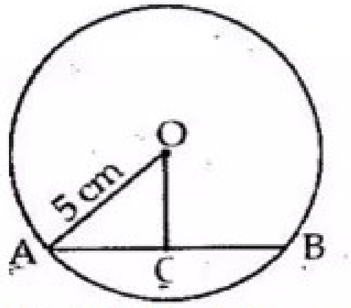
Chapter 17. Circle

Exercise 17(A)

Solution 1:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AC = CB = 3 cm

In ΔOCA,

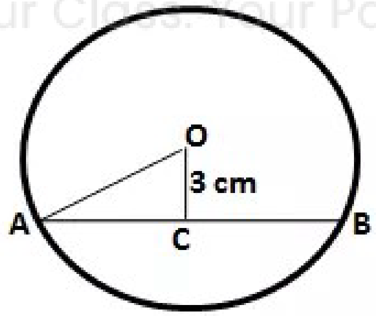
$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

$\Rightarrow OC^2 = (5)^2 - (3)^2 = 16$

$\Rightarrow OC = 4$ cm

Solution 2:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AB=8 cm

$\Rightarrow AC=CB=\frac{AB}{2}$

$\Rightarrow AC=CB=\frac{8}{2}$

$\Rightarrow AC=CB=4$ cm

In ΔOCA,

$OA^2 = OC^2 + AC^2$ (By Pythagoras theorem)

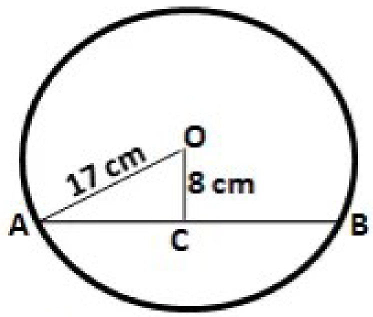
$\Rightarrow OA^2 = (4)^2 + (3)^2 = 25$

$\Rightarrow OA = 5$ cm

Hence, radius of the circle is 5 cm.

Solution 3:

Let AB be the chord and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.



We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AC = CB

In ΔOCA,

$$OA^2 = OC^2 + AC^2 \quad \text{(By Pythagoras theorem)}$$

$$\Rightarrow \quad AC^2 = (17)^2 - (8)^2 = 225$$

$$\Rightarrow \quad AC = 15 \text{ cm}$$

$$\therefore \quad AB = 2\,AC = 2 \times 15 = 30 \text{ cm.}$$

Solution 4:

Let AB be the chord of length 24 cm and O be the centre of the circle.
Let OC be the perpendicular drawn from O to AB.
We know, that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AC = CB = 12 cm

In ΔOCA,

$$OA^2 = OC^2 + AC^2 \quad \text{(By Pythagoras theorem)}$$

$$= (5)^2 + (12)^2 = 169$$

$$\Rightarrow \quad OA = 13 \text{ cm}$$

$$\therefore \quad \text{radius of the circle} = 13 \text{ cm.}$$

Let A'B' be the new chord at a distance of 12 cm from the centre.

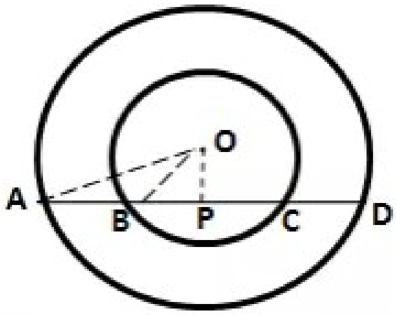
$$\therefore \quad (OA')^2 = (OC')^2 + (A'C')^2$$

$$\Rightarrow \quad (A'C')^2 = (13)^2 - (12)^2 = 25$$

$$\therefore \quad A'C' = 5 \text{ cm}$$

Hence, length of the new chord $= 2 \times 5 = 10$ cm.

Solution 5:



For the inner circle, BC is a chord and $OP \perp BC$.

We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ BP = PC

By Pythagoras Theorem,

$$OB^2 = OP^2 + BP^2$$

$$\Rightarrow \quad BP^2 = (20)^2 - (16)^2 = 144$$

$$\therefore \quad BP = 12 \text{ cm}$$

For the outer circle, AD is the chord and $OP \perp AD$.

We know that the perpendicular to a chord, from the centre of a circle, bisects the chord.

$\therefore$ AP = PD

By Pythagoras Theorem,

$OA^2 = OP^2 + AP^2$

=> $AP^2 = (34)^2 - (16)^2 = 900$

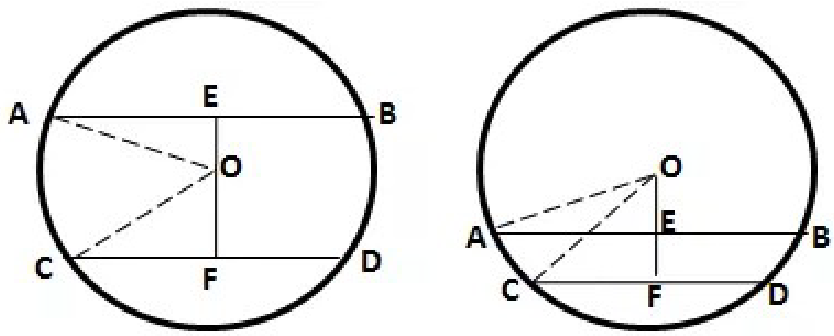
=> AP = 30 cm

AB = AP - BP = 30 - 12 = 18 cm

Solution 6:

Let O be the centre of the circle and AB and CD be the two parallel chords of length 30 cm and 16 cm respectively.

Drop OE and OF perpendicular on AB and CD from the centre O.



$OE \perp AB$ and $OF \perp CD$

$\therefore$ OE bisects AB and OF bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad AE = \frac{30}{2} = 15\text{ cm}; \quad CF = \frac{16}{2} = 8\text{ cm}$

In right ΔOAE,

$OA^2 = OE^2 + AE^2$

$\Rightarrow \quad OE^2 = OA^2 - AE^2 = (17)^2 - (15)^2 = 64$

$\therefore \quad OE = 8\text{ cm}$

In right ΔOCF,

$OC^2 = OF^2 + CF^2$

$\Rightarrow \quad OF^2 = OC^2 - CF^2 = (17)^2 - (8)^2 = 225$

$\therefore \quad OF = 15\text{ cm}$

(i) The chords are on the opposite sides of the centre:

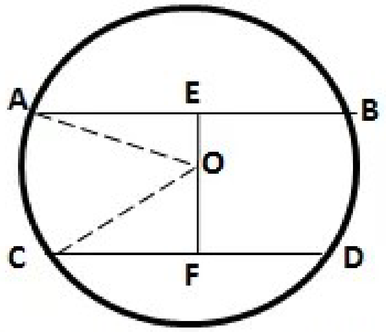
$\therefore \quad EF = EO + OF = (8+15) = 23\text{ cm}$

(ii) The chords are on the same side of the centre:

$\therefore \quad EF = OF - OE = (15 - 8) = 7\text{ cm}$

Solution 7:

Since the distance between the chords is greater than the radius of the circle (15 cm), so the chords will be on the opposite sides of the centre.



Let O be the centre of the circle and AB and CD be the two parallel chords such that AB = 24 cm.
Let length of CD be 2x cm.
Drop OE and OF perpendicular on AB and CD from the centre O.

$OE \perp AB$ and $OF \perp CD$

$\therefore$ OE bisects AB and OF bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad AE = \frac{24}{2} = 12 \text{ cm}; \quad CF = \frac{2x}{2} = x \text{ cm}$

In right ΔOAE,

$OA^2 = OE^2 + AE^2$

$\Rightarrow \quad OE^2 = OA^2 - AE^2 = (15)^2 - (12)^2 = 81$

$\therefore \quad OE = 9 \text{ cm}$

$\therefore \quad OF = EF - OE = (21 - 9) = 12 \text{ cm}$

In right ΔOCF,

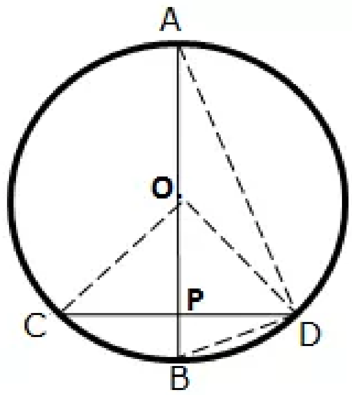
$OC^2 = OF^2 + CF^2$

$\Rightarrow \quad x^2 = OC^2 - OF^2 = (15)^2 - (12)^2 = 81$

$\therefore \quad x = 9 \text{ cm}$

Hence, length of chord CD $= 2x = 2 \times 9 = 18$ cm

Solution 8:



(i) OP $\perp$ CD

$\therefore$ OP bisects CD

(Perpendicular drawn from the centre of a circle to a chord bisects it)

$\Rightarrow \quad CP = \frac{CD}{2}$

In right ΔOPC,

$OC^2 = OP^2 + CP^2$

$\Rightarrow \quad CP^2 = OC^2 - OP^2 = (15)^2 - (9)^2 = 144$

$\therefore \quad CP = 12 \text{ cm}$

$\therefore \quad CD = 12 \times 2 = 24 \text{ cm}$

(ii) Join BD.

$\therefore \quad BP = OB - OP = 15 - 9 = 6 \text{ cm}$

In right ΔBPD,

$BD^2 = BP^2 + PD^2$

$= (6)^2 + (12)^2 = 180$

In ΔADB, $\angle ADB = 90^0$

(Angle in a semicircle is a right angle)

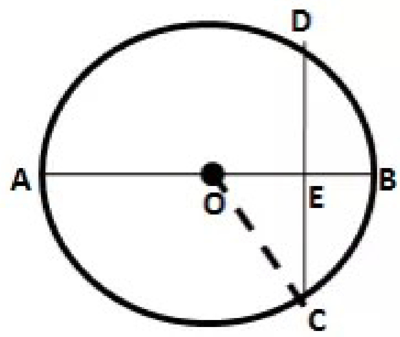
$\therefore \quad AB^2 = AD^2 + BD^2$

$\Rightarrow \quad AD^2 = AB^2 - BD^2 = (30)^2 - 180 = 720$

$\therefore \quad AD = \sqrt{720} = 26.83 \text{ cm}$

(iii) Also, $BC = BD = \sqrt{180} = 13.42$ cm

Solution 9:



Let the radius of the circle be r cm.

$\therefore$ $OE = OB - EB = r - 4$

Join OC.

In right ΔOEC,

$OC^2 = OE^2 + CE^2$

$\Rightarrow r^2 = (r-4)^2 + (8)^2$

$\Rightarrow r^2 = r^2 - 8r + 16 + 64$

$\Rightarrow 8r = 80$

$\therefore$ $r = 10$ cm

Hence, radius of the circle is 10 cm.

Solution 10:

(i) AB is the chord of the circle and OM is perpendicular to AB.

So, AM = MB = 12 cm (Since $\perp$ bisects the chord)

In right ΔOMA,

$OA^2 = OM^2 + AM^2$

$\Rightarrow OA^2 = 5^2 + 12^2$

$\Rightarrow OA = 13$ cm

So, radius of the circle is 13 cm.

(ii) So, OA = OC = 13 cm (radii of the same circle)

In right ΔONC,

$NC^2 = OC^2 - ON^2$

$\Rightarrow NC^2 = 13^2 - 12^2$

$\Rightarrow NC = 5$ cm

So, CD = 2NC = 10 cm