EXERCISE 33.2

A coin is tossed. Find the total number of elementary events and also the total number of events associated with the random experiment.

Solution:

Given: A coin is tossed.

When a coin is tossed, there will be two possible outcomes, Head (H) and Tail (T).

Since, the no. of elementary events is 2 {H, T}

We know, if there are n elements in a set, then the number of total element in its subset is 2n.

So, the total number of the experiment is 4,

There are 4 subset of S = {H}, {T}, {H, T} and Φ

∴ There are total 4 events in a given experiment.

1. List all events associated with the random experiment of tossing of two coins. How many of them are elementary events?

Solution:

Given: Two coins are tossed once.

We know, when two coins are tossed then the no. of possible outcomes are $2^2 = 4$

So, the Sample spaces are {HH, HT, TT, TH}

∴ There are total 4 events associated with the given experiment.

2. Three coins are tossed once. Describe the following events associated with this random experiment:

A = Getting three heads, B = Getting two heads and one tail, C = Getting three tails, D = Getting a head on the first coin.

(i) Which pairs of events are mutually exclusive?

(ii) Which events are elementary events?

(iii) Which events are compound events?

Solution:

Given: There are three coins tossed once.

When three coins are tossed, then the sample spaces are:

S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

So, according to the question,

A = {HHH}

B = {HHT, HTH, THH}

C = {TTT}

D = {HHH, HHT, HTH, HTT}

Now, $A \cap B = \Phi$,
$A \cap C = \Phi$,
$A \cap D = \{HHH\}$

$B \cap C = \Phi$,
$B \cap D = \{HHT, HTH\}$

$C \cap D = \Phi$
We know that, if the intersection of two sets are null or empty it means both the sets are Mutually Exclusive.

(i) Events A and B, Events A and C, Events B and C and events C and D are mutually exclusive.

(ii) Here, We know, if an event has only one sample point of a sample space, then it is called elementary events.
So, A and C are elementary events.

(iii) If there is an event that has more than one sample point of a sample space, it is called a compound event.
Since, $B \cap D = \{HHT, HTH\}$
So, B and D are compound events.

3. In a single throw of a die describe the following events:
(i) A = Getting a number less than 7
(ii) B = Getting a number greater than 7
(iii) C = Getting a multiple of 3
(iv) D = Getting a number less than 4
(v) E = Getting an even number greater than 4.
(vi) F = Getting a number not less than 3.
Also, find $A \cup B$, $A \cap B$, $B \cap C$, $E \cap F$, $D \cap F$ and $\bar{F}$.
Solution:
Given: A dice is thrown once.
Let us find the given events, and also find $A \cup B$, $A \cap B$, $B \cap C$, $E \cap F$, $D \cap F$ and $\bar{F}$.
$S = \{1, 2, 3, 4, 5, 6\}$
According to the subparts of the question, we have certain events as:

(i) A = getting a number below 7

So, the sample spaces for A are:
A = {1, 2, 3, 4, 5, 6}

(ii) B = Getting a number greater than 7
So, the sample spaces for B are:
B = {Φ}

(iii) C = Getting multiple of 3
So, the Sample space of C is
C = {3, 6}

(iv) D = Getting a number less than 4
So, the sample space for D is
D = {1, 2, 3}

(v) E = Getting an even number greater than 4.
So, the sample space for E is
E = {6}

(vi) F = Getting a number not less than 3.
So, the sample space for F is
F = {3, 4, 5, 6}

Now,
A = {1, 2, 3, 4, 5, 6} and B = {Φ}
A ∪ B = {1, 2, 3, 4, 5, 6}

A = {1, 2, 3, 4, 5, 6} and B = {Φ}
A ∩ B = {Φ}

B = {Φ} and C = {3, 6}
B ∩ C = {Φ}

F = {3, 4, 5, 6} and E = {6}
E ∩ F = {6}

E = {6} and D = {1, 2, 3}
D ∩ F = {3}

And, for $\bar{F} = S - F$
$S = \{1, 2, 3, 4, 5, 6\}$ and $F = \{3, 4, 5, 6\}$
$\bar{F} = \{1, 2\}$
∴ These are the events for given experiment.

4. Three coins are tossed. Describe
(i) two events A and B which are mutually exclusive.
(ii) three events A, B and C which are mutually exclusive and exhaustive.
(iii) two events A and B which are not mutually exclusive.
(iv) two events A and B which are mutually exclusive but not exhaustive.
Solution:
Given: Three coins are tossed.
When three coins are tossed, then the sample space is
S = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

Now, the subparts are:
(i) The two events which are mutually exclusive are when,
A: getting no tails
B: getting no heads
Then, A = {HHH} and B = {TTT}
So, the intersection of this set will be null. Or, the sets are disjoint.

(ii) Three events which are mutually exclusive and exhaustive are:
A: getting no heads
B: getting exactly one head
C: getting at least two head
So, A = {TTT} B = {TTH, THT, HTT} and C = {HHH, HHT, HTH, THH}
Since, $A \cap B = B \cap C = C \cap A = \Phi$ and
$A \cup B \cup C = S$

(iii) The two events that are not mutually exclusive are:
A: getting three heads
B: getting at least 2 heads
So, A = {HHH} B = {HHH, HHT, HTH, THH}
Hence, $A \cap B = \{HHH\} = \Phi$

(iv) The two events which are mutually exclusive but not exhaustive are:
A: getting exactly one head

B: getting exactly one tail
So, A = {HTT, THT, TTH} and B = {HHT, HTH, THH}
It is because $A \cap B = \Phi$ but $A \cup B \neq S$

5. A die is thrown twice. Each time the number appearing on it is recorded. Describe the following events:
(i) A = Both numbers are odd.
(ii) B = Both numbers are even
(iii) C = sum of the numbers is less than 6.
Also, find A ∪ B, A ∩ B, A ∪ C, A ∩ C. Which pairs of events are mutually exclusive?
Solution:
Given: A dice is thrown twice. And each time number appearing on it is recorded.
When the dice is thrown twice then the number of sample spaces are $6^2 = 36$
Now,
The possibility both odd numbers are:
A = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5)}

Since, possibility of both even numbers is:
B = {(2, 2), (2, 4), (2, 6), (4, 2), (4, 4), (4, 6), (6, 2), (6, 4), (6, 6)}
And, possible outcome of sum of the numbers is less than 6.

C = {(1, 1)(1, 2)(1, 3)(1, 4)(2, 1)(2, 2)(2, 3)(3, 1)(3, 2)(4, 1)}
Hence,
(AUB) = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5) (2, 2)(2, 4)(2, 6)(4, 2)(4, 4)(4, 6)(6, 2)(6, 4)(6, 6)}

$(A \cap B) = \{\Phi\}$

(AUC) = {(1, 1), (1, 3), (1, 5), (3, 1), (3, 3), (3, 5), (5, 1), (5, 3), (5, 5) (1, 2)(1, 4)(2, 1)(2, 2)(2, 3)(3, 1)(3, 2)(4, 1)}

$(A \cap C)$ = {(1, 1), (1, 3), (3, 1)}

$\therefore (A \cap B) = \Phi$ and $(A \cap C) \neq \Phi$, A and B are mutually exclusive, but A and C are not.