EXERCISE PAGE NO: 178

SHORT ANSWER TYPE:

1. Find the equation of the straight line which passes through the point (1, – 2) and cuts off equal intercepts from axes.

Solution:

The equation of line in intercept form is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Where a and b are the intercepts on the axis.

Given that a = b

$$\Rightarrow \frac{x}{a} + \frac{y}{a} = 1$$

The above equation can be written as

$$\Rightarrow \frac{x + y}{a} = 1$$

On cross multiplication we get

$\Rightarrow x + y = a$ …..1

If equation 1 passes through the point (1, - 2), we get

x = 1 and y = -2

1 + (-2) = a

$\Rightarrow 1 - 2 = a$

$\Rightarrow a = -1$

Putting the value of a in equation 1, we get

x + y = -1

$\Rightarrow x + y + 1 = 0$

Hence, the equation of straight line is x + y + 1 = 0 which passes through the point (1, – 2).

2. Find the equation of the line passing through the point (5, 2) and perpendicular to the line joining the points (2, 3) and (3, – 1).

Solution:

Given points are A (5, 2), B (2, 3) and C (3, -1)

Firstly, we find the slope of the line joining the points (2, 3) and (3, -1)

Slope of the line joining two points $= \dfrac{y_2 - y_1}{x_2 - x_1}$

$\therefore m_{BC} = \dfrac{-1-3}{3-2} = -\dfrac{4}{1} = -4$

It is given that line passing through the point (5, 2) is perpendicular to BC

$\because m_1 m_2 = -1$

$\Rightarrow -4 \times m_2 = -1$

$\Rightarrow m_2 = \frac{1}{4}$

Therefore slope of the required line = $\frac{1}{4}$

Now, we have to find the equation of line passing through point (5, 2)

Equation of line: $y - y_1 = m(x - x_1)$

$\Rightarrow y - 2 = \dfrac{1}{4}(x - 5)$

$\Rightarrow 4y - 8 = x - 5$

$\Rightarrow x - 5 - 4y + 8 = 0$

$\Rightarrow x - 4y + 3 = 0$

Hence, the equation of line passing through the point (5, 2) is $x - 4y + 3 = 0$

3. Find the angle between the lines $y = \left(2-\sqrt{3}\right)(x+5)$ **and** $y = \left(2+\sqrt{3}\right)(x-7)$

Solution:

Given equations are $y = (2 - \sqrt{3})(x + 5)$ and $(2 + \sqrt{3})(x - 7)$

The given equation can be written as

$\Rightarrow y = (2 - \sqrt{3})x + (2 - \sqrt{3})5$ 1

$\Rightarrow y = (2 + \sqrt{3})x - 7(2 + \sqrt{3})$ 2

Now, we have to find the slope of equation 1

Since, the equation 1 is in $y = mx + b$ form, we can easily see that the slope (m_1) is $(2 - \sqrt{3})$

Now, the slope (m_2) of equation 2 is $(2 + \sqrt{3})$

Let θ be the angle between the given two lines.

$\tan\theta = \left|\dfrac{m_1 - m_1}{1 + m_1 m_2}\right|$

Putting the values of m_1 and m_2 in above equation, we get

$$\Rightarrow \tan\theta = \left|\frac{2-\sqrt{3}-(2+\sqrt{3})}{1+(2-\sqrt{3})(2+\sqrt{3})}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{2-\sqrt{3}-2-\sqrt{3}}{1+[(2)^2-(\sqrt{3})^2]}\right|$$

[∵ $(a - b)(a + b) = (a^2 - b^2)$]

$$\Rightarrow \tan\theta = \left|\frac{-2\sqrt{3}}{1+[4-3]}\right|$$

On simplifying we get

$$\Rightarrow \tan\theta = \left|\frac{-2\sqrt{3}}{1+1}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{-2\sqrt{3}}{2}\right|$$

⇒ Tan θ = √3 or - √3

⇒ $\theta = \tan^{-1}(\sqrt{3})$ or $(-\sqrt{3})$

⇒ θ = 60° or 120°

Hence, the required angle is 60° or 120°.

4. Find the equation of the lines which passes through the point (3, 4) and cuts off intercepts from the coordinate axes such that their sum is 14.

Solution:

The equation of line in intercept form is

$\frac{x}{a} + \frac{y}{b} = 1$ 1

Where a and b are the intercepts on the axis.

Given that a + b = 14

The above equation can be written as

⇒ b = 14 – a

Substituting the value of a and b in equation 1 we get

So, equation of line is

$$\frac{x}{a} + \frac{y}{14-a} = 1$$

Taking LCM

$\Rightarrow \frac{x(14-a)+ay}{(a)(14-a)} = 1$

$\Rightarrow 14x - ax + ay = 14a - a^2$ 2

If equation 2 passes through the point (3, 4) then

$14(3) - a(3) + a\ 4) = 14a - a^2$

$\Rightarrow 42 - 3a + 4a - 14a + a^2 = 0$

$\Rightarrow a^2 - 13a + 42 = 0$

$\Rightarrow a^2 - 7a - 6a + 42 = 0$

$\Rightarrow a(a-7) - 6(a-7) = 0$

$\Rightarrow (a-6)(a-7) = 0$

$\Rightarrow a - 6 = 0$ or $a - 7 = 0$

$\Rightarrow a = 6$ or $a = 7$

If a = 6, then

$6 + b = 14$

$\Rightarrow b = 14 - 6 = 8$

If a = 7, then

$7 + b = 14$

$\Rightarrow b = 14 - 7$

$= 7$

If a = 6 and b = 8, then equation of line is

$\frac{x}{6} + \frac{y}{8} = 1$

$\Rightarrow \frac{4x + 3y}{24} = 1$

$\Rightarrow 4x + 3y = 24$

If a = 7 and b = 7, then equation of line is

$\frac{x}{7} + \frac{y}{7} = 1$

$\Rightarrow x + y = 7$

5. Find the points on the line x + y = 4 which lie at a unit distance from the line 4x + 3y = 10.

Solution:

Let (x_1, y_1) be any point lying in the equation x+ y = 4

$\therefore x_1 + y_1 = 4$ 1

Distance of the point (x_1, y_1) from the equation $4x + 3y = 10$

$$d = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$\Rightarrow 1 = \left|\frac{4x_1 + 3y_1 - 10}{\sqrt{(4)^2 + (3)^2}}\right|$$

On simplification we get

$$\Rightarrow 1 = \left|\frac{4x_1 + 3y_1 - 10}{\sqrt{16 + 9}}\right|$$

$$\Rightarrow 1 = \left|\frac{4x_1 + 3y_1 - 10}{5}\right|$$

$\Rightarrow 4x_1 + 3y_1 - 10 = \pm 5$

$4x_1 + 3y_1 - 10 = 5$ or $4x_1 + 3y_1 - 10 = -5$

$4x_1 + 3y_1 = 5 + 10$ or $4x_1 + 3y_1 = -5 + 10$

$4x_1 + 3y_1 = 15$ 2

Or $4x_1 + 3y_1 = 5$ 3

From equation 1, we have $y_1 = 4 - x_1$ 4

Putting the value of y_1 in equation 2, we get

$4x_1 + 3(4 - x_1) = 15$

$\Rightarrow 4x_1 + 12 - 3x_1 = 15$

$\Rightarrow x_1 = 15 - 12$

$\Rightarrow x_1 = 3$

Putting the value of x_1 in equation 4, we get

$y_1 = 4 - 3$

$\Rightarrow y_1 = 1$

Putting the value of $y_1 = 4 - x_1$ in equation 3, we get

$4x_1 + 3(4 - x_1) = 5$

$\Rightarrow 4x_1 + 12 - 3x_1 = 5$

$\Rightarrow x_1 = 5 - 12$

$\Rightarrow x_1 = -7$

Putting the value of x_1 in equation 4, we get

$y_1 = 4 - (-7)$

$\Rightarrow y_1 = 4 + 7$

$\Rightarrow y_1 = 11$

Hence, the required points on the given line are (3, 1) and (-7, 11)

6. Show that the tangent of an angle between the lines $\frac{x}{a}+\frac{y}{b}=1$ and $\frac{x}{a}-\frac{y}{b}=1$ is $\frac{2ab}{a^2-b^2}$

Solution:

Given

$\frac{x}{a}+\frac{y}{b}=1$ ……1

$\frac{x}{a}-\frac{y}{b}=1$ …….2

Firstly, we find the slope of the given lines

$\frac{x}{a}+\frac{y}{b}=1$

Above equation can be written as

$\Rightarrow \frac{y}{b}=1-\frac{x}{a}$

On rearranging we get

$\Rightarrow y=b-\frac{b}{a}x$

$\Rightarrow y=\left(-\frac{b}{a}\right)x+b$

Since, the above equation is in y = mx + b form.

So, Slope of the equation 1 is

$m_1=-\frac{b}{a}$

Now, finding the slope of the equation 2

$\frac{x}{a}-\frac{y}{b}=1$

The above equation can be written as

$\Rightarrow -\frac{y}{b}=1-\frac{x}{a}$

$\Rightarrow -y=b-\frac{b}{a}x$

On rearranging we get

$$\Rightarrow y = \left(\frac{b}{a}\right)x - b$$

$$\Rightarrow y = \left(\frac{b}{a}\right)x + (-1)b$$

Since, the above equation is in y = mx + b form.

So, Slope of the eq. (ii) is

$$m_2 = \frac{b}{a}$$

Let θ be the angle between the given two lines.

$$\tan\theta = \left|\frac{m_1 - m_1}{1 + m_1 m_2}\right|$$

Putting the values of m_1 and m_2 in above eq., we get

$$\Rightarrow \tan\theta = \left|\frac{-\frac{b}{a} - \frac{b}{a}}{1 + \left(-\frac{b}{a}\right)\left(\frac{b}{a}\right)}\right|$$

On simplifying we get

$$\Rightarrow \tan\theta = \left|\frac{-2\left(\frac{b}{a}\right)}{1 - \left(\frac{b^2}{a^2}\right)}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{-2\left(\frac{b}{a}\right)}{\frac{a^2 - b^2}{a^2}}\right|$$

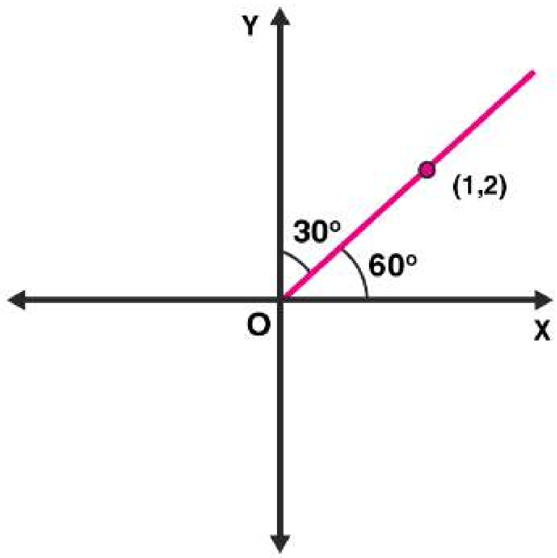
$$\Rightarrow \tan\theta = \left|\frac{-2ab}{a^2 - b^2}\right|$$

$$\Rightarrow \tan\theta = \frac{2ab}{a^2 - b^2}$$

Hence the proof.

7. Find the equation of lines passing through (1, 2) and making angle 30° with y-axis.

Solution:



Given that line passing through (1, 2) making an angle 30° with y – axis.
Angle made by the line with x – axis is (90° - 30°) = 60°
∴ Slope of the line, m = tan 60°
= √3
So, the equation of the line passing through the point (x_1, y_1) and having slope 'm' is
$y – y_1 = m (x – x_1)$
Here, (x_1, y_1) = (1, 2) and m = √3
⇒ y – 2 = √3(x – 1)
⇒ y – 2 = √3x - √3
⇒ y - √3x + √3 - 2 = 0

8. Find the equation of the line passing through the point of intersection of 2x + y = 5 and x + 3y + 8 = 0 and parallel to the line 3x + 4y = 7.

Solution:
Given lines are
2x + y = 5 ……1
x + 3y = -8 ……2
Firstly, we find the point of intersection of equation 1 and equation 2
Multiply the equation 2 by 2, we get
2x + 6y = -16 …….3
On subtracting equation 3 from 1, we get
2x + y – 2x – 6y = 5 – (-16)
On simplifying we get

⇒ -5y = 5 + 16

⇒ -5y = 21

$\Rightarrow y = -\frac{21}{5}$

Putting the value of y in equation 1, we get

$2x + \left(-\frac{21}{5}\right) = 5$

On rearranging we get

$\Rightarrow 2x = 5 + \frac{21}{5}$

$\Rightarrow 2x = \frac{25 + 21}{5}$

⇒ 10x = 46

$\Rightarrow x = \frac{46}{10} = \frac{23}{5}$

Hence, the point of intersection is $\left(\frac{23}{5}, -\frac{21}{5}\right)$

Now, we find the slope of the given equation 3x + 4y = 7

We know that the slope of an equation is

m = - a/b

$\Rightarrow m = -\frac{3}{4}$

So, the slope of a line which is parallel to this line is also $-\frac{3}{4}$

Then the equation of the line passing through the point $\left(\frac{23}{5}, -\frac{21}{5}\right)$ having slope $-\frac{3}{4}$ is:

$y - y_1 = m (x - x_1)$

Substituting the values we get

$\Rightarrow y - \left(-\frac{21}{5}\right) = -\frac{3}{4}\left(x - \frac{23}{5}\right)$

Computing and simplifying

$\Rightarrow y + \frac{21}{5} = -\frac{3}{4}x + \frac{69}{20}$

$\Rightarrow \frac{3}{4}x + y = \frac{69}{20} - \frac{21}{5}$

$$\Rightarrow \frac{3x + 4y}{4} = \frac{69 - 84}{20}$$

$$\Rightarrow 3x + 4y = -\frac{15}{5}$$

$$\Rightarrow 3x + 4y + 3 = 0$$

9. For what values of a and b the intercepts cut off on the coordinate axes by the line ax + by + 8 = 0 are equal in length but opposite in signs to those cut off by the line 2x – 3y + 6 = 0 on the axes.

Solution:

Given equation is ax + by + 8 = 0

It can also be re-written as ax + by = -8

Now, dividing by -8 to both the sides, we get

$$\frac{a}{-8}x + \frac{b}{-8}y = 1$$

$$\Rightarrow \frac{x}{-\frac{8}{a}} + \frac{y}{-\frac{8}{b}} = 1$$

So, the intercepts on the axes are $-\frac{8}{a}$ and $-\frac{8}{b}$

Now, the second equation which is given is 2x – 3y + 6 = 0

It can also be re-written as 2x – 3y = -6

Now, dividing by -6 to both the sides, we get

$$\frac{2}{-6}x - \frac{3}{-6}y = 1$$

$$\Rightarrow \frac{x}{-3} + \frac{y}{2} = 1$$

So, the intercepts are -3 and 2

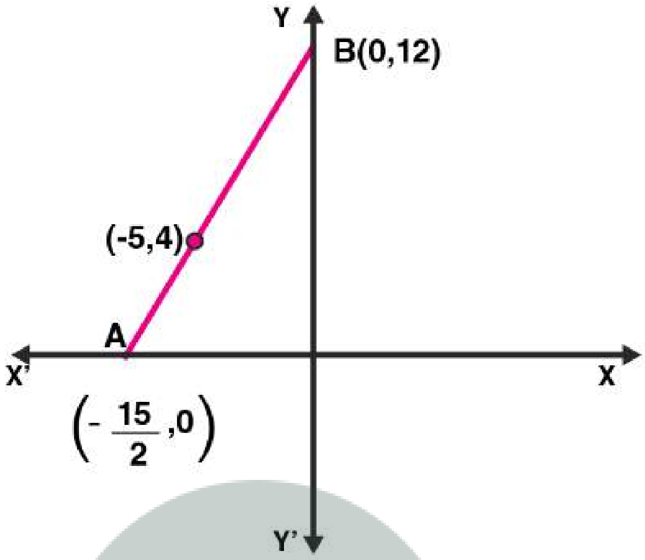
Now, according to the question intercepts cut off on the coordinate axes by the line ax + by + 8 = 0 are equal in length but opposite in signs to those cut off by the line 2x – 3y + 6 = 0 on the axes

Therefore, $-\frac{8}{a} = 3$ and $-\frac{8}{b} = -2$

$\Rightarrow a = -\frac{8}{3}$ And $\Rightarrow b = 4$

10. If the intercept of a line between the coordinate axes is divided by the point (–5, 4) in the ratio 1:2, then find the equation of the line.

Solution:



Let a and b be the intercepts on the given line

∴ Coordinates of A and B are (a, 0) and (0, b) respectively.

Given that coordinate axes is divided by the point (-5, 4) in the ratio 1:2.

Now, using the section formula, we find the value of a and b

$$(x, y) = \left(\frac{m_1x_2 + m_2x_1}{m_1 + m_2}, \frac{m_1y_2 + m_2y_1}{m_1 + m_2}\right)$$

$$\therefore (-5, 4) = \left(\frac{1 \times 0 + 2 \times a}{1 + 2}, \frac{1 \times b + 2 \times 0}{1 + 2}\right)$$

On simplifying we get

$$\Rightarrow (-5, 4) = \left(\frac{2a}{3}, \frac{b}{3}\right)$$

$$\Rightarrow -5 = \frac{2a}{3} \text{ and } 4 = \frac{b}{3}$$

$\Rightarrow$-15 = 2a and b = 12

$$\Rightarrow a = -\frac{15}{2} \text{ and } b = 12$$

∴ Coordinates of A and B are $\left(-\frac{15}{2}, 0\right)$ and $(0,12)$

Now, we have to find the equation of line AB.

Now, we have to find the equation of line AB.
Equation of line when two points are given:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Substituting the values, we get

$$y - 0 = \frac{12 - 0}{0 - \left(-\frac{15}{2}\right)}\left(x - \left(-\frac{15}{2}\right)\right)$$

On simplifying we get

$$\Rightarrow y = \frac{12}{\frac{15}{2}}\left(x + \frac{15}{2}\right)$$

$$\Rightarrow y = \frac{24}{15}\left(x + \frac{15}{2}\right)$$

$$\Rightarrow y = \frac{8}{5}x + 12$$

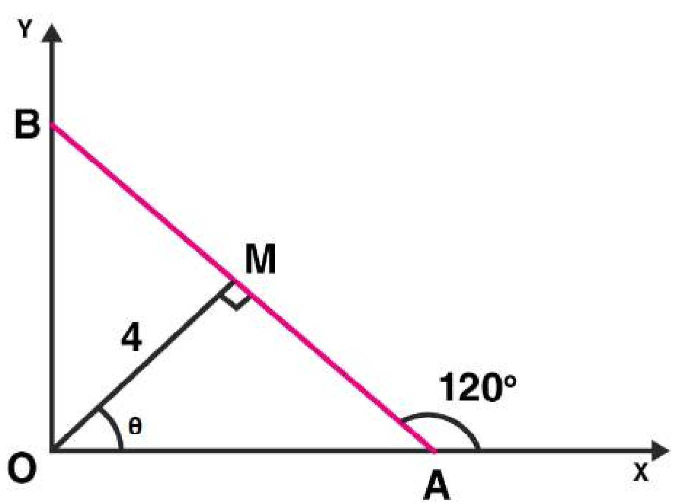
⇒ 5y = 8x + 60

⇒ 8x – 5y + 60 = 0

Hence, the required equation is 8x – 5y + 60 = 0

11. Find the equation of a straight line on which length of perpendicular from the origin is four units and the line makes an angle of 120° with the positive direction of x-axis.

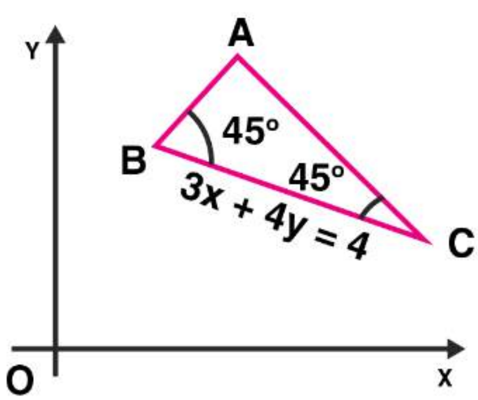
Solution:



Given length of the perpendicular from the origin (OM) = 4 units
And line makes an angle with positive direction of x – axis
$\angle BAX = 120°$
$\therefore \angle BAO = 180° - 120° = 60°$
$\therefore \angle MAO = 60°$
Now, In Δ AMO,
We know that sum of angles of a triangle is 180°
$\angle MAO + \angle AOM + \angle OMA = 180°$
$\Rightarrow 60° + \theta + 90° = 180°$
$\Rightarrow 150° + \theta = 180°$
$\Rightarrow \theta = 180° - 150°$
$\Rightarrow \theta = 30°$
$\therefore \angle AOM = 30°$
Now, we find the equation in normal form
$X \cos \theta + y \sin \theta = p$
Given p = 4, substituting this we get
$\Rightarrow x \cos (30°) + y \sin (30°) = 4$
$$\Rightarrow x\left(\frac{\sqrt{3}}{2}\right) + y\left(\frac{1}{2}\right) = 4$$
$\Rightarrow \sqrt{3}x + y = 8$
Hence, the required equation is $\sqrt{3}x + y = 8$

12. Find the equation of one of the sides of an isosceles right angled triangle whose hypotenuse is given by 3x + 4y = 4 and the opposite vertex of the hypotenuse is (2, 2).

Solution:



Given that equation of the hypotenuse is 3x + 4y = 4

And also given that opposite vertex of the hypotenuse is (2, 2)

Firstly, we find the slope of the given equation

3x + 4y = 4

It can be re-written as 4y = 4 – 3x

$$\Rightarrow y = \frac{-3}{4}x + 1$$

Since, the above equation is in y = mx + b form

So, slope = - ¾

Now, let the slope of AC be m

Now, we find the value of m, by using the formula

$$\tan\theta = \left|\frac{m_1 - m_1}{1 + m_1 m_2}\right|$$

Substituting the values of m_1 and m_2 in above equation, we get

$$\tan 45° = \left|\frac{m - \left(-\frac{3}{4}\right)}{1 + m \times \left(-\frac{3}{4}\right)}\right|$$

We know that tan 45° = 1

$$\Rightarrow 1 = \left|\frac{m + \frac{3}{4}}{1 - \frac{3}{4}m}\right|$$

$$\Rightarrow 1 = \left|\frac{4m + 3}{4 - 3m}\right|$$

$$\Rightarrow 1 = \pm\frac{4m + 3}{4 - 3m}$$

Taking (+) sign, we get

$$\frac{4m + 3}{4 - 3m} = 1$$

On cross multiplication we get

⇒ 4m + 3 = 4 – 3m

⇒ 4m + 3m = 4 – 3

⇒ 7m = 1

$$\Rightarrow m = \frac{1}{7}$$

Taking (-) sign, we get

$-\dfrac{4m+3}{4-3m} = 1$

⇒ 4m + 3 = - (4 – 3m)

⇒ 4m + 3 = - 4 + 3m

⇒ 4m – 3m = - 4 – 3

⇒ m = -7

If m = 1/7, then equation of AC is

$y - y_1 = m (x - x_1)$

$\Rightarrow y - 2 = \dfrac{1}{7}(x-2)$

⇒ 7y – 14 = x – 2

⇒ x – 7y – 2 + 14 = 0

⇒ x – 7y + 12 = 0

If m = -7, then equation of AC is

y – 2 = (-7) (x – 2)

⇒ y – 2 = -7x + 14

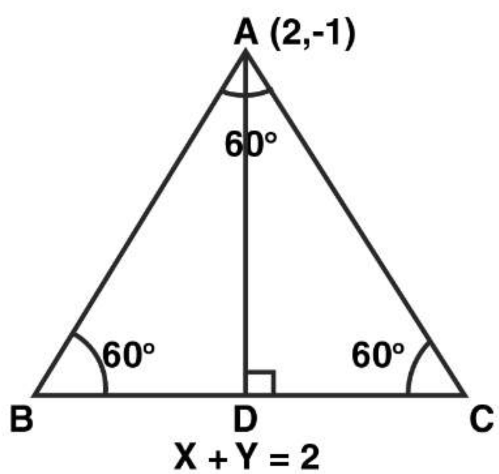
⇒ 7x + y = 16

Hence, the required equations are x – 7y + 12 = 0 and 7x + y = 16

LONG ANSWER TYPE

13. If the equation of the base of an equilateral triangle is x + y = 2 and the vertex is (2, – 1), then find the length of the side of the triangle.

Solution:



Let Δ ABC be an equilateral triangle.
Given equation of the base BC is x + y = 2
We know that, in an equilateral triangle all angles are of 60°
So, in Δ ABD

$$\sin 60° = \frac{AD}{AB}$$

$$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AD}{AB} \left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$$

$$\Rightarrow AD = \frac{\sqrt{3}}{2} AB$$

We know that, the distance d of a point P(x_0, y_0) from the line Ax + By + C = 0 is given by

$$d = \left|\frac{Ax_0 + By_0 + C}{\sqrt{A^2 + B^2}}\right|$$

Now, length of perpendicular from vertex A (2, -1) to the line x + y = 2 is

$$AD = \left|\frac{1 \times 2 + 1 \times (-1) - 2}{\sqrt{(1)^2 + (1)^2}}\right|$$

$$\Rightarrow \frac{\sqrt{3}}{2} AB = \left|\frac{2 - 1 - 2}{\sqrt{2}}\right|$$

On simplification we get

$$\Rightarrow \frac{\sqrt{3}}{2} AB = \frac{1}{\sqrt{2}}$$

Squaring both the sides, we get

$$\Rightarrow \frac{3}{4} AB^2 = \frac{1}{2}$$

On cross multiplication we get

$$\Rightarrow AB^2 = \frac{4}{3} \times \frac{1}{2}$$

$$\Rightarrow AB^2 = \frac{2}{3}$$

$$\Rightarrow AB = \sqrt{\frac{2}{3}}$$

Hence, the required length of side is $\sqrt{\frac{2}{3}}$

14. A variable line passes through a fixed point P. The algebraic sum of the perpendiculars drawn from the points (2, 0), (0, 2) and (1, 1) on the line is zero. Find the coordinates of the point P.

Solution:

Let the variable line be ax + by = 1

We know that, length of the perpendicular from (p, q) to the line ax + by + c = 0 is

$$d = \left|\frac{ap + bq + c}{\sqrt{a^2 + b^2}}\right|$$

Now, perpendicular distance from A (2, 0)

$$d_1 = \left|\frac{2 \times a + 0 \times b - 1}{\sqrt{a^2 + b^2}}\right|$$

$$= \frac{2a - 1}{\sqrt{a^2 + b^2}}$$

Now, perpendicular distance from B (0, 2)

$$d_2 = \left|\frac{0 \times a + 2 \times b - 1}{\sqrt{a^2 + b^2}}\right|$$

$$= \frac{2b - 1}{\sqrt{a^2 + b^2}}$$

Now, perpendicular distance from C (1, 1)

$$d_3 = \left|\frac{1 \times a + 1 \times b - 1}{\sqrt{a^2 + b^2}}\right|$$

$$= \frac{a + b - 1}{\sqrt{a^2 + b^2}}$$

It is given that the algebraic sum of the perpendicular from the given points (2, 0), (0, 2) and (1, 1) to this line is zero.

$d_1 + d_2 + d_3 = 0$

Substituting the values we get

$$\therefore \frac{2a - 1}{\sqrt{a^2 + b^2}} + \frac{2b - 1}{\sqrt{a^2 + b^2}} + \frac{a + b - 1}{\sqrt{a^2 + b^2}} = 0$$

$\Rightarrow 2a - 1 + 2b - 1 + a + b - 1 = 0$

$\Rightarrow 3a + 3b - 3 = 0$

$\Rightarrow a + b - 1 = 0$

$\Rightarrow a + b = 1$

So, the equation ax + by = 1 represents a family of straight lines passing through a fixed point.

Comparing the equation ax + by = 1 and a + b = 1, we get

x = 1 and y = 1

So, the coordinates of fixed point is (1, 1)

15. In what direction should a line be drawn through the point (1, 2) so that its point of intersection with the line x + y = 4 is at a distance √6/3 from the given point.

Solution:

Let the given line x + y = 4 and the required line 'l' intersect at B (a, b)

Slope of line 'l' is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{b-2}{a-1}$$

And we also know that, m = tan θ

$\therefore \tan\theta = \frac{b-2}{a-1}$ ... (i)

Given that AB = √6/3

So, by distance formula for point A (1, 2) and B (a, b), we get

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\Rightarrow \frac{\sqrt{6}}{3} = \sqrt{(a-1)^2 + (b-2)^2}$$

On squaring both the sides, we get

$$\Rightarrow \frac{6}{9} = (a-1)^2 + (b-2)^2$$

Using $(a - b)^2$ formula we get

$$\Rightarrow \frac{2}{3} = a^2 + 1 - 2a + b^2 + 4 - 4b$$

On cross multiplication we get

$\Rightarrow 2 = 3a^2 + 3 - 6a + 3b^2 + 12 - 12b$

$\Rightarrow 2 = 3a^2 + 3b^2 - 6a - 12b + 15$

$\Rightarrow 3a^2 + 3b^2 - 6a - 12b + 13 = 0$ ... (ii)

Point B (a, b) also satisfies the equation x + y = 4

$\therefore a + b = 4$

$\Rightarrow b = 4 - a$ ... (iii)

Putting the value of b in equation (ii), we get

$3a^2 + 3(4 - a)^2 - 6a - 12(4 - a) + 13 = 0$

Computing and simplifying we get

$\Rightarrow 3a^2 + 3(16 + a^2 - 8a) - 6a - 48 + 12a + 13 = 0$

$\Rightarrow 3a^2 + 48 + 3a^2 - 24a - 6a - 48 + 12a + 13 = 0$

$\Rightarrow 6a^2 - 18a + 13 = 0$

Now, we solve the above equation by using this formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{-(-18) \pm \sqrt{(-18)^2 - 4 \times 6 \times 13}}{2 \times 6}$$

$$a = \frac{18 \pm \sqrt{324 - 312}}{12}$$

$$a = \frac{18 \pm \sqrt{12}}{12}$$

$$a = \frac{9 \pm \sqrt{3}}{6}$$

$$a = \frac{\sqrt{3}(3\sqrt{3} \pm 1)}{\sqrt{3}(2\sqrt{3})}$$

$$\Rightarrow a = \frac{3\sqrt{3} \pm 1}{2\sqrt{3}}$$

$$\Rightarrow a = \frac{3\sqrt{3} + 1}{2\sqrt{3}} \text{ or } a = \frac{3\sqrt{3} - 1}{2\sqrt{3}}$$

Putting the value of a in equation (iii), we get

$$b = 4 - \frac{3\sqrt{3} \pm 1}{2\sqrt{3}}$$

Taking LCM and simplifying we get

$$\Rightarrow b = \frac{8\sqrt{3} - 3\sqrt{3} \pm 1}{2\sqrt{3}}$$

$$\Rightarrow b = \frac{5\sqrt{3} \pm 1}{2\sqrt{3}}$$

$$\Rightarrow b = \frac{5\sqrt{3} + 1}{2\sqrt{3}} \text{ or } b = \frac{5\sqrt{3} - 1}{2\sqrt{3}}$$

Now, putting the value of a and b in equation (i), we get

$$\tan\theta = \frac{b-2}{a-1}$$

$$\Rightarrow \tan\theta = \frac{\frac{5\sqrt{3}\pm 1}{2\sqrt{3}} - 2}{\frac{3\sqrt{3}\pm 1}{2\sqrt{3}} - 1}$$

Taking LCM and simplifying we get

$$\Rightarrow \tan\theta = \frac{\frac{5\sqrt{3}\pm 1 - 4\sqrt{3}}{2\sqrt{3}}}{\frac{3\sqrt{3}\pm 1 - 2\sqrt{3}}{2\sqrt{3}}}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{3}\pm 1}{\sqrt{3}\pm 1}$$

$$\Rightarrow \tan\theta = \frac{\sqrt{3}+1}{\sqrt{3}-1} \quad \text{......1}$$

$$\tan\theta = \frac{\sqrt{3}-1}{\sqrt{3}+1} \quad \text{........2}$$

We solve the equation 1 to get the value of θ, we get

We know that,

$$\tan^{-1}x - \tan^{-1}y = tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

$$\tan^{-1}\left(\frac{x-y}{1+xy}\right)$$

if x = √3 and y = 1

$$= \tan^{-1}\left(\frac{\sqrt{3}-1}{1+(\sqrt{3})(1)}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}-1}{1+\sqrt{3}}\right)$$

We have,

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$$

$\theta = \tan^{-1}(\sqrt{3}) - \tan^{-1}(1)$

$\theta = \tan^{-1}(\tan 60°) - \tan^{-1}(\tan 45°)$

$\theta = 60° - 45°$

θ = 15°

Now, we solve the equation 2

We know that,

$$\tan^{-1} x + \tan^{-1} y = tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

$$\tan^{-1}\left(\frac{x+y}{1-xy}\right)$$

if x = √3 and y = 1

$$= \tan^{-1}\left(\frac{\sqrt{3}+1}{1-(\sqrt{3})(1)}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{3}+1}{1-\sqrt{3}}\right)$$

We have,

$$\theta = \tan^{-1}\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$$

$\theta = \tan^{-1}(\sqrt{3}) + \tan^{-1}(1)$

$\theta = \tan^{-1}(\tan 60°) + \tan^{-1}(\tan 45°)$

θ = 60° + 45°

θ = 105°

16. A straight line moves so that the sum of the reciprocals of its intercepts made on axes is constant. Show that the line passes through a fixed point.

Solution:

We know that intercepts form of a straight line is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Where a and b are the intercepts on the axes

Given that $\frac{1}{a} + \frac{1}{b} = \frac{1}{k}$(let)

On cross multiplication we get

$$\Rightarrow \frac{k}{a} + \frac{k}{b} = 1$$

This shows that the line is passing through the fixed point (k, k)

17. Find the equation of the line which passes through the point (– 4, 3) and the

portion of the line intercepted between the axes is divided internally in the ratio 5 : 3 by this point.

Solution:

Let AB be a line passing through a point (-4, 3) and meets x – axis at A (a, 0) and y – axis at B (0, b)

Using the section formula for internal division, we have

$(x, y) = \left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ …… (i)

Here, $m_1 = 5$, $m_2 = 3$

$(x_1, y_1) = (a, 0)$ and $(x_2, y_2) = (0, b)$

Substituting the above values in the above formula, we get

$\Rightarrow x = \frac{5(0)+3(a)}{5+3}, y = \frac{5(b)+3(0)}{5+3}$

$\Rightarrow -4 = \frac{3a}{8}, 3 = \frac{5b}{8}$

⇒ -32 = 3a or 24 = 5b

$\Rightarrow a = -\frac{32}{3}$ Or $b = \frac{24}{5}$

We know that intercept form of the line is

$\frac{x}{a} + \frac{y}{b} = 1$

Substituting the value of a and b in above equation, we get

$\frac{x}{-\frac{32}{3}} + \frac{y}{\frac{24}{5}} = 1$

On simplification we get

$\Rightarrow -\frac{3x}{32} + \frac{5y}{24} = 1$

Taking LCM

$\Rightarrow \frac{-72x + 160y}{(32)(24)} = 1$

On cross multiplication we get

⇒ -72x + 160y = 768

⇒ -36x + 80y = 384

⇒ 18x – 40y + 192 = 0

⇒ 9x – 20y + 96 = 0

Hence, the required equation is 9x – 20y + 96 = 0

18. Find the equations of the lines through the point of intersection of the lines x – y + 1 = 0 and 2x – 3y + 5 = 0 and whose distance from the point (3, 2) is 7/5.

Solution:

Given two lines are x – y + 1 = 0 ...... (i)

And 2x – 3y + 5 = 0 ... (ii)

Now, point of intersection of these lines can be find out as:

Multiplying equation (i) by 2, we get

2x – 2y + 2 = 0 ...... (iii)

On subtracting equation (iii) from (ii), we get

2x – 2y + 2 – 2x + 3y – 5 = 0

⇒ y – 3 = 0

⇒ y = 3

On putting value of y in equation (ii), we get

2x – 3(3) + 5 = 0

⇒ 2x – 9 + 5 = 0

⇒ 2x – 4 = 0

⇒ 2x = 4

⇒ x = 2

So, the point of intersection of given two lines is (x, y) = (2, 3)

Let m be the slope of the required line

∴ Equation of the line is

y – 3 = m(x – 2)

⇒ y – 3 = mx – 2m

⇒ mx – y – 2m + 3 = 0 ... (iv)

Since, the perpendicular distance from the point (3, 2) to the line is 7/5 then

$$d = \left|\frac{m(3) - 2 + 3 - 2m}{\sqrt{(m)^2 + (1)^2}}\right|$$

$$\Rightarrow \frac{7}{5} = \left|\frac{3m + 1 - 2m}{\sqrt{m^2 + 1}}\right|$$

$$\Rightarrow \frac{7}{5} = \frac{m + 1}{\sqrt{m^2 + 1}}$$

Squaring both the sides, we get

$\Rightarrow \frac{49}{25} = \frac{(m+1)^2}{m^2+1}$

On cross multiplication we get

$\Rightarrow 49(m^2 + 1) = 25(m + 1)^2$

Computing and simplifying we get

$\Rightarrow 49m^2 + 49 = 25(m^2 + 1 + 2m)$

$\Rightarrow 49m^2 + 49 = 25m^2 + 25 + 50m$

$\Rightarrow 25m^2 + 25 + 50m - 49m^2 - 49 = 0$

$\Rightarrow - 24m^2 + 50m - 24 = 0$

$\Rightarrow - 12m^2 + 25m - 12 = 0$

$\Rightarrow 12m^2 - 25m + 12 = 0$

$\Rightarrow 12m^2 - 16m - 9m + 12 = 0$

Taking m common we get

$\Rightarrow 4m (3m - 4) - 3(3m - 4) = 0$

$\Rightarrow (3m - 4) (4m - 3) = 0$

$\Rightarrow 3m - 4 = 0$ or $4m - 3 = 0$

$\Rightarrow 3m = 4$ or $4m = 3$

$\Rightarrow m = \frac{4}{3}$ Or $m = \frac{3}{4}$

$\therefore m = \frac{4}{3}, \frac{3}{4}$

Putting the value of m = 4/3 in equation (iv), we get

$\frac{4}{3}x - y - 2\left(\frac{4}{3}\right) + 3 = 0$

Simplifying and computing we get

$\Rightarrow \frac{4}{3}x - y - \frac{8}{3} + 3 = 0$

$\Rightarrow \frac{4}{3}x - y = \frac{8-9}{3}$

$\Rightarrow \frac{4}{3}x - y = -\frac{1}{3}$

$\Rightarrow \frac{4}{3}x - y + \frac{1}{3} = 0$

$\Rightarrow 4x - 3y + 1 = 0$

Putting the value of m = ¾ in equation (iv), we get

$\frac{3}{4}x - y - 2\left(\frac{3}{4}\right) + 3 = 0$

Simplifying and computing we get

$\Rightarrow \frac{3}{4}x - y - \frac{3}{2} + 3 = 0$

$\Rightarrow \frac{3}{4}x - y = \frac{3}{2} - 3$

$\Rightarrow \frac{3}{4}x - y = \frac{3-6}{2}$

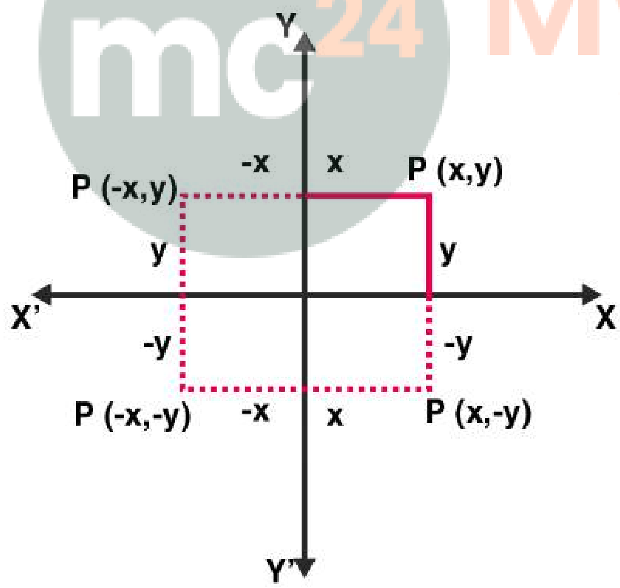
$\Rightarrow \frac{3}{4}x - y + \frac{3}{2} = 0$

$\Rightarrow 3x - 4y + 6 = 0$

Hence, the required equation are $4x - 3y + 1 = 0$ and $3x - 4y + 6 = 0$

19. If the sum of the distances of a moving point in a plane from the axes is 1, then find the locus of the point.

Solution:



Let the coordinates of a moving point P be (a, b)

Given that the sum of the distance from the axes to the point is always 1

$\therefore |x| + |y| = 1$

$\Rightarrow \pm x \pm y = 1$

$\Rightarrow -x - y = 1, x + y = 1, -x + y = 1$ and $x - y = 1$

Hence, these equations gives us the locus of the point P which is a square.

20. P_1, P_2 are points on either of the two lines $y - \sqrt{3}|x| = 2$ at a distance of 5 units from their point of intersection. Find the coordinates of the foot of perpendiculars drawn from P_1, P_2 on the bisector of the angle between the given lines.

Solution:

Given lines are $y - \sqrt{3}|x| = 2$

If $x \geq 0$, then

$y - \sqrt{3}x = 2$ ….. (i)

If $x < 0$, then

$y + \sqrt{3}x = 2$ ….. (ii)

On adding equation (i) and (ii), we get

$y - \sqrt{3}x + y + \sqrt{3}x = 2 + 2$

$\Rightarrow 2y = 4$

$\Rightarrow y = 2$

Substituting the value of $y = 2$ in equation (ii), we get

$2 + \sqrt{3}x = 2$

$\Rightarrow \sqrt{3}x = 2 – 2$

$\Rightarrow x = 0$

∴ Point of intersection of given lines is (0, 2)

Now, we find the slopes of given lines.

Slope of equation (i) is

$y = \sqrt{3}x + 2$

Comparing the above equation with $y = mx + b$, we get

$m = \sqrt{3}$

And we know that, $m = \tan \theta$

∴ $\tan \theta = \sqrt{3}$

$\Rightarrow \theta = 60°$ [∵ $\tan 60° = \sqrt{3}$]

Slope of equation (ii) is

$y = - \sqrt{3}x + 2$

Comparing the above equation with $y = mx + b$, we get

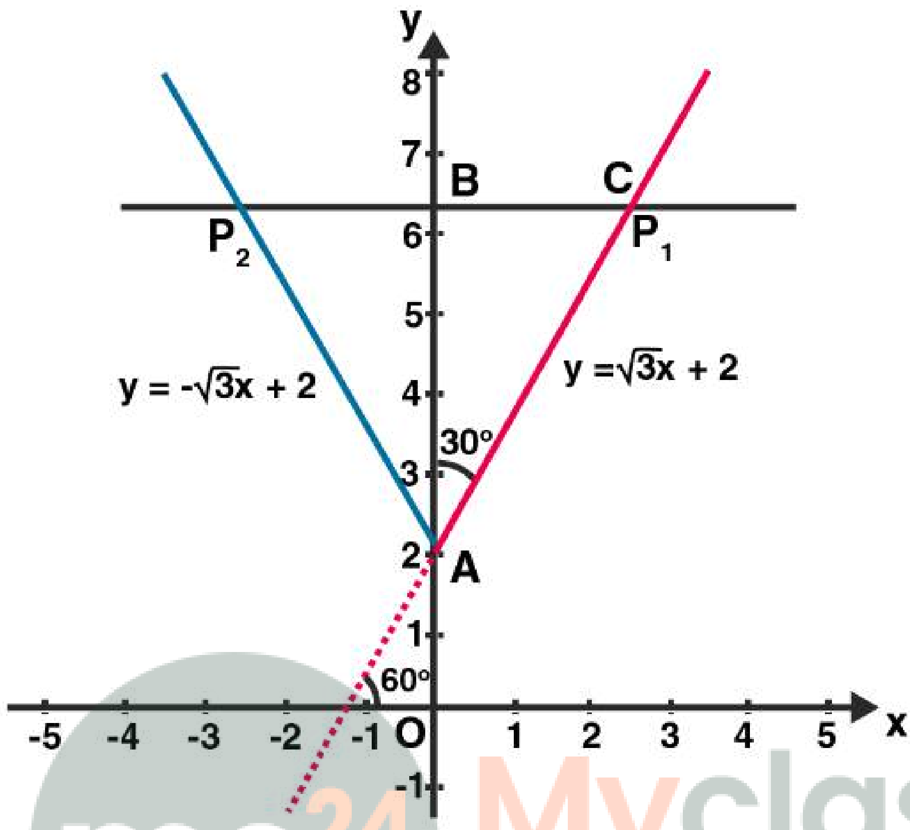
$m = -\sqrt{3}$

And we know that, $m = \tan \theta$

∴ $\tan \theta = -\sqrt{3}$

$\Rightarrow \theta = (180° - 60°)$

$\Rightarrow \theta = 120°$



In ΔACB,

$\cos 30^\circ = \frac{BA}{AC}$

Given AC = 5 units

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{BA}{5}$

$\Rightarrow BA = \frac{5\sqrt{3}}{2}$

∴ OB = OA + AB

$= 2 + \frac{5\sqrt{3}}{2}$

Hence, the coordinates of the foot of perpendicular $= \left(0, 2 + \frac{5\sqrt{3}}{2}\right)$

21. If p is the length of perpendicular from the origin on the line $\frac{x}{a} + \frac{y}{b} = 1$ and a_2, p_2, b_2 are in A.P, then show that $a_4 + b_4 = 0$.

Solution:

Given equation is

$$\frac{x}{a} + \frac{y}{b} = 1$$

Since, p is the length of perpendicular drawn from the origin to the given line

$$\therefore p = \left| \frac{\frac{0}{a} + \frac{0}{b} - 1}{\sqrt{\frac{1}{a^2} + \frac{1}{b^2}}} \right|$$

Squaring both the sides, we have

$$p^2 = \left| \frac{1}{\frac{1}{a^2} + \frac{1}{b^2}} \right|$$

$$\Rightarrow \frac{1}{p^2} = \frac{1}{a^2} + \frac{1}{b^2} \text{ ... (i)}$$

Since, a^2, b^2 and p^2 are in AP

$\therefore 2p^2 = a^2 + b^2$

$$\Rightarrow p^2 = \frac{a^2 + b^2}{2}$$

$$\Rightarrow \frac{1}{p^2} = \frac{2}{a^2 + b^2} \text{ ... (ii)}$$

Form equation (i) and (ii), we get

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{a^2 + b^2}$$

$$\Rightarrow \frac{b^2 + a^2}{a^2b^2} = \frac{2}{a^2 + b^2}$$

$\Rightarrow (a^2 + b^2)(a^2 + b^2) = 2(a^2b^2)$

$\Rightarrow a^4 + b^4 + a^2b^2 + a^2b^2 = 2a^2b^2$

$\Rightarrow a^4 + b^4 = 0$

Hence Proved

OBJECTIVE TYPE QUESTIONS

22. A line cutting off intercept – 3 from the y-axis and the tangent at angle to the x-axis is 3/5, its equation is

A. 5y – 3x + 15 = 0

B. 3y – 5x + 15 = 0
C. 5y – 3x – 15 = 0
D. None of these

Solution:
A. 5y – 3x + 15 = 0

Explanation:
Given that $\tan\theta = \frac{3}{5}$
We know that,
Slope of a line, m = tan θ
$\Rightarrow$ Slope of line, $m = \frac{3}{5}$
Since, the lines cut off intercepts – 3 on y – axis then the line is passing through the point (0, -3).
So, the equation of line is
$y - y_1 = m(x - x_1)$
$\Rightarrow y - (-3) = \frac{3}{5}(x - 0)$
$\Rightarrow y + 3 = \frac{3}{5}x$
$\Rightarrow$ 5y + 15 = 3x
$\Rightarrow$ 5y – 3x + 15 = 0
Hence, the correct option is (a)

23. Slope of a line which cuts off intercepts of equal lengths on the axes is
A. – 1
B. – 0
C. 2
D. √3

Solution:
A. – 1

Explanation:

We know that the equation of line in intercept form is

$$\frac{x}{a}+\frac{y}{b}=1$$

Where a and b are the intercepts on the axis.

Given that a = b

$$\Rightarrow \frac{x}{a}+\frac{y}{a}=1$$

$$\Rightarrow \frac{x+y}{a}=1$$

⇒ x + y = a

⇒ y = - x + a

⇒ y = (-1) x + a

Since, the above equation is in y = mx + b form

So, the slope of the line is – 1.

24. The equation of the straight line passing through the point (3, 2) and perpendicular to the line y = x is

A. x – y = 5

B. x + y = 5

C. x + y = 1

D. x – y = 1

Solution:

B. x + y = 5

Explanation:

Given that straight line passing through the point (3, 2)

And perpendicular to the line y = x

Let the equation of line 'L' is

$y - y_1 = m(x - x_1)$

Since, L is passing through the point (3, 2)

∴ y – 2 = m(x – 3) … (i)

Now, given eq. is y = x

Since, the above equation is in y = mx + b form

So, the slope of this equation is 1

It is also given that line L and y = x are perpendicular to each other.

We know that, when two lines are perpendicular, then

$m_1 \times m_2 = -1$
$\therefore m \times 1 = -1$
$\Rightarrow m = -1$
Putting the value of m in equation (i), we get
$y – 2 = (-1) (x – 3)$
$\Rightarrow y – 2 = -x + 3$
$\Rightarrow x + y = 3 + 2$
$\Rightarrow x + y = 5$
Hence, the correct option is (b)

25. The equation of the line passing through the point (1, 2) and perpendicular to the line x + y + 1 = 0 is
A. y – x + 1 = 0
B. y – x – 1 = 0
C. y – x + 2 = 0
D. y – x – 2 = 0

Solution:
B. y – x – 1 = 0

Explanation:
Given that line passing through the point (1, 2)
And perpendicular to the line x + y + 1 = 0
Let the equation of line 'L' is
$x – y + k = 0$ … (i)
Since, L is passing through the point (1, 2)
$\therefore 1 – 2 + k = 0$
$\Rightarrow k = 1$
Putting the value of k in equation (i), we get
$x – y + 1 = 0$
Or $y – x – 1 = 0$
Hence, the correct option is (b)

26. The tangent of angle between the lines whose intercepts on the axes are a, – b and b, – a, respectively, is

A. $\frac{a^2 - b^2}{ab}$

B. $\frac{b^2 - a^2}{2}$

C. $\frac{b^2 - a^2}{2ab}$

D. None of these

Solution:

C. $\frac{b^2 - a^2}{2ab}$

Explanation:

Let the first equation of line having intercepts on the axes a, -b is

$\frac{x}{a} + \frac{y}{-b} = 1$

$\Rightarrow \frac{x}{a} - \frac{y}{b} = 1$

⇒ bx – ay = ab … (i)

Let the second equation of line having intercepts on the axes b, -a is

$\frac{x}{b} + \frac{y}{-a} = 1$

$\Rightarrow \frac{x}{b} - \frac{y}{a} = 1$

⇒ ax – by = ab … (ii)

Now, we find the slope of equation (i)

bx – ay = ab

⇒ ay = bx – ab

$\Rightarrow y = \frac{b}{a}x - b$

Since, the above equation is in y = mx + b form

So, the slope of equation (i) is

$$m_1 = \frac{b}{a}$$

Now, we find the slope of equation (ii)

ax – by = ab

⇒ by = ax – ab

$$\Rightarrow y = \frac{a}{b}x - a$$

Since, the above equation is in y = mx + b form

So, the slope of eq. (i) is

$$m_2 = \frac{a}{b}$$

Let θ be the angle between the given two lines.

$$\tan\theta = \left|\frac{m_1 - m_1}{1 + m_1 m_2}\right|$$

Putting the values of m_1 and m_2 in above equation, we get

$$\Rightarrow \tan\theta = \left|\frac{\frac{b}{a} - \frac{a}{b}}{1 + \left(\frac{b}{a}\right)\left(\frac{a}{b}\right)}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{\frac{b^2 - a^2}{ab}}{1 + 1}\right|$$

$$\Rightarrow \tan\theta = \left|\frac{b^2 - a^2}{2ab}\right|$$

$$\Rightarrow \tan\theta = \frac{b^2 - a^2}{2ab}$$

Hence, the correct option is (c)

27. If the line $\frac{x}{a} + \frac{y}{b} = 1$ passes through the points (2, –3) and (4, –5), then (a, b) is

A. (1, 1)

B. (– 1, 1)

C. (1, – 1)

D. (– 1, –1)

Solution:
D. (– 1, –1)

Explanation:
Given points are (2, -3) and (4, -5)
Firstly, we find the equation of line.
We know that,
Equation of line when two points are given:

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Putting the values, we get

$$y - (-3) = \frac{-5 - (-3)}{4 - 2}(x - 2)$$

$$\Rightarrow y + 3 = \frac{-5 + 3}{2}(x - 2)$$

$$\Rightarrow y + 3 = \frac{-2}{2}(x - 2)$$

$\Rightarrow y + 3 = -1\ (x - 2)$
$\Rightarrow y + 3 = -x + 2$
$\Rightarrow x + y = 2 - 3$
$\Rightarrow x + y = -1$
$\Rightarrow \frac{x}{-1} + \frac{y}{-1} = 1$ (Intercept form)

Comparing the above equation with the given equation $\frac{x}{a} + \frac{y}{b} = 1$, we get the value of a and b
a = -1 and b = -1
Hence, the correct option is (d)

28. The distance of the point of intersection of the lines 2x – 3y + 5 = 0 and 3x + 4y = 0 from the line 5x – 2y = 0 is

A. $\frac{130}{17\sqrt{29}}$

B. $\frac{13}{7\sqrt{29}}$

C. $\frac{130}{7}$

D. None of these

Solution:

A. $\frac{130}{17\sqrt{29}}$

Explanation:

Given two lines are 2x – 3y + 5 = 0 … (i)

And 3x + 4y = 0 … (ii)

Now, point of intersection of these lines can be find out as

Multiplying equation (i) by 3, we get

6x – 9y + 15 = 0 … (iii)

Multiplying equation (ii) by 2, we get

6x + 8y = 0 … (iv)

On subtracting equation (iv) from (iii), we get

6x – 9y + 15 – 6x – 8y = 0

⇒ – 17y + 15 = 0

⇒ - 17y = -15

$\Rightarrow y = \frac{15}{17}$

On putting value of y in equation (ii), we get

$3x + 4\left(\frac{15}{17}\right) = 0$

$\Rightarrow 3x = -\frac{60}{17}$

$\Rightarrow x = -\frac{20}{17}$

So, the point of intersection of given two lines is

$(x, y) = \left(-\frac{20}{17}, \frac{15}{17}\right)$

Now, perpendicular distance from the point $\left(-\frac{20}{17}, \frac{15}{17}\right)$ to the given line 5x – 2y = 0

$$d = \left|\frac{5\left(-\frac{20}{17}\right) - 2\left(\frac{15}{17}\right)}{\sqrt{(5)^2 + (-2)^2}}\right|$$

$$\Rightarrow d = \left|\frac{-\frac{100}{17} - \frac{30}{17}}{\sqrt{25 + 4}}\right|$$

$$\Rightarrow d = \frac{130}{17\sqrt{29}}$$

Hence, the correct option is (a)