

EXERCISE 15.3

Solve each of the following system of equations in \mathbb{R} .

1. $|x + 1/3| > 8/3$

Solution:

Let 'r' be a positive real number and 'a' be a fixed real number. Then,

$$|x + a| > r \Leftrightarrow x > r - a \text{ or } x < - (a + r)$$

Here, $a = 1/3$ and $r = 8/3$

$$x > 8/3 - 1/3 \text{ or } x < - (8/3 + 1/3)$$

$$x > (8-1)/3 \text{ or } x < - (8+1)/3$$

$$x > 7/3 \text{ or } x < - 9/3$$

$$x > 7/3 \text{ or } x < - 3$$

$$x \in (7/3, \infty) \text{ or } x \in (-\infty, -3)$$

$$\therefore x \in (-\infty, -3) \cup (7/3, \infty)$$

2. $|4 - x| + 1 < 3$

Solution:

$$|4 - x| + 1 < 3$$

Let us subtract 1 from both the sides, we get

$$|4 - x| + 1 - 1 < 3 - 1$$

$$|4 - x| < 2$$

Let 'r' be a positive real number and 'a' be a fixed real number. Then,

$$|a - x| < r \Leftrightarrow a - r < x < a + r$$

Here, $a=4$ and $r=2$

$$4 - 2 < x < 4 + 2$$

$$2 < x < 6$$

$$\therefore x \in (2, 6)$$

3. $|(3x - 4)/2| \leq 5/12$

Solution:

Given:

$$|(3x - 4)/2| \leq 5/12$$

We can rewrite it as

$$|3x/2 - 4/2| \leq 5/12$$

$$|3x/2 - 2| \leq 5/12$$

Let 'r' be a positive real number and 'a' be a fixed real number. Then,

$$|x - a| \leq r \Leftrightarrow a - r \leq x \leq a + r$$

Here, $a = 2$ and $r = 5/12$

$$2 - 5/12 \leq 3x/2 \leq 2 + 5/12$$

$$(24-5)/12 \leq 3x/2 \leq (24+5)/12$$

$$19/12 \leq 3x/2 \leq 29/12$$

Now, multiplying the whole inequality by 2 and dividing by 3, we get

$$19/18 \leq x \leq 29/18$$

$$\therefore x \in [19/18, 29/18]$$

4. $|x - 2| / (x - 2) > 0$

Solution:

Given:

$$|x - 2| / (x - 2) > 0$$

Clearly it states, $x \neq 2$ so two case arise:

Case 1: $x - 2 > 0$

$$x > 2$$

In this case $|x - 2| = x - 2$

$$x \in (2, \infty) \dots (1)$$

Case 2: $x - 2 < 0$

$$x < 2$$

In this case, $|x - 2| = -(x - 2)$

$$-(x - 2) / (x - 2) > 0$$

$$-1 > 0$$

Inequality doesn't get satisfy

This case gets nullified.

$$\therefore x \in (2, \infty) \text{ from (1)}$$

5. $1 / (|x| - 3) < 1/2$

Solution:

We know that, if we take reciprocal of any inequality we need to change the inequality as well.

$$\text{Also, } |x| - 3 \neq 0$$

$$|x| > 3 \text{ or } |x| < 3$$

For $|x| < 3$

$$-3 < x < 3$$

$$x \in (-3, 3) \dots (1)$$

The equation can be re-written as

$$|x| - 3 > 2$$

Let us add 3 on both the sides, we get



$$|x| - 3 + 3 > 2 + 3$$

$$|x| > 5$$

Let 'a' be a fixed real number. Then,

$$|x| > a \Leftrightarrow x < -a \text{ or } x > a$$

Here, $a = 5$

$$x < -5 \text{ or } x > 5 \dots (2)$$

From (1) and (2)

$$x \in (-\infty, -5) \text{ or } x \in (5, \infty)$$

$$\therefore x \in (-\infty, -5) \cup (-3, 3) \cup (5, \infty)$$

6. $(|x + 2| - x) / x < 2$

Solution:

Given:

$$(|x + 2| - x) / x < 2$$

Let us rewrite the equation as

$$|x + 2|/x - x/x < 2$$

$$|x + 2|/x - 1 < 2$$

By adding 1 on both sides, we get

$$|x + 2|/x - 1 + 1 < 2 + 1$$

$$|x + 2|/x < 3$$

By subtracting 3 on both sides, we get

$$|x + 2|/x - 3 < 3 - 3$$

$$|x + 2|/x - 3 < 0$$

Clearly it states, $x \neq 0$ so two case arise:

Case 1: $x + 2 > 0$

$$x > -2$$

In this case $|x+2| = x + 2$

$$x + 2/x - 3 < 0$$

$$(x + 2 - 3x)/x < 0$$

$$-(2x - 2)/x < 0$$

$$(2x - 2)/x < 0$$

Let us consider only the numerators, we get

$$2x - 2 > 0$$

$$x > 1$$

$$x \in (1, \infty) \dots (1)$$

Case 2: $x + 2 < 0$

$$x < -2$$

In this case, $|x+2| = -(x+2)$

$$-(x+2)/x - 3 < 0$$

$$(-x - 2 - 3x)/x < 0$$

$$-(4x + 2)/x < 0$$

$$(4x + 2)/x < 0$$

Let us consider only the numerators, we get

$$4x + 2 > 0$$

$$x > -\frac{1}{2}$$

But $x < -2$

From the denominator we have,

$$x \in (-\infty, 0) \dots(2)$$

From (1) and (2)

$$\therefore x \in (-\infty, 0) \cup (1, \infty)$$



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