EXERCISE 19.7

Integrate the following integrals:

1. $\int \sin 4x \cos 7x \, dx$

Solution:

Given

$$\int \sin 4x \cos 7x \, dx$$

We know that 2 Sin A cos B = sin (A + B) + sin (A – B)

Now by substituting this formula in given question we get

$\therefore$ Sin 4x cos 7x = $\frac{\sin 11x + \sin(-3x)}{2}$

We know sin (– θ) = – sin θ

Hence Sin (– 3x) = – sin3x

$\therefore$ the above equation becomes

$$\Rightarrow \int \frac{1}{2}(\sin 11x - \sin 3x)\,dx$$

$$\Rightarrow \frac{1}{2}(\int \sin 11x\,dx - \int \sin 3x\,dx)$$

We know $\int \sin ax\,dx = \frac{-1}{a}\cos ax + c$

$$\Rightarrow \frac{1}{2}\left(\frac{-1}{11}\cos 11x + \frac{1}{3}\cos 3x\right)$$

$$= -\frac{1}{22}\cos 11x + \frac{1}{6}\cos 3x + c$$

2. $\int \cos 3x \cos 4x \, dx$

Solution:

Given

$$\int \cos 4x \cos 3x \, dx$$

Multiply and divide the given equation by 2

$$= \frac{1}{2}\int 2\cos 4x \ \cos \ 3x \ dx$$

We know that 2 cos A cos B = cos (A + B) + cos (A – B)

$$= \frac{1}{2}\int [\cos (4x + 3x) + \cos \ (4x - 3x)]\, dx$$

Now by simplifying we get

$$= \frac{1}{2}\int (\cos (7x) + \cos x)\, dx$$

On integration we get

$$= \frac{1}{2}\left[\frac{\sin 7x}{7} + \sin x\right] + C$$

$$= \frac{1}{14}\sin 7x + \frac{1}{2}\sin x + C$$

3. $\int$ **cos mx cos nx dx, m ≠ n**

Solution:

Given

$$\int \cos mx \cos nx \, dx, \, m \neq n$$

We know 2cosAcosB = cos (A – B) + cos (A + B)

Now substituting the above formula we get,

∴ Cos mx cos n x = $\frac{\cos(m-n)x + \cos(m+n)x}{2}$

∴ The above equation becomes

$\Rightarrow \int \frac{1}{2}(\cos(m-n)x + \cos(m+n)x)dx$

We know $\int \cos ax\,dx = \frac{1}{a}\sin ax + c$

Applying the above

$\Rightarrow \frac{1}{2}\left(\frac{1}{m-n}\sin(m-n)x + \frac{1}{m+n}\sin(m+n)x\right)$

$\Rightarrow \frac{1}{2}\left(\frac{(m+n)\sin(m-n)x + (m-n)\sin(m+n)x}{m^2-n^2}\right) + c$

We know that $a^2 - b^2 = (a + b)(a - b)$

By substituting the above formula and simplifying we get

$\frac{1}{2}\left\{\frac{\sin(m+n)x}{m+n} + \frac{\sin(m-n)x}{m-n}\right\} + c$