

# NCERT Solutions for Class-XII Maths

## Chapter-9.4

### NCERT Math Class 12

1.  $\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$

1. The given differential equation is:

$$\frac{dy}{dx} = \frac{1 - \cos x}{1 + \cos x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{2 \sin^2 \frac{x}{2}}{2 \cos^2 \frac{x}{2}} = \tan^2 \frac{x}{2}$$

$$\Rightarrow \frac{dy}{dx} = \left( \sec^2 \frac{x}{2} - 1 \right)$$

Separating the variables, we get:

$$dy = \left( \sec^2 \frac{x}{2} - 1 \right) dx$$

Now, integrating both sides of this equation, we get:

$$\int dy = \int \left( \sec^2 \frac{x}{2} - 1 \right) dx = \int \sec^2 \frac{x}{2} dx - \int dx$$

$$\Rightarrow y = 2 \tan \frac{x}{2} - x + C$$

This is the required general solution of the given differential equation.

2.  $\frac{dy}{dx} = \sqrt{4 - y^2} \quad (-2 < y < 2)$

2.  $\Rightarrow \frac{dy}{dx} = \sqrt{4 - y^2}$

$$\Rightarrow \frac{dy}{\sqrt{4 - y^2}} = dx$$

$$\Rightarrow \int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}$$

$$\Rightarrow \sin^{-1} \frac{y}{2} = x + c$$

3.  $\frac{dy}{dx} + y = 1 (y \neq 1)$

3. The given differential equation is:

$$\frac{dy}{dx} + y = 1$$

$$\Rightarrow dy + ydx = dx$$

$$\Rightarrow dy = (1 - y)dx$$

Separating the variables, we get:

$$\Rightarrow \frac{dy}{1 - y} = dx$$

Now, integrating both sides, we get:

$$\int \frac{dy}{1 - y} = \int dx$$

$$\Rightarrow \log(1 - y) = x + \log C$$

$$\Rightarrow -\log C - \log(1 - y) = x$$

$$\Rightarrow \log C(1 - y) = -x$$

$$\Rightarrow C(1 - y) = e^{-x}$$

$$\Rightarrow 1 - y = \frac{1}{C} e^{-x}$$

$$\Rightarrow y = 1 - \frac{1}{C} e^{-x}$$

$$\Rightarrow y = 1 + Ae^{-x} \left( \text{where } A = -\frac{1}{C} \right)$$

This is the required general solution of the given differential equation.

4.  $\sec^2 x \tan y dx + \sec^2 y \tan x dy = 0$

4.  $\Rightarrow \sec^2 x \tan y dx + \sec^2 y \tan x dy$

Dividing both sides by  $(\tan x)(\tan y)$

$$\therefore \frac{\sec^2 x \tan y dx}{\tan x \tan y} + \frac{\sec^2 y \tan x dy}{\tan x \tan y} = 0$$

$$\Rightarrow \frac{\sec^2 x dx}{\tan x} + \frac{\sec^2 y dy}{\tan y} = 0$$

Integrating both sides,

$$\Rightarrow \int \frac{\sec^2 x dx}{\tan x} = \int \frac{\sec^2 y dy}{\tan y}$$

$\Rightarrow$  let  $\tan x = t$  &  $\tan y = u$

$$\therefore \sec^2 x dx = dt \text{ \& } \sec^2 y dy = du$$

$$\therefore \int \frac{dt}{t} = - \int \frac{du}{u}$$

$$\Rightarrow \log t = -\log u + \log c$$

Or,

$$\Rightarrow \log(\tan x) = -\log(\tan y) + \log c$$

$$\Rightarrow \log \tan x = \log \frac{c}{\tan y}$$

Or

$$\Rightarrow (\tan x) (\tan y) = c$$

5.  $(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$

5. The given differential equation is:

$$(e^x + e^{-x})dy - (e^x - e^{-x})dx = 0$$

$$\Rightarrow (e^x + e^{-x})dy = (e^x - e^{-x})dx$$

$$\Rightarrow dy = \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx$$

Integrating both sides of this equation, we get:

$$\int dy = \int \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx + C$$

$$\Rightarrow y = \int \left[ \frac{e^x - e^{-x}}{e^x + e^{-x}} \right] dx + C \quad \dots(1)$$

Let  $(e^x + e^{-x}) = t$ .

Differentiating both sides with respect to x, we get:

$$\frac{d}{dx}(e^x + e^{-x}) = \frac{dt}{dx}$$

$$\Rightarrow e^x - e^{-x} = \frac{dt}{dx}$$

$$\Rightarrow (e^x - e^{-x})dx = dt$$

Substituting this value in equation (1), we get:

$$\Rightarrow y = \int \frac{1}{t} dt + C$$

$$\Rightarrow y = \log(t) + C$$

$$\Rightarrow y = \log(e^x + e^{-x}) + C$$

This is the required general solution of the given differential equation.

6.  $\frac{dy}{dt} = (1+x^2)(1+y^2)$

6.  $\Rightarrow \frac{dy}{dx} = (1+x^2)(1+y^2)$

Separating variables,

$$\Rightarrow \frac{dy}{1+y^2} = dx(1+x^2)$$

Integrating both sides,

$$\Rightarrow \int \frac{dy}{1+y^2} = \int dx + \int x^2 dx$$

$$\Rightarrow \tan^{-1} y = x + \frac{x^3}{3} + c$$

7.  $y \log y dx - x dy = 0$

7. The given differential equation is:

$$y \log y dx - x dy = 0$$

$$\Rightarrow y \log y dx - x dy$$

$$\Rightarrow \frac{dy}{y \log y} = \frac{dx}{x}$$

Integrating both sides, we get:

$$\int \frac{dy}{y \log y} = \int \frac{dx}{x} \quad \dots(1)$$

Let  $\log y = t$ .

$$\therefore \frac{d}{dy}(\log y) = \frac{dt}{dy}$$

$$\Rightarrow \frac{1}{y} = \frac{dt}{dy}$$

$$\Rightarrow \frac{1}{y} dy = dt$$

Substituting this value in equation (1), we get:

8.  $x^5 \frac{dy}{dx} = -y^5$

8.  $\Rightarrow x^5 \frac{dy}{dx} = -y^5$

Separating variables,

$$\Rightarrow \frac{dy}{y^5} = \frac{-dx}{x^5}$$

Or,

$$\Rightarrow \frac{dy}{y^5} + \frac{dx}{x^5} = 0$$

Integrating both sides,

$$\Rightarrow \int \frac{dy}{y^5} + \int \frac{dx}{x^5} = a$$

Let "a" be a constant,

$$\Rightarrow \int y^{-5} dy + \int x^{-5} dx = a$$

$$\Rightarrow -4y^{-4} - 4x^{-4} + c = a$$

Or,

$$\Rightarrow -x^{-4} - y^{-4} = c$$

Or,

$$\Rightarrow \frac{1}{x^4} + \frac{1}{y^4} = c$$

9.  $\frac{dy}{dx} = \sin^{-1} x$

9. The given differential equation is:

$$\frac{dy}{dx} = \sin^{-1} x$$

$$\Rightarrow dy = \sin^{-1} x dx$$

Integrating both sides, we get:

$$\int dy = \int \sin^{-1} x dx$$

$$\Rightarrow y = \int (\sin^{-1} x \cdot 1) dx$$

$$\Rightarrow y = \sin^{-1} x \cdot \int (1) dx - \int \left[ \left( \frac{d}{dx} (\sin^{-1} x) \right) \cdot \int (1) dx \right] dx$$

$$\Rightarrow y = \sin^{-1} x \cdot x - \int \left( \frac{1}{\sqrt{1-x^2}} \cdot x \right) dx$$

$$\Rightarrow y = x \sin^{-1} x + \int \frac{-x}{\sqrt{1-x^2}} dx \quad \dots(1)$$

Let  $1 - x^2 = t$ .

$$\Rightarrow \frac{d}{dx} (1 - x^2) = \frac{dt}{dx}$$

$$\Rightarrow -2x = \frac{dt}{dx}$$

$$\Rightarrow x dx = -\frac{1}{2} dt$$

Substituting this value in equation (1), we get:

$$y = x \sin^{-1} x + \int \frac{1}{2\sqrt{t}} dt$$

$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2} \cdot \int (t)^{-\frac{1}{2}} dt$$

$$\Rightarrow y = x \sin^{-1} x + \frac{1}{2} \cdot \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$$

$$\Rightarrow y = x \sin^{-1} x + \sqrt{t} + C$$

$$\Rightarrow y = x \sin^{-1} x + \sqrt{1-x^2} + C$$

This is the required general solution of the given differential equation.

10.  $e^x \tan y dx + (1 - e^x) \sec^2 y dy = 0$

10.  $\Rightarrow e^x \tan y dx + 1(1 - e^x) \sec^2 y dy = 0$

$$\Rightarrow (1 - e^x) \sec^2 y dy = -e^x \tan y dy = 0$$

Separating the variables,

$$\Rightarrow \frac{\sec^2 y}{\tan y} dy = -\frac{e^x}{1-e^x} dx$$

Now Integrating both sides,

$$\Rightarrow \int \frac{\sec^2 y}{\tan y} dy = \int \frac{e^{-x}}{1-e^x} dx$$

Let  $\tan y = t$  &  $1 - e^x = u$

Then,

$$(\sec^2 y dy = dt) \& (e^x dx = du)$$

Then,

$$\therefore \int \frac{dt}{t} = \int \frac{du}{u}$$

Or,

$$\Rightarrow \log t = \log u + \log c$$

Substituting the values of  $t$  and  $u$  on above equation.

$$\Rightarrow \log(\tan y) = \log(1 - e^x) + \log c$$

$$\Rightarrow \log \tan y = \log c(1 - e^x)$$

Or,

$$\Rightarrow \tan y = c(1 - e^x)$$

11.  $(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x; y = 1$  when  $x = 0$

11. The given differential equation is:

$$(x^3 + x^2 + x + 1) \frac{dy}{dx} = 2x^2 + x$$

$$\Rightarrow \frac{dy}{dx} = \frac{2x^2 + x}{(x^3 + x^2 + x + 1)}$$

$$\Rightarrow dy = \frac{2x^2 + x}{(x+1)(x^2+1)} dx$$

Integrating both sides, we get:

$$\int dy = \int \frac{2x^2 + x}{(x+1)(x^2+1)} dx \quad \dots(1)$$

$$\text{Let } \frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}. \quad \dots(2)$$

$$\Rightarrow \frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{Ax^2 + A + (Bx+C)(x+1)}{(x+1)(x^2+1)}$$

$$\Rightarrow 2x^2 + x = Ax^2 + A + Bx^2 + Bx + Cx + C$$

$$\Rightarrow 2x^2 + x = (A+B)x^2 + (B+C)x + (A+C)$$

Comparing the coefficients of  $x^2$  and  $x$ , we get:

$$A + B = 2$$

$$B + C = 1$$

$$A + C = 0$$

Solving these equations, we get:

$$\frac{2x^2 + x}{(x+1)(x^2+1)} = \frac{1}{2} \cdot \frac{1}{x+1} + \frac{1(3x-1)}{2(x^2+1)}$$

Therefore, equation (1) becomes:

$$\begin{aligned} \int dy &= \frac{1}{2} \int \frac{1}{x+1} dx + \frac{1}{2} \int \frac{3x-1}{x^2+1} dx \\ \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{2} \int \frac{x}{x^2+1} dx - \frac{1}{2} \int \frac{1}{x^2+1} dx \\ \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{4} \int \frac{2x}{x^2+1} dx - \frac{1}{2} \tan^{-1} x + C \\ \Rightarrow y &= \frac{1}{2} \log(x+1) + \frac{3}{4} \log(x^2+1) - \frac{1}{2} \tan^{-1} x + C \\ \Rightarrow y &= \frac{1}{4} [2 \log(x+1) + 3 \log(x^2+1)] - \frac{1}{2} \tan^{-1} x + C \\ \Rightarrow y &= \frac{1}{4} [(x+1)^2 (x^2+1)^3] - \frac{1}{2} \tan^{-1} x + C \quad \dots(3) \end{aligned}$$

Now,  $y = 1$  when  $x = 0$ .

$$\Rightarrow 1 = \frac{1}{4} \log(1) - \frac{1}{2} \tan^{-1} 0 + C$$

$$\Rightarrow 1 = \frac{1}{4} \times 0 - \frac{1}{2} \times 0 + C$$

$$\Rightarrow C = 1$$

Substituting  $C = 1$  in equation (3), we get:

$$y = \frac{1}{4} [\log(x+1)^2 (x^2+1)^3] - \frac{1}{2} \tan^{-1} x + 1$$

12.  $x(x^2 - 1) \frac{dy}{dx} = 1; y = 0$  when  $x = 2$

12.  $x(x^2 + 1) \frac{dy}{dx} = 1$

Separating variables,

$$\Rightarrow dy = \frac{dx}{x(x^2+1)}$$

Or,

$$\Rightarrow dy = \frac{dx}{x(x+1)(x-1)}$$

Integrating both sides,

$$\Rightarrow \int dy = \int \frac{dx}{x(x+1)(x-1)} \quad - i)$$

Now let,

$$\Rightarrow \frac{1}{x(x+1)(x-1)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-1} \quad - ii)$$

$$\Rightarrow \frac{1}{x(x+1)(x-1)} = \frac{A(x-1)(x+1)+B(x)(x-1)+C(x)(x+1)}{x(x+1)(x-1)}$$

Or

$$\Rightarrow \frac{1}{x(x+1)(x-1)} = \frac{(A+B+C)x^2+(B-C)x-A}{x(x+1)(x-1)}$$

Now comparing the values of A,B,C

$$A+B+C=0$$

$$B-C=0$$

$$A=-1$$

Solving these we will get that  $B = \frac{1}{2}$  and  $C = \frac{1}{2}$

Now putting the values of A,B,C in ii)

$$\Rightarrow \frac{1}{x(x+1)(x-1)} = -\frac{1}{x} + \frac{1}{2} \left( \frac{1}{x+1} \right) + \frac{1}{2} \left( \frac{1}{x-1} \right)$$

Now integrating it,

$$\Rightarrow \int dy = -\int \frac{1}{x} dx + \frac{1}{2} \int \left( \frac{1}{x+1} \right) dx + \frac{1}{2} \int \left( \frac{1}{x-1} \right) dx$$

$$\Rightarrow y = \frac{1}{2} \log \left[ \frac{c^2(x-1)(x+1)}{x^2} \right] \quad - iii)$$

Now we are given that  $y = 0$  when  $x = 2$

$$0 = \frac{1}{2} \log \left[ \frac{c^2(2-1)(2+1)}{4} \right]$$

$$\Rightarrow \log \frac{3c^2}{4} = 0$$

Or,

$$\Rightarrow \frac{3c^2}{4} = 1$$

$$\Rightarrow 3c^2 = 4$$

$$\Rightarrow c^2 = 4/3$$

Now putting the value of  $c^2$  in – iii)

Then,

$$\Rightarrow y = \frac{1}{2} \log \left[ \frac{4(x-1)(x+1)}{3x^2} \right]$$

$$\Rightarrow y = \frac{1}{2} \log \left[ \frac{4(x^2-1)}{3x^2} \right]$$

13.  $\cos\left(\frac{dy}{dx}\right) = a$  ( $a \in \mathbb{R}$ );  $y = 1$  when  $x = 0$

13.  $\cos\left(\frac{dy}{dx}\right) = a$

$$\Rightarrow \frac{dy}{dx} = \cos^{-1} a$$

$$\Rightarrow dy = \cos^{-1} a dx$$

Integrating both sides, we get:

$$\int dy = \cos^{-1} a \int dx$$

$$\Rightarrow y = \cos^{-1} a \cdot x + C$$

$$\Rightarrow y = x \cos^{-1} a + C$$

Now,  $y = 1$  when  $x = 0$ .

$$\Rightarrow 1 = 0 \cdot \cos^{-1} a + C$$

$$\Rightarrow C = 1$$

Substituting  $C = 1$  in equation (1), we get:

$$y = x \cos^{-1} a + 1$$

$$\Rightarrow \frac{y-1}{x} = \cos^{-1} a$$

$$\Rightarrow \cos\left(\frac{y-1}{x}\right) = a$$

14.  $\frac{dy}{dx} = y \tan x$ ;  $y = 1$  when  $x = 0$

14.  $\frac{dy}{dx} = y \tan x$

Separating variables,

$$\Rightarrow \frac{dy}{y} = \tan x dx$$

Integrating both sides,

$$\Rightarrow \int \frac{dy}{y} = \int \tan x dx$$

$$\Rightarrow \log y = -\log(\cos x) + \log c$$

Or,

$$\Rightarrow \log y = \log (\sec x) + \log c$$

$$\Rightarrow \log y = \log c (\sec x)$$

$$\Rightarrow y = c (\sec x) \quad \text{-i)}$$

Now we are given that  $y=1$  when  $x=0$

$$\Rightarrow 1 = c (\sec 0)$$

$$\Rightarrow 1 = c \times 1$$

$$\Rightarrow c = 1$$

Putting the value of  $c$  in i)

$$\Rightarrow y = \sec x$$

15. Find the equation of a curve passing through the point  $(0, 0)$  and whose differential equation is  $y' = e^x \sin x$ .

15. The differential equation of the curve is:

$$y' = e^x \sin x$$

$$\Rightarrow \frac{dy}{dx} = e^x \sin x$$

$$\Rightarrow dy = e^x \sin x$$

Integrating both sides, we get:

$$\int dy = \int e^x \sin x dx \quad \dots(1)$$

$$\text{Let } I = \int e^x \sin x dx .$$

$$\Rightarrow I = \sin x \int e^x dx - \int \left( \frac{d}{dx} (\sin x) \cdot \int e^x dx \right) dx$$

$$\Rightarrow I = \sin x \cdot e^x - \int \cos x \cdot e^x dx$$

$$\Rightarrow I = \sin x \cdot e^x - \left[ \cos x \cdot \int e^x dx - \int \left( \frac{d}{dx} (\cos x) \cdot \int e^x dx \right) dx \right]$$

$$\Rightarrow I = \sin x \cdot e^x - \left[ \cos x \cdot e^x - \int (-\sin x) \cdot e^x dx \right]$$

$$\Rightarrow I = e^x \sin x - e^x \cos x - I$$

$$\Rightarrow 2I = e^x (\sin x - \cos x)$$

$$\Rightarrow I = \frac{e^x (\sin x - \cos x)}{2}$$

Substituting this value in equation (1), we get:

$$y = \frac{e^x (\sin x - \cos x)}{2} + C \quad \dots(2)$$

Now, the curve passes through point  $(0, 0)$ .

$$\therefore 0 = \frac{e^0(\sin 0 - \cos 0)}{2} + C$$

$$\Rightarrow 0 = \frac{1(0-1)}{2} + C$$

$$\Rightarrow C = \frac{1}{2}$$

Substituting  $C = \frac{1}{2}$  in equation (2), we get:

$$y = \frac{e^x(\sin x - \cos x)}{2} + \frac{1}{2}$$

$$\Rightarrow 2y = e^x(\sin x - \cos x) + 1$$

$$\Rightarrow 2y - 1 = e^x(\sin x - \cos x)$$

Hence, the required equation of the curve is  $2y - 1 = e^x(\sin x - \cos x)$ .

16. For the differential equation  $xy \frac{dy}{dx} = (x+2)(y+2)$ , find the solution curve passing through the point (1, -1).

16. For this question, we need to find the particular solution at point(1,-1) for the given differential equation.

Given differential equation is

$$\Rightarrow xy \frac{dy}{dx} = (x+2)(y+2)$$

Separating variables,

$$\Rightarrow \frac{y}{y+2} dy = \frac{(x+2)dx}{x}$$

Or,

$$\Rightarrow \left(1 - \frac{2}{y+2}\right) dy = \left(1 + \frac{2}{x}\right) dx$$

Integrating both sides,

$$\Rightarrow \int \left(1 - \frac{2}{y+2}\right) dy = \int \left(1 + \frac{2}{x}\right) dx$$

$$\Rightarrow \int dy - 2 \int \frac{1}{y+2} dy = \int dx + 2 \int \frac{1}{x} dx$$

$$\Rightarrow y - 2 \log(y+2) = x + 2 \log x + c$$

Now separating like terms on each side,

$$\Rightarrow y - x - c = 2 \log x + 2 \log(y+2)$$

$$\Rightarrow y - x - c = \log x^2 + \log(y+2)^2$$

Or,

$$\Rightarrow y - x - c = \log\{x^2(y+2)^2\} \quad - i)$$

Now we are given that, the curve passes through (1, -1)

$\therefore$  putting the values of  $x$  &  $y$ , to find the value of  $c$ .

$$\Rightarrow -1 - 1 - c = \log\{1(-1 + 2)^2\}$$

$$\Rightarrow -2 - c = \log(1)$$

$$\Rightarrow c = -2 + 0 \quad (\because \log(1) = 0)$$

So  $c = -2$

Putting the value of  $c$  in

$$y - x - c = \log\{x^2(y + 2)^2\}$$

$$y - x + 2 = \log\{x^2(y + 2)^2\}$$

17. Find the equation of a curve passing through the point  $(0, -2)$  given that at any point  $(x, y)$  on the curve, the product of the slope of its tangent and  $y$ -coordinate of the point is equal to the  $x$ -coordinate of the point.

17. Let  $x$  and  $y$  be the  $x$ -coordinate and  $y$ -coordinate of the curve respectively. We know that the slope of a tangent to the curve in the coordinate axis is given by the relation,

$$\frac{dy}{dx}$$

According to the given information, we get:

$$y \cdot \frac{dy}{dx} = x$$

$$\Rightarrow y dy = x dx$$

Integrating both sides, we get:

$$\int y dy = \int x dx$$

$$\Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C$$

$$\Rightarrow y^2 - x^2 = 2C \quad \dots(1)$$

Now, the curve passes through point  $(0, -2)$ .

$$\therefore (-2)^2 - 0^2 = 2C$$

$\Rightarrow 2C = 4$  Substituting  $2C = 4$  in equation (1), we get:

$$y^2 - x^2 = 4$$

This is the required equation of the curve.

18. At any point  $(x, y)$  of a curve, the slope of the tangent is twice the slope of the line segment joining the point of contact to the point  $(-4, -3)$ . Find the equation of the curve given that it passes through  $(-2, 1)$ .

18. We know that  $(x, y)$  is the point of contact of curve and its tangent.

slope( $m_1$ ) for line joining  $(x, y)$  and  $(-4, -3)$  is  $\frac{y+3}{x+4}$ .

Also we know that slope of tangent of a curve is  $\frac{dy}{dx}$ .

$$\therefore \text{slope (m2) of tangent} = \frac{dy}{dx} \quad - ii)$$

Now, according to the question,

$$(m2) = 2(m1)$$

$$\Rightarrow \frac{dy}{dx} = \frac{2(y+3)}{x+4}$$

Separating variables,

$$\Rightarrow \frac{dy}{y+3} = \frac{2dx}{x+4}$$

Integrating both sides,

$$\Rightarrow \int \frac{dy}{y+3} = 2 \int \frac{dx}{x+4}$$

$$\Rightarrow \log(y+3) = 2\log(x+4) + \log c$$

$$\Rightarrow \log(y+3) = \log c(x+4)^2$$

$$\Rightarrow y+3 = c(x+4)^2 \quad - iii)$$

Now, this equation passes through the point (-2,1).

$$\Rightarrow 1+3 = c(-2+4)^2$$

$$\Rightarrow 4 = 4c$$

$$\Rightarrow c = 1$$

Substitute the value of c in iii)

$$\Rightarrow y+3 = (x+4)^2$$

19. The volume of spherical balloon being inflated changes at a constant rate. If initially its radius is 3 units and after 3 seconds it is 6 units. Find the radius of balloon after t seconds.

19. Let the rate of change of the volume of the balloon be k (where k is a constant).

$$\Rightarrow \frac{dv}{dt} = k$$

$$\Rightarrow \frac{d}{dt} \left( \frac{4}{3} \pi r^3 \right) = k \quad \left[ \text{Volume of sphere} = \frac{4}{3} \pi r^3 \right]$$

$$\Rightarrow \frac{4}{3} \pi \cdot 3r^2 \cdot \frac{dr}{dt} = k$$

$$\Rightarrow 4\pi^2 dr = kdt$$

Integrating both sides, we get:

$$4\pi \int r^2 dr = k \int dt$$

$$\Rightarrow 4\pi \cdot \frac{r^3}{3} = kt + C$$

$$\Rightarrow 4\pi r^3 = 3(kt + C) \quad \dots(1)$$

Now, at t = 0, r = 3:

$$\Rightarrow 4\pi \times 3^3 = 3(k \times 0 + C)$$

$$\Rightarrow 108\pi = 3C$$

$$\Rightarrow C = 36\pi$$

At  $t = 3$ ,  $r = 6$ :

$$\Rightarrow 4\pi \times 6^3 = 3(k \times 3 + C)$$

$$\Rightarrow 864\pi = 3(3k + 36\pi)$$

$$\Rightarrow 3k = -288\pi - 36\pi = 252\pi$$

$$\Rightarrow k = 84\pi$$

Substituting the values of  $k$  and  $C$  in equation (1), we get:

$$4\pi r^3 = 3[84\pi t + 36\pi]$$

$$\Rightarrow 4\pi r^3 = 4\pi(63t + 27)$$

$$\Rightarrow r^3 = 63t + 27$$

$$\Rightarrow r = (63t + 27)^{\frac{1}{3}}$$

Thus, the radius of the balloon after  $t$  seconds is  $(63t + 27)^{\frac{1}{3}}$ .

20. In a bank, principal increases continuously at the rate of  $r\%$  per year. Find the value of  $r$  if Rs 100 doubles itself in 10 years ( $\log_e 2 = 0.6931$ ).

20. let  $t$  be time

$p$  be principal

$r$  be rate of interest

according the information principal increases at the rate of  $r\%$  per year.

$$\therefore \frac{dp}{dt} = \left(\frac{r}{100}\right)p$$

Separating variables,

$$\Rightarrow \frac{dp}{p} = \left(\frac{r}{100}\right) dt$$

Integrating both sides,

$$\Rightarrow \int \frac{dp}{p} = \frac{r}{100} \int dt$$

$$\Rightarrow \log p = \frac{rt}{100} + k$$

$$\Rightarrow p = e^{\frac{rt}{100} + k} \quad - i)$$

Given that  $t=0$ ,  $p=100$ .

$$\Rightarrow 100 = e^k \quad - ii)$$

Now, if  $t = 10$ , then  $p = 2 \times 100 = 200$

So,

$$\Rightarrow 200 = e^{\frac{rt}{100} + k}$$

$$\Rightarrow 200 = e^{\frac{rt}{100}} \cdot e^k$$

$$\Rightarrow 200 = e^{\frac{rt}{100}} \times 100 \quad \text{from ii)}$$

$$\Rightarrow e^{\frac{r}{10}} = 2$$

$$\Rightarrow \frac{r}{10} = \log 2$$

$$\Rightarrow r = 6.93$$

So r is 6.93%.

21. In a bank, principal increases continuously at the rate of 5% per year. An amount of Rs 1000 is deposited with this bank, how much will it worth after 10 years ( $e^{0.5} = 1.648$ ).

21. Let p and t be the principal and time respectively.

It is given that the principal increases continuously at the rate of 5% per year.

$$\Rightarrow \frac{dp}{dt} = \left(\frac{5}{100}\right)p$$

$$\Rightarrow \frac{dp}{p} = \frac{5}{20} dt$$

$$\Rightarrow \frac{dp}{p} = \frac{dt}{20}$$

Integrating both sides, we get:

$$\int \frac{dp}{p} = \frac{1}{20} \int dt$$

$$\Rightarrow \log p = \frac{t}{20} + C$$

$$\Rightarrow p = e^{\frac{t}{20} + C} \dots(1)$$

Now, when t = 0, p = 1000.

$$\Rightarrow 1000 = e^C \dots(2)$$

At t = 10, equation (1) becomes:

$$p = e^{\frac{10}{20} + C}$$

$$\Rightarrow p = e^{0.5} \times e^C$$

$$\Rightarrow p = 1.648 \times 1000$$

$$\Rightarrow p = 1648$$

Hence, after 10 years the amount will worth Rs 1648.

22. In a culture, the bacteria count is 1,00,000. The number is increased by 10% in 2 hours. In how many hours will the count reach 2,00,000, if the rate of growth of bacteria is proportional to the number present?

22. let y be the number of bacteria at any instant t.

Given that the rate of growth of bacteria is proportional to the number present

$$\therefore \frac{dy}{dt} \propto y$$

$$\Rightarrow \frac{dy}{dt} = ky \quad (k \text{ is a constant})$$

Separating variables,

$$\Rightarrow \frac{dy}{y} = k dt$$

Integrating both sides,

$$\Rightarrow \int \frac{dy}{y} = k \int dt$$

$$\Rightarrow \log y = kt + c \quad \text{-i)}$$

Let  $y'$  be the number of bacteria at  $t=0$ .

$$\Rightarrow \log y' = c$$

Substituting the value of  $c$  in  $i)$

$$\Rightarrow \log y = kt + \log y'$$

$$\Rightarrow \log y - \log y' = kt$$

$$\Rightarrow \log \frac{y}{y'} = kt \quad \text{- ii)}$$

Also, given that number of bacteria increases by 10% in 2 hours.

$$\Rightarrow y = \frac{110}{100} y'$$

$$\Rightarrow \frac{y}{y'} = \frac{11}{10} \quad \text{- iii)}$$

Substituting this value in  $ii)$

$$\Rightarrow k \times 2 = \log \frac{11}{10}$$

$$\Rightarrow k = \frac{1}{2} \log \frac{11}{10}$$

So,  $ii)$  becomes

$$\Rightarrow \frac{1}{2} \log \frac{11}{10} \times t = \log \frac{y}{y'}$$

$$\Rightarrow t = \frac{2 \log \frac{y}{y'}}{\log \frac{11}{10}} \quad \text{- iv)}$$

Now, let the time when number of bacteria increase from 100000 to 200000 be  $t'$ .

$$\Rightarrow y = 2y' \text{ at } t = t'$$

So from  $iv)$

$$\Rightarrow t' = \frac{2 \log \frac{y}{y'}}{\log \frac{11}{10}} = \frac{2 \log 2}{\log \frac{11}{10}}$$

So bacteria increases from 100000 to 200000 in  $\frac{2 \log 2}{\log \frac{11}{10}}$  hours.

**23.** The general solution of the differential equation  $\frac{dy}{dx} = e^{x+y}$  is

(a)  $e^x + e^{-y} = C$       (b)  $e^x + e^y = C$

$$(c) e^{-x} + e^y = C \quad (d) e^{-x} + e^{-y} = C$$

$$23. \frac{dy}{dx} = e^{x+y} = e^x \cdot e^y$$

$$\Rightarrow \frac{dy}{e^y} = e^x dx$$

$$\Rightarrow e^{-y} dy = e^x dx$$

Integrating both sides, we get:

$$\int e^{-y} dy = \int e^x dx$$

$$\Rightarrow -e^{-y} = e^x + k$$

$$\Rightarrow e^x + e^{-y} = -k$$

$$\Rightarrow e^x + e^{-y} = c \quad (c = -k)$$

Hence, the correct answer is A.



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