EXERCISE 12A

Use direct method to evaluate the following products:

(i) (x+8)(x+3)

Solution:-

$(x+8)(x+3) = (x \times x) + (x \times 3) + (8 \times x) + (8 \times 3) = x^2 + 3x + 8x + 24 = x^2 + 11x + 24$

(ii) (y+5)(y-3)

Solution:-

$(y+5)(y-3) = (y \times y) + (y \times -3) + (5 \times y) + (5 \times -3) = y^2 + (-3y) + (5y) - 15$
$= y^2 - 3y + 5y - 15 = y^2 + 2y - 15$

(iii) (a-8)(a+2)

Solution:-

$(a-8)(a+2) = (a \times a) + (a \times 2) + (-8) \times a + (-8)(2) = a^2 + 2a - 8a - 16 = a^2 - 6a - 16$

(iv) (b-3)(b-5)

Solution:-

$(b \times b) + (b \times -5) + (-3 \times b) + (-3)(-5) = b^2 - 5b - 3b + 15 = b^2 - 8b + 15$

(v) (3 x-2 y)(2 x+y)

Solution:-

$(3x - 2y)(2x + y) = (3x \times 2x) + (3x \times y) + (-2y \times 2x) + (-2y \times y) = 6x^2 + 3xy - 4xy - 2y^2$
$= 6x^2 - xy - 2y^2$

(vi) (5 a+16)(3 a-7)

Solution:-

(5 a+16)(3 a-7) = (5a×3a)+(5a×-7)+(16×3a)+(16×-7)

$= 15a^2 + (-35a) + 48a + (-112) = 15a^2 - 35a + 48a - 112 = 15a^2 + 13a - 112$

(vii) (8-b)(3+b)

Solution:-

(8-b)(3+b) = (8×3)+(8×b)+(-b×3)+(-b×b)

$= 24 + 8b - 3b - b^2 = 24 + 5b - b^2$

Question 2.

Use direct method to evaluate:

(i) (x+1)(x-1)

Solution:-

$(x+1)(x-1) = (x^2) - 1^2 = x^2 - 1$

(ii) (2+a)(2-a)

Solution:-

$(2+a)(2-a) = (2)^2 - (a^2) = 4 - a^2$

(iii) $(3b-1)(3b+1)$

Solution:-

$(3b-1)(3b+1) = (3b)^2 - (1)^2 = 9b^2 - 1$

(iv) (4+5x)(4-5x)

Solution:-

$(4+5x)(4-5x) = (4)^2 - (5)^2 = 16 - 25x^2$

(v) (2a+3)(2a-3)

Solution:-

$(2a+3)(2a-3) = 2a^2 - 3^2 = 4a^2 - 9$

(vi) (xy+4)(xy-4)

Solution:-

$(xy+4)(xy-4) = xy^2 - 4^2 = x^2y^2 - 16$

(vii) $(ab+x^2)\ (ab-x^2)$

Solution:-

$(ab+x^2)\ (ab-x^2) = (ab)^2 - (x^2)^2 = a^2b^2 - x^4$

(viii) $(3x^2+5y^2)\ (3x^2-5y^2)$

Solution:-

$(3x^2+5y^2)\ (3x^2-5y^2) = (3x^2)^2 - (5y^2)^2 = 9x^4 - 25y^4$

(ix) $(z-\frac{2}{3})\ (z+\frac{2}{3})$

Solution:-

$(z-\frac{2}{3})\ (z+\frac{2}{3}) = (z)^2 - (\frac{2}{3})^2 = z^2 - \frac{4}{9}$

(x) $(\frac{3}{5}a+\frac{1}{2})\ (\frac{3}{5}a-\frac{1}{2})$

Solution:-

$= (\frac{3}{5}a)^2 - (\frac{1}{2})^2 = \frac{9}{25}a^2 - \frac{1}{4}$

(xi) (0·5-2a)(0·5+2a)

Solution:-

$= (0 \cdot 5)^2 - (2a)^2 = 0 \cdot 25 - 4a^2$

(xii) $\left(\frac{a}{2}-\frac{b}{3}\right)\left(\frac{a}{2}+\frac{b}{3}\right)$

Solution:-

$=\frac{a^2}{4}-\frac{b^2}{9}$

Question 3.

Evaluate:

(i) $(a+1)(a-1)\left(a^2+1\right)$

Solution:-

$=\left[(a)^2-(1)^2\right]\left(a^2+1\right)=\left(a^2-1\right)\left(a^2+1\right)=\left(a^2\right)^2-(1)^2=a^4-1$

(ii) $(a+b)(a-b)\left(a^2+b^2\right)$

Solution:-

$=\left(a^2-b^2\right)\left(a^2+b^2\right)=\left(a^2\right)^2-\left(b^2\right)^2=a^4-b^4$

(iii) $(2a-b)(2a+b)\left(4a^2+b^2\right)$

Solution:-

$=\left[(2a)^2-(b)^2\right]\left(4a^2+b^2\right)=\left(4a^2-b^2\right)\left(4a^2+b^2\right)=\left(4a^2\right)^2-\left(b^2\right)^2=16a^4-b^4$

(iv) $(3-2x)(3+2x)\left(9+4x^2\right)$

Solution:-

$=\left[(3)^2-(2x)^2\right]\left(9+4x^2\right)=\left(9-4x^2\right)\left(9+4x^2\right)=(9)^2-\left(4x^2\right)^2=81-16x^4$

(v) $(3x-4y)(3x+4y)\left(9x^2+16y^2\right)$

Solution:-

$=\left[(3x)^2-(4y)^2\right]\left(9x^2+16y^2\right)=\left(9x^2-16y^2\right)\left(9x^2+16y^2\right)=\left(9x^2\right)^2-\left(16y^2\right)^2$

$=81x^4-256y^4$

Question 4.

Use the product $(a+b)(a-b)=a^2-b^2$ to evaluate:

(i) (21×19)

Solution:-

$=21\times19=(20+1)(20-1)=(20)^2-(1)^2=400-1=399$

(ii) (33×27)

Solution:-

$= 33 \times 27 = (30 + 3)(30 - 3) = (30)^2 - (3)^2 = 900 - 9 = 891$

(iii) (103×97)

Solution:-

$(103 \times 97 = (100 + 3)(100 - 3) = (100)^2 - (3)^2 = 10000 - 9 = 9991$

(iv) (9.8×10.2)

Solution:-

$= 9.8 \times 10.2 = (10 - .2)(10 + .2) = (10)^2 - (.2)^2 = 100 - .04 = 99.96$

(v) (7.7×8.3)

Solution:-

$= 7.7 \times 8.3 = (8 - .3)(8 + .3) = (8)^2 - (.3)^2 = 64 - .09 = 63.91$

(vi) (4.6×5.4)

Solution:-

$= 4.6 \times 5.4 = (5 - .4)(5 + .4) = (5)^2 - (.4)^2 = 25 - .16 = 24.84$

Question 5.

Evaluate:

(i) (6-xy)(6+xy)

Solution:-

(6-xy)(6+xy)=6(6+xy)-xy(6+xy)

$= 36 + 6xy - 6xy + (xy)^2 = 36 - x^2y^2$

(ii) $\left(7x + \frac{2}{3}y\right)\left(7x - \frac{2}{3}y\right)$

Solution:-

$= 7x\left(7x - \frac{2}{3}y\right) + \frac{2}{3}y\left(7x - \frac{2}{3}y\right) = 49x^2 - \frac{14}{3}xy + \frac{14}{3}xy - \frac{4}{9}y^2 = 49x^2 - \frac{4}{9}y^2$

(iii) $\left(\frac{a}{2b} + \frac{2b}{a}\right)\left(\frac{a}{2b} - \frac{2b}{a}\right)$

Solution:-

$= \frac{a}{2b}\left(\frac{a}{2b} - \frac{2b}{a}\right) + \frac{2b}{a}\left(\frac{a}{2b} - \frac{2b}{a}\right) = \frac{a^2}{4b^2} - 1 + 1 - \frac{4b^2}{a^2} = \frac{a^2}{4b^2} - \frac{4b^2}{a^2}$

(iv) $\left(3x - \frac{1}{2y}\right)\left(3x + \frac{1}{2y}\right)$

Solution:-

$= 3x\left(3x + \frac{1}{2y}\right) - \frac{1}{2y}\left(3x + \frac{1}{2y}\right) = 9x^2 + \frac{3x}{2y} - \frac{3x}{2y} - \frac{1}{4y^2} = 9x^2 - \frac{1}{4y^2}$

(v) $(2a+3)(2a-3)\left(4a^2+9\right)$

Solution:-

$=\left[(2a)^2-(3)^2\right]\left(4a^2+9\right)\left[(a+b)(a-b)=a^2-b^2\right]=\left(4a^2-9\right)\left(4a^2+9\right)$
$=\left(4a^2\right)^2-(9)^2\left[(a+b)(a-b)=a^2-b^2\right]=16a^4-81$
(vi) $(a+bc)(a-bc)\left(a^2+b^2c^2\right)$

Solution:-

$=\left[(a)^2-(bc)^2\right]\left(a^2+b^2c^2\right)=\left[(a+b)(a-b)=a^2-b^2\right]=\left(a^2-b^2c^2\right)\left(a^2+b^2c^2\right)$
$=\left(a^2\right)^2-\left(b^2c^2\right)^2\left[\because \quad (a+b)(c-b)=a^2-b^2\right]=\left(a^2\right)^2-\left(b^2c^2\right)^2\left[\because (a+b)(c-b)=a^2-b^2\right]$
$=a^4-b^4c^4$
(vii) (5x+8y)(3x+5y)

Solution:-

=5x(3x+5y)+8y(3x+5y)

$=15x^2+25xy+24xy+40y^2=15x^2+49xy+40y^2$
(viii)(7x+15y)(5x-4y)

Solution:-

=7x(5x-4y)+15y(5x-4y)

$=35x^2-28xy+75xy-60y^2=35x^2+47xy-60y^2$
(ix)(2a-3b)(3a+4b)

Solution:-

=2a(3a+4b)-3b(3a+4b)

$=6a^2+8ab-9ab-12b^2=6a^2-ab-12b^2$
(x)(9a-7b)(3a-b)

Solution:-

=9a(3a-b)-7b(3a-b)

$=27a^2-9ab-21ab+7b^2=27a^2-30ab+7b^2$