# EXERCISE 23A

**1. Find the value of:**

**(i) $\sin 30^\circ \cos 30^\circ$**

**(ii) $\tan 30^\circ \tan 60^\circ$**

**(iii) $\cos^2 60^\circ + \sin^2 30^\circ$**

**(iv) $\text{cosec}^2 60^\circ - \tan^2 30^\circ$**

**(v) $\sin^2 30^\circ + \cos^2 30^\circ + \cot^2 45^\circ$**

**(vi) $\cos^2 60^\circ + \sec^2 30^\circ + \tan^2 45^\circ$.**

**Solution:**

(i) Given $\sin 30^\circ \cos 30^\circ$

By substituting the values, we get

$\sin 30^\circ \cos 30^\circ = \frac{1}{2} (\sqrt{3}/2)$

$= \sqrt{3}/4$

(ii) Given $\tan 30^\circ \tan 60^\circ$

By substituting the values, we get

$\tan 30^\circ \tan 60^\circ = 1/\sqrt{3} (\sqrt{3})$

$= 1$

(iii) Given $\cos^2 60^\circ + \sin^2 30^\circ$

By substituting the values, we get

$\cos^2 60^\circ + \sin^2 30^\circ = (\frac{1}{2})^2 + (\frac{1}{2})^2$

$= \frac{1}{4} + \frac{1}{4}$

$= \frac{1}{2}$

(iv) Given $\text{cosec}^2 60^\circ - \tan^2 30^\circ$

By substituting the values, we get

$\text{cosec}^2 60^\circ - \tan^2 30^\circ = (2/\sqrt{3})^2 – (1/\sqrt{3})^2$

$= 4/3 – 1/3$

$= 1$

(v) Given $\sin^2 30^\circ + \cos^2 30^\circ + \cot^2 45^\circ$

By substituting the values, we get

$\sin^2 30^\circ + \cos^2 30^\circ + \cot^2 45^\circ = (\frac{1}{2})^2 + (\sqrt{3}/2)^2 + 1^2$

$= \frac{1}{4} + \frac{3}{4} + 1$

$= 2$

(vi) Given $\cos^2 60^\circ + \sec^2 30^\circ + \tan^2 45^\circ$

By substituting the values, we get

$\cos^2 60^\circ + \sec^2 30^\circ + \tan^2 45^\circ = (\frac{1}{2})^2 + (2/\sqrt{3})^2 + 1^2$

$= \frac{1}{4} + 4/3 + 1$

$= 31/12$

**2. Find the value of:**

**(i) $\tan^2 30^o + \tan^2 45^o + \tan^2 60^o$**

**(ii)** $\frac{\tan 45^o}{\operatorname{cosec} 30^o} + \frac{\sec 60^o}{\cot 45^o} - \frac{5 \sin 90^o}{2 \cos 0^o}$

**(iii) $3 \sin^2 30^o + 2 \tan^2 60^o - 5 \cos^2 45^o$.**

**Solution:**

(i) Given $\tan^2 30^o + \tan^2 45^o + \tan^2 60^o$

By substituting the values, we get

$\tan^2 30^o + \tan^2 45^o + \tan^2 60^o = (1//\sqrt{3})^2 + 1^2 + (/\sqrt{3})^2$

$= 1/3 + 1 + 3$

$= 13/3$

$= 4\ 1/3$

(ii) Given

$$\frac{\tan 45^o}{\operatorname{cosec} 30^o} + \frac{\sec 60^o}{\cot 45^o} - \frac{5 \sin 90^o}{2 \cos 0^o}$$

By substituting the values, we get

$= ½ + 2/1 – 5/2$

$= (1 + 4 – 5)/2$

$= 0$

(iii) Given $3 \sin^2 30^o + 2 \tan^2 60^o - 5 \cos^2 45^o$.

By substituting the values, we get

$3 \sin^2 30^o + 2 \tan^2 60^o - 5 \cos^2 45^o. = 3 (½)^2 + 2 (\sqrt{3})^2 + 5 (1/\sqrt{3})^2$

$= ¾ + 6 – 5/2$

$= (3 + 24 – 10)/4$

$= 4\ ¼$

**3. Prove that:**

**(i) $\sin 60^o \cos 30^o + \cos 60^o. \sin 30^o = 1$**

**(ii) $\cos 30^o. \cos 60^o - \sin 30^o. \sin 60^o = 0$**

**(iii) $\operatorname{cosec}^2 45^o - \cot^2 45^o = 1$**

**(iv) $\cos^2 30^o - \sin^2 30^o = \cos 60^o$.**

**(v)** $\left(\frac{\tan 60^o + 1}{\tan 60^o - 1}\right)^2 = \frac{1 + \cos 30^o}{1 - \cos 30^o}$

**(vi) $3 \operatorname{cosec}^2 60^o - 2 \cot^2 30^o + \sec^2 45^o = 0$.**

**Solution:**

(i) Given $\sin 60^o \cos 30^o + \cos 60^o. \sin 30^o$

LHS = sin $60^o$ cos $30^o$ + cos $60^o$. sin $30^o$
Now we have to prove that RHS = 1
= (√3/2) (√3/2) + ½ ½
= ¾ + ¼
= 1
= RHS

(ii) Given cos $30^o$. cos $60^o$ - sin $30^o$. sin $60^o$ = 0
LHS = cos $30^o$. cos $60^o$ - sin $30^o$. sin $60^o$
= (√3/2) ½ - ½ (√3/2)
= (√3/4) - (√3/4)
= 0
= RHS

(iii) Given $\text{cosec}^2 45^o - \cot^2 45^o = 1$
LHS = $\text{cosec}^2 45^o - \cot^2 45^o = 1$
= $(\sqrt{2})^2 - 1^2$
= 2 – 1
= 1
= RHS



(iv) Given $\cos^2 30^o - \sin^2 30^o = \cos 60^o$.
LHS = $\cos^2 30^o - \sin^2 30^o = \cos 60^o$.
= $(\sqrt{3}/2)^2 - (½)^2$
= ¾ - ¼
= ½
= cos $60^o$
= RHS

(v) LHS= $\left(\dfrac{\tan 60^o + 1}{\tan 60^o - 1}\right)^2$

$$= \left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)^2 = \frac{4+2\sqrt{3}}{4-2\sqrt{3}} = \frac{1+\frac{\sqrt{3}}{2}}{1-\frac{\sqrt{3}}{2}} = \frac{1+\cos 30^0}{1-\cos 30^0} = \text{RHS}$$

(vi) Given 3 $\text{cosec}^2 60^o$ - 2 $\cot^2 30^o$ + $\sec^2 45^o$ = 0.
LHS = 3 $\text{cosec}^2 60^o$ - 2 $\cot^2 30^o$ + $\sec^2 45^o$ = 0.
= $3 (2/\sqrt{3})^2 - 2 (\sqrt{3})^2 + (\sqrt{2})^2$
= 4 – 6 + 2
= 0
= RHS

**4. Prove that**

(i) $\sin(2 \times 30^\circ) = \dfrac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

(ii) $\cos(2 \times 30^\circ) = \dfrac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ}$

(iii) $\tan(2 \times 30^\circ) = \dfrac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

**Solution:**

(i) RHS =

$$\frac{2\tan 30^\circ}{1+\tan^2 30^\circ} = \frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$$

On rearranging we get

$$= \frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{\sqrt{3}}{2}$$

LHS = $\sin(2 \times 30^\circ)$

= $\sin 60^\circ$

= $\sqrt{3}/2$

Therefore LHS = RHS

(ii) RHS =

$$\frac{1-\tan^2 30^\circ}{1+\tan^2 30^\circ}$$

Now substituting the values, we get

= (1 – 1/3)/ (1 + 1/3)

= ½

Consider LHS

$\cos(2 \times 30^\circ)$

= $\cos 60^\circ$

= ½

Therefore, LHS = RHS

(iii) RHS

$$\frac{2\tan 30°}{1-\tan^2 30°}$$

Now substituting the values, we get

$$\frac{2\frac{1}{\sqrt{3}}}{1-\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$$

= √3

LHS,

tan (2 × 30$^o$)

= tan 60$^o$

= √3

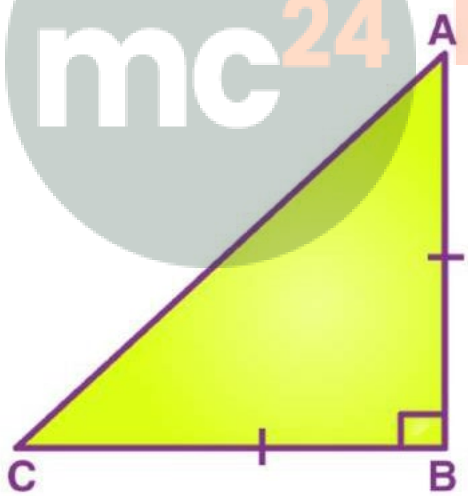
Therefore, LHS = RHS

**5. ABC is an isosceles right-angled triangle. Assuming of AB = BC = x, find the value of each of the following trigonometric ratios:**

**(i) sin 45$^o$**

**(ii) cos 45$^o$**

**(iii) tan 45$^o$**






**Solution:**

Given that AB = BC = x

$$AC = \sqrt{AB^2 + BC^2} = \sqrt{x^2 + x^2} = x\sqrt{2}$$

(i) sin 45$^o$ = AB/AC

= x/x√2

= 1/√2

(ii) cos 45$^o$ = BC/AC

= x/x√2

$= 1/\sqrt{2}$

(iii) $\tan 45^\circ = AB/BC$
$= x/x$
$= 1$

**6. Prove that:**
**(i) $\sin 60^\circ = 2 \sin 30^\circ \cos 30^\circ$.**
**(ii) $4 (\sin^4 30^\circ + \cos^4 60^\circ) - 3 (\cos^2 45^\circ - \sin^2 90^\circ) = 2$**

**Solution:**
(i) LHS = $\sin 60^\circ$
$= \sqrt{3}/2$
RHS = $2 \sin 30^\circ \cos 30^\circ$
$= 2 (\sqrt{3}/2) (½)$
$= \sqrt{3}/2$
Therefore LHS = RHS

(ii) LHS = $4 (\sin^4 30^\circ + \cos^4 60^\circ) - 3 (\cos^2 45^\circ - \sin^2 90^\circ)$
Now by substituting the values we get
$= 4[(½)^4 + (½)^4] – 3 [(1/\sqrt{2})^2 + 1^4]$
$= 4(1/16 + 1/16) – 3 (½ - 1)$
$= 8/16 + 3/2$
$= 2$
LHS = RHS

**7. (i) If sin x = cos x and x is acute, state the value of x.**
**(ii) If sec A = cosec A and $0^\circ \le A \le 90^\circ$, state the value of A.**
**(iii) If $\tan \theta = \cot \theta$ and $0^\circ \le \theta \le 90^\circ$, state the value of $\theta$.**
**(iv) If sin x = cos y; write the relation between x and y, if both the angles x and y are acute.**

**Solution:**
(i) The angle, x is acute and hence we have, $0 < x$
We know that
$\cos^2 x + \sin^2 x = 1$
Since cos x = sin x
Above equation will become
$2 \sin^2 x = 1$
$\sin x = 1/\sqrt{2}$
Therefore, $x = 45^\circ$

(ii) sec A = cosec A
Cos A = sin A

$\cos^2 A = \sin^2 A$
$\cos^2 x + \sin^2 x = 1$
Above equation will become
$\cos^2 A = 1 - \cos^2 A$
$2 \cos^2 A = 1$
$\cos A = 1/\sqrt{2}$
$A = 45^o$

(iii) $\tan \theta = \cot \theta$
$\tan \theta = 1/\tan \theta$
$\tan^2 \theta = 1$
$\tan \theta = 1$
$\tan \theta = \tan 45^o$
$\theta = 45^o$

(iv) $\sin x = \cos y = \sin (90^o - y)$
If x and y are acute angles
$x = 90^o - y$
which implies,
$x + y = 90^o$
hence x and y are complementary angles.



**8. (i) If $\sin x = \cos y$, then $x + y = 45^o$; write true of false.**
**(ii) $\sec \theta$. $\cot \theta = \text{cosec } \theta$; write true or false.**
**(iii) For any angle $\theta$, state the value of:**
**$\sin^2 \theta + \cos^2 \theta$.**

**Solution:**
(i) $\sin x = \cos y = \sin (\pi/2 - y)$
If x and y acute angles,
$x = (\pi/2 - y)$
$x + y = \pi/2$
$x + y = 45^o$ is false

(ii) $\sec \theta. \cot \theta = 1/\cos \theta . \cos \theta / \sin \theta$
$= \text{cosec } \theta$
$\sec \theta . \cot \theta = \text{cosec } \theta$
is true.

(iii) $\sin^2 \theta + \cos^2 \theta = \sin^2 \theta + 1 - \sin^2 \theta$.
$= 1$

**9. State for any acute angle $\theta$ whether:**

**(i) sin θ increases or decreases as θ increases:**
**(ii) cos θ increases or decreases as θ increases.**
**(iii) tan θ increases or decreases as θ decreases.**

**Solution:**
(i) For acute angles, remember what sine means: opposite over hypotenuse. If we increase the angle, then the opposite side gets larger. That means "opposite/hypotenuse" gets larger or increases.

(ii) For acute angles, remember what cosine means: base over hypotenuse. If we increase the angle, then the hypotenuse side gets larger. That means "base/hypotenuse" gets smaller or decreases.

(iii) For acute angles, remember what tangent means: opposite over base. If we decrease the angle, then the opposite side gets smaller. That means "opposite /base" gets decreases.

**10. If √3 = 1.732, find (correct to two decimal place) the value of each of the following:**
**(i) sin 60°**
**(ii) 2/ tan 30°**

**Solution:**
(i) sin 60° = √3 /2
= 1.732/2
= 0.87

(ii) 2/ tan 30° = 2/ (1/√3)
= 2√3
= 2 (1.732)
= 3.46