

A. $\frac{2}{3}\tan^{-1}\frac{3x}{2} + C$

B. $\frac{1}{6}\tan^{-1}3x + C$

C. $\frac{1}{6}\tan^{-1}\frac{3x}{2} + C$

D. none of these

Answer

Consider $\int \frac{dx}{(3x)^2+2^2}$,

3x=t

3dx=dt

$dx = \frac{dt}{3}$

$= \frac{1}{3}\int \frac{dt}{t^2 + 2^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{1}{6}\tan^{-1}\frac{t}{2} + c$

put t=3x

$= \frac{1}{6}\tan^{-1}\frac{3x}{2} + c$

15. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{dx}{\left(4x^2 - 4x + 3\right)} = ?$

A. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

B. $\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

C. $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{2x-1}{\sqrt{2}}\right) + C$

D. none of these

Answer

Consider $\int \frac{dx}{4x^2-4x+3}$,

Completing the square

$4x^2 - 4x + 3 = 4(x^2 - x + \frac{3}{4})$

$= 4(x^2 - x + \frac{3}{4} + \frac{1}{4} - \frac{1}{4})$

$= 4(\,(x - \frac{1}{2})^2 + \frac{1}{2})$

$= \frac{1}{4}\int \frac{dx}{(\,(x-\frac{1}{2})^2 + \frac{1}{2})}$

$Let\; x - \frac{1}{2} = t$

dx=dt

$= \frac{1}{4}\int \frac{dt}{t^2 + \frac{1}{\sqrt{2}}^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$$= \frac{\sqrt{2}}{4}\tan^{-1}\frac{t}{\frac{1}{\sqrt{2}}} + c$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\sqrt{2}t + c$$

put t=x- �

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\frac{2x-1}{\sqrt{2}} + c$$

16. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{dx}{\left(\sin^4 x + \cos^4 x\right)} = ?$$

A. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x - 1}{\sqrt{2}\tan x}\right) + C$

B. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{\tan^2 x - 1}{\tan x}\right) + C$

C. $\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}\tan x}\right) + C$

D. None of these

Answer

$$\int \frac{dx}{sin^4x + cos^4x} = \int \frac{1}{cos^4x(tan^4x+1)}\,dx$$

$$= \int \frac{sec^4x}{tan^4x+1}\,dx$$

$$= \int \frac{sec^2x\,sec^2x}{tan^4x+1}\,dx$$

$$= \int \frac{sec^2x\,(1 + tan^2x)}{tan^4x+1}\,dx$$

tan x=t

sec^2 x dx=dt

$$= \int \frac{1 + t^2}{t^4+1}\,dt$$

$$= \int \frac{t^2+1}{t^4+1}\,dt$$

$$= \int \frac{1+t^{-2}}{t^2+t^{-2}}dt$$

$$= \int \frac{1+t^{-2}}{t^2+t^{-2}+2-2}dt$$

$= \int \frac{1+t^{-2}}{(t-t^{-1})^2+2} dt$

Let t-t^{-1} =u

1+x^{-2} dt=du

$= \int \frac{du}{(u)^2+\sqrt{2}^2}$

We know, $\int \frac{1}{x^2+a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a} + c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}} + c$

put u=t-t^{-1}

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t-t^{-1}}{\sqrt{2}} + c$

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{t^2-1}{\sqrt{2}t} + c$

put t=tan x

$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{tan^2 x-1}{\sqrt{2}\tan x} + c$

17. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\left(x^2+1\right)}{\left(x^4+x^2+1\right)} dx = ?$

A. $\tan^{-1}\frac{\left(x^2-1\right)}{\sqrt{3}} + C$

B. $\frac{1}{\sqrt{3}}\tan^{-1}\frac{\left(x^2-1\right)}{\sqrt{3}} + C$

C. $\frac{1}{\sqrt{3}}\tan^{-1}\frac{\left(x^2-1\right)}{\sqrt{3}x} + C$

D. none of these

Answer

$\int \frac{(x^2+1)}{(x^4+x^2+1)} dx = \int \frac{1+x^{-2}}{x^2+1+x^{-2}} dx$

$= \int \frac{1+x^{-2}}{x^2+1+x^{-2}+2-2} dx$

$= \int \frac{1+x^{-2}}{(x-x^{-1})^2+3} dx$

Let x-x^{-1} =t

$1+x^{-2}\,dx=dt$

$$=\int \frac{dt}{(t)^2+\sqrt{3}^2}$$

We know, $\int \frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\frac{t}{\sqrt{3}}+c$$

put $t=x-x^{-1}$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\frac{x-x^{-1}}{\sqrt{3}}+c$$

$$=\frac{1}{\sqrt{3}}\tan^{-1}\frac{x^2-1}{\sqrt{3}x}+c$$

18. Question

Mark (√) against the correct answer in each of the following:

$$\int \frac{\sin\ 2x}{\left(\sin^4 x+\cos^4 x\right)}dx=?$$

A. $\tan^{-1}(\tan^2 x)+C$

B. x^2+C

C. - tan-1 $(\tan^2 x)$ +C

D. none of these

Answer

$$\int \frac{\sin 2x}{sin^4x+cos^4x}dx=\int \frac{2\sin x\cos x}{cos^4x(tan^4x+1)}dx$$

$$=\int \frac{2\tan x\ sec^2x}{(tan^2x)^2+1}dx$$

$$=\int \frac{2\tan x\ sec^2x}{(sec^2x-1)^2+1}dx$$

Let $\sec^2 x-1=t$

$2\sec x\sec x\tan x\,dx=dt$

$$=\int \frac{dt}{(t)^2+1}$$

We know, $\int \frac{1}{x^2+a^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

$=\tan^{-1} t+c$

put $t=\sec^2 x-1$

$=\tan^{-1}\sec^2 x-1+c$

$=\tan^{-1}\tan^2 x+c$

19. Question

Mark (√) against the correct answer in each of the following: