Chapter 6

Chapter 6: Triangles

EXERCISE 6.2:

1. In Fig. 6.17, (i) and (ii), DE || BC. Find EC in (i) and AD in (ii)

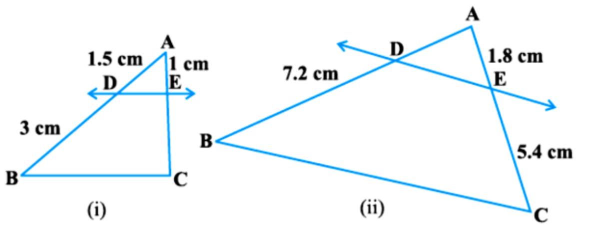


Fig. 6.17

B

Solution: (i) Let us take EC = x cm

Given: DE || BC.

Now, using basic proportionality theorem, we get:

$\frac{AD}{DB} = \frac{AE}{EC}$

$$\frac{1.5}{3} = \frac{1}{x}$$

x = $3 * \frac{1}{1.5}$

x = 2 cm

Hence, EC = 2 cm

(ii) Let us take AD = x cm

Given: DE || BC

Now, using basic proportionality theorem, we get

$$\frac{AD}{DB} = \frac{AE}{EC}$$

$$\frac{x}{7.2} = \frac{1.8}{5.4}$$

x = $1.8 * \frac{7.2}{5.4}$

Hence, AD = 2.4 cm

2. E and F are points on the sides PQ and PR respectively of a Δ PQR. For each of the following cases, state whether EF || QR :

   (i) PE = 3.9 cm, EQ = 3 cm, PF = 3.6 cm and FR = 2.4 cm

   (ii) PE = 4 cm, QE = 4.5 cm, PF = 8 cm and RF = 9 cm

   (iii) PQ = 1.28 cm, PR = 2.56 cm, PE = 0.18 cm and PF = 0.36 cm

Solution: (i) Given : PE = 3.9 cm,

EQ = 3 cm,

PF = 3.6 cm,
FR = 2.4 cm
Now,

$$\frac{PE}{EQ} = \frac{3.9}{3} = 1.3$$
$$\frac{PF}{FR} = \frac{3.6}{2.4} = 1.5$$
$$Hence,$$
$$\frac{PE}{EQ} \neq \frac{PF}{FR}$$

Therefore, EF is not parallel to QR

(ii) In this,

$$\frac{PE}{EQ} = \frac{4}{4.5} = \frac{8}{9}$$
$$\frac{PF}{FR} = \frac{8}{9}$$

Hence,

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore, EF is parallel to QR

(iii) In this,

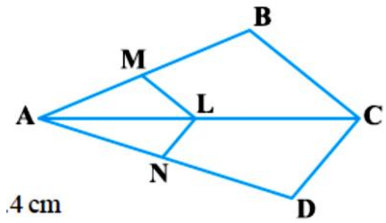
$$\frac{PE}{PQ} = \frac{0.18}{1.28} = \frac{8}{128} = \frac{9}{64}$$
$$\frac{PF}{PR} = \frac{0.36}{2.56} = \frac{9}{64}$$

Hence,

$$\frac{PE}{PQ} = \frac{PF}{PR}$$

EF is parallel to QR

3. In Fig. 6.18, if LM || CB and LN || CD, prove that $\frac{AM}{AB} = \frac{AN}{AD}$.



Solution: From the given figure:
LM || CB

Now, using basic proportionality theorem, we get:

$\frac{AM}{AB} = \frac{AL}{AC}$ (i)

$$Similarly, LN\ parallel\ to\ CD$$

Therefore,

$\frac{AN}{AD} = \frac{AL}{AC}$ (ii)

From (i) and (ii), we obtain

$$\frac{AM}{AB} = \frac{AN}{AD}$$

4. In Fig. 6.19, DE || AC and DF || AE. Prove that

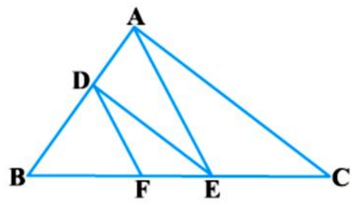


Fig. 6.19

$\frac{BF}{FE} = \frac{BE}{EC}$.

Solution: Given thatIn triangle ABC, DE is parallel to AC

Therefore,

$\frac{BD}{DA} = \frac{BE}{EC}$ (Basic proportionality theorem)

Now, $In\ triangle\ BAE, DF\ is\ parallel\ to\ AE$

Therefore,

$\frac{BD}{DA} = \frac{BF}{FE}$ (Basic proportionality theorem)

$$From\ above, we\ get$$

$$\frac{BE}{EC} = \frac{BF}{FE}$$

5. In Fig. 6.20, DE || OQ and DF || OR. Show thatEF || QR.

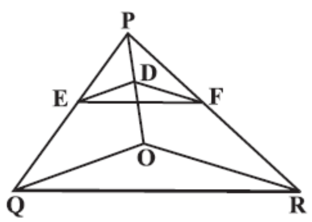


Fig. 6.20

Solution: Given that In triangle POQ, DE parallel to OQ

Hence,

$\frac{PE}{EQ} = \frac{PD}{DO}$ (Basic proportionality theorem) (i)

Now,

$$In\ triangle\ POR, DF\ parallel\ OR$$

Hence,

$\frac{PF}{FR} = \frac{PD}{DO}$ (Basic proportionality theorem) (ii)

From (i) and (ii), we get

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

Therefore,

EF is parallel to QR (Converse of basic proportionality theorem)

6. In Fig. 6.21, A, B and C are points on OP, OQ andOR respectively such that AB || PQ and AC || PR.Show that BC || QR.

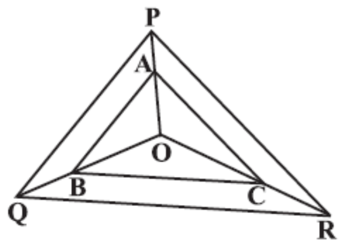


Fig. 6.21

Solution: Given that in triangle POQ, AB parallel to PQ

Hence,

$\frac{OA}{AP} = \frac{OB}{BQ}$ (Basic proportionality theorem)

Now, $in\ triangle\ POR, AC\ parallel\ PR$

Therefore,

Using Basic proportionality theorem, we get:

$$\frac{OA}{AP} = \frac{OC}{CR}$$

From above equations, we get

$$\frac{OB}{BQ} = \frac{OC}{CR}$$

BC is parallel to QR (By the converse of Basic proportionality theorem)

7. Using Theorem 6.1, prove that a line drawn throughthe mid-point of one side of a triangle parallel toanother side bisects the third side. (Recall that you have proved it in Class IX)

Solution: Consider the given figure in which PQ is a line segment drawn through the mid-point P of line AB, such that PQ is parallel to BC

Now, using basic proportionality theorem, we get

$$\frac{AQ}{QC} = \frac{AP}{PB}$$

$\frac{AQ}{QC} = \frac{1}{1}$ (P is the Mid-point of AB, AP = PB)

$$Hence,$$

AQ = QC

Or, Q is the mid-point of AC

8. Using Theorem 6.2, prove that the line joining themid-points of any two sides of a triangle is parallelto the third side. (Recall that you have done it in Class IX)

Solution: Let us take the given figure in which PQ is a line segment which joins the mid-points P and Q of line AB and AC respectively

i.e., AP = PB and AQ = QC

We observe that,

$$\frac{AP}{PB} = \frac{1}{1}$$

And,

$$\frac{AQ}{QC} = \frac{1}{1}$$

Therefore,

$$\frac{AP}{PB} = \frac{AQ}{QC}$$

Hence, using basic proportionality theorem we get:

PQ parallel to BC

9. ABCD is a trapezium in which AB || DC and itsdiagonals intersect each other at the point O. Showthat $\frac{AO}{BO} = \frac{CO}{DO}$.

Solution: Construct a line EF through point O, such that EF is parallel to CD

In ΔADC, EO is parallel to CD

Now, using basic proportionality theorem, we obtain

AE/ED = AO/OC (i)

In ΔABD, OE is parallel to AB

So, using basic proportionality theorem, we get

ED/AE = OD/BO

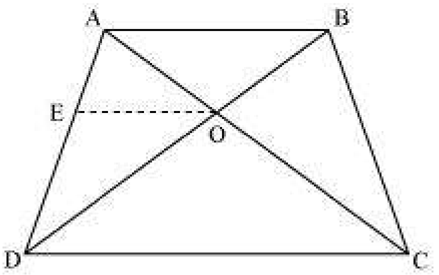
AE/ED = BO/OD (ii)

From (i) and (ii), we get

$$\frac{AO}{OC} = \frac{BO}{OD}$$
$$\frac{AO}{BO} = \frac{OC}{OD}$$

10. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO}=\frac{CO}{DO}$. Show that ABCD is a trapezium

Solution: Draw a line OE parallel to AB



Given: In triangle ABD, OE is parallel to AB

Now, using basic proportionality theorem, we obtain

$\frac{AE}{ED}=\frac{BO}{OD}$ (i)

It is given that,

$\frac{AO}{OC}=\frac{OB}{OD}$ (ii)

From (i) and (ii), we get

$$\frac{AE}{ED}=\frac{AO}{OC}$$

EO || DC (By the converse of basic proportionality theorem)

⇒ AB || OE || DC

⇒ AB || CD

Hence,

ABCD is a trapezium