NCERT Solutions for Class-XII Maths

Chapter-7.2

NCERT Maths Class 12

1. $\frac{2x}{1+x^2}$

1. Let $1 + x^2 = t$

$\therefore$ $2x\,dx = dt$

$\Rightarrow \int \frac{2x}{1+x^2}dx = \int \frac{1}{t}dt$

$= \log|t| + C$

$= \log|1+x^2| + C$

$= \log(1+x^2) + C$

2. $\frac{(\log x)^2}{x}$

2. Let $\log|x| = t$

$\Longrightarrow \frac{1}{x}dx = dt$

Now, $\int \frac{(logx)^2}{x} = \int t^2 dt$

$= \frac{t^3}{3} + C$

$= \frac{(log|x|)^3}{3} + C$

3. $\frac{1}{x + x\log x}$

3. $\frac{1}{x + x\log x} = \frac{1}{x(1+\log x)}$

$= \log|t| + C$

$= \log|1 + \log x| + C$

4. sin x sin(cos x)

4. Let cosx = t

⇨ -sinxdx = dt

⇨ $\int sinx.\sin(cosx)\,dx = -\int sintdt$

$=-[-\cos t] + C$

$= \cos t + C$

$= \cos(\cos x) + C$

5. $\sin(ax + b)\cos(ax + b)$

5. $\sin(ax+b)\cos(ax+b) = \frac{2\sin(ax+b)\cos(ax+b)}{2} = \frac{\sin 2(ax+b)}{2}$

Let $2(ax + b) = t$

$\therefore\ 2adx = dt$

$\Rightarrow \int \frac{\sin 2(ax+b)}{2}dx = \frac{1}{2}\int \frac{\sin t dt}{2a}$

$= \frac{1}{4a}[-\cos t] + C$

$= \frac{-1}{4a}\cos 2(ax+b) + C$

6. $\sqrt{ax+b}$

6. Let $ax + b = t$

⇨ $adx = dt$

⇨ $dx = \frac{1}{a}dt$

⇨ $\int (ax+b)^{\frac{1}{2}} = \frac{1}{a}\int t^{\frac{1}{2}}dt$

$= \frac{1}{a}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{2}{3a}(ax+b)^{\frac{3}{2}} + C$

7. $x\sqrt{x+2}$

7. Let $+ x + 2 = t$

$\therefore dx = dt$

$\Rightarrow \int x\sqrt{x+2}dx = \int (t-2)\sqrt{r}dt$

$= \int\left(t^{\frac{3}{2}} - 2t^{\frac{1}{2}}\right)dt$

$= \int t^{\frac{3}{2}}dt - 2\int t^{\frac{1}{2}}dt$

$= \frac{t^{\frac{5}{2}}}{\frac{5}{2}} - 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{2}{5} t^{\frac{5}{2}} - \frac{4}{3} t^{\frac{3}{2}} + C$

$= \frac{2}{5}(x+2)^{\frac{4}{2}} - \frac{4}{3}(x+2)^{\frac{3}{2}} + C$

8. $x\sqrt{1+2x^2}$

8. Let $1 + 2x^2 = t$

$\Rightarrow 4xdx = dt$

$\Longrightarrow \int x\sqrt{x + 2x^2}dx = \int \frac{\sqrt{t}dt}{4}$

$= \frac{1}{4}\int t^{\frac{1}{2}}dt$

$= \frac{1}{4}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{1}{6}(1 + 2x^2)^{\frac{3}{2}} + C$

9. $(4x+2)\sqrt{x^2+x+1}$

9. Let $x^2 + x + 1 = t$

$\therefore (2x+1)dx = dt$

$\int (4x+2)\sqrt{x^2+x+1}dx$

$= \int 2\sqrt{t}dt$

$= 2\int \sqrt{t}dt$

$= 2\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{4}{3}(x^2+x+1)^{\frac{3}{2}} + C$

10. $\frac{1}{x-\sqrt{x}}$

10. $\frac{1}{x-\sqrt{x}} = \frac{1}{\sqrt{x}(\sqrt{x}-1)}$

Now, Let $(\sqrt{x} - 1) = t$

$\Longrightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\Longrightarrow \int \frac{1}{\sqrt{x}(\sqrt{x}-1)}dx = \int \frac{2}{t}dt$

$= 2\log|t| + C$

$= 2\log|\sqrt{x} - 1| + C$

11. $\frac{x}{\sqrt{x+4}}.x > 0$

11. Let x + 4 = t

$\therefore dx = dt$

$\int \frac{x}{\sqrt{x+4}} dx \int \frac{(t-4)}{\sqrt{t}} dt$

$= \int \left(\sqrt{t} - \frac{4}{\sqrt{t}} \right) dt$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 4 \left(\frac{t^{\frac{1}{2}}}{\frac{1}{2}} \right) + C$

$= \frac{2}{3}(t)^{\frac{3}{2}} - 8(t)^{\frac{1}{2}} + C$

$= \frac{2}{3} t - t^{\frac{1}{2}} - 8t^{\frac{1}{2}} + C$

$= \frac{2}{3} t^{\frac{1}{2}} (t - 12) + C$

$= \frac{2}{3} (x+4)^{\frac{1}{2}} (x + 4 - 12) + C$

$= \frac{2}{3} \sqrt{x+4} (x - 8) + C$

12. $\left(x^3 - 1\right)^{\frac{1}{3}} x^5$

12. Let $x^3 - 1 = t$

⇨ $3x^2 dx = dt$

$\Longrightarrow \int (x^3 - 1)^{\frac{1}{3}} x^5 dx = \int (x^3 - 1)^{\frac{1}{3}} x^3 . x^2 dx$

$= \int t^{\frac{1}{3}} (t + 1) \frac{dt}{3}$

$= \frac{1}{3} \int (t^{\frac{4}{3}} + t^{\frac{1}{3}}) dt$

$= \frac{1}{3} \left[\frac{t^{\frac{7}{3}}}{\frac{7}{3}} + \frac{t^{\frac{4}{3}}}{\frac{4}{3}} \right] + C$

$= \frac{1}{3} \left[\frac{3}{7} t^{\frac{7}{3}} + \frac{3}{4} t^{\frac{4}{3}} \right] + C$

$= \frac{1}{7} (x^3 - 1)^{\frac{7}{3}} + \frac{1}{4} (x^3 - 1)^{\frac{4}{3}} + C$

13. $\dfrac{x^2}{\left(2+3x^2\right)}$

13. Let $2 + 3x^3 = t$

$\therefore\ 9x^2\,dx = dt$

$$\Rightarrow \int \frac{x^2}{(2+3x^3)^3}dx = \frac{1}{9}\int \frac{dt}{(t)^3}$$

$$= \frac{1}{9}\left[\frac{r^{-2}}{-2}\right] + C$$

$$= \frac{-1}{18}\left(\frac{1}{r^2}\right) + C$$

$$= \frac{-1}{18(2+3x^3)^2} + C$$

14. $\dfrac{1}{x(\log x)^m}, x > 0$

14. Let $\log x = t$

$$\Longrightarrow \frac{1}{x}dx = dt$$

$$\Longrightarrow \int \frac{1}{x(logx)^m}dx = \int \frac{dt}{t^m}$$

$$= \left(\frac{t^{-m+1}}{1-m}\right) + C$$

$$= \frac{(logx)^{1-m}}{(1-m)} + C$$

15. $\dfrac{x}{9-4x^2}$

15. Let $9 - 4x = t$

$\therefore -8xdx = dt$

$$\Rightarrow \int \frac{x}{9-4x^2}dx = \frac{-1}{8}\int \frac{1}{t}dt$$

$$= \frac{-1}{8}\log|t| + C$$

$$= \frac{-1}{8}\log|9-4x^2| + C$$

16. e^{2x+3}

16. Let $2x + 3 = t$

$=> 2dx = dt$

$$\Longrightarrow \int e^{2x+3}dx = \frac{1}{2}\int e^t dt$$

$$= \frac{1}{2}e^t + C$$

$= \frac{1}{2} e^{2x+3} + C$

17. $\frac{x}{e^{x^2}}$

17. Let $x^2 = t$

$\therefore 2x dx = dt$

$\Rightarrow \int \frac{x}{e^{x^2}} dx = \frac{1}{2} \int \frac{1}{e^t} dt$

$= \frac{1}{2} \int e^{-1} dt$

$= \frac{1}{2} \left(\frac{e^{-t}}{-1} \right) + C$

$= \frac{1}{2} e^{-x^2} + C$

$= \frac{-1}{2e^{x2}} + C$

18. $\frac{e^{\tan -1x}}{1+x^2}$

18. let tan-1 x = t

$=> \frac{1}{1+x^2} dx = dt$

$\Rightarrow \int \frac{e^{tan^{-1}x}}{1+x^2} dx = \int e^t dt$

$= e^t + C$

$= e^{tan^{-1}x} + C$

19. $\frac{e^{2x} - 1}{e^{2x} + 1}$

19 $\frac{e^{2x} - 1}{e^{2x} + 1}$

Dividing numerator and denominator by e^x, we obtain

$\frac{\frac{(e^{2x} - 1)}{e^x}}{\frac{(e^{2x} + 1)}{e^x}} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

Let $e^x + e^{-x} = t$

$(e^x - e^{-x}) dx = dt$

$\Rightarrow \int \frac{e^{2x} - 1}{e^{2x} + 1} dx = \int \frac{ex - e - x}{e^x + e^{-x}} dx$

$= \int \frac{dt}{t}$

$= \log|t| + C$

$= \log|e^x + e^{-x}| + C$

20. $\frac{e^{2x} - e^{-2x}}{e^{2x} + e^{-2x}}$

20. Let $e^{2x} + e^{-2x} = t$

$\Rightarrow (2e^{2x} - 2e^{-2x})dx = dt$

$\Rightarrow 2(e^{2x} - e^{-2x})dx = dt$

$\Rightarrow \int \frac{e^{2x} \pm}{e^{2x} + e^{-2x}} dx$

$= \int \frac{dt}{2t}$

$= \frac{1}{2} \int \frac{dt}{t}$

$= \frac{1}{2} \log|t| + C$

$= \frac{1}{2} \log|e^{2x} + e^{-2x}| + C$

21. $\tan^2(2x - 3)$

21. $\tan^2(2x - 3) = \sec^2(2x - 3) - 1$

Let $2x - 3 = t$

$\therefore 2dx = dt$

$\Rightarrow \int \tan^2(2x-3)dx = \int\left[\left(\sec^2(2x-3)\right) - 1\right]dx$

$= \frac{1}{2}\int\left(\sec^2 t\right)dt - \int 1dx$

$= \frac{1}{2}\int \sec^2 t dt - \int 1dx$

$= \frac{1}{2}\tan t - x + C$

$= \frac{1}{2}\tan(2x-3) - x + C$

22. $\sec^2(7 - 4x)$

22. Let $7 - 4x = t$

$\Rightarrow -4dx = dt$

$\Rightarrow \int sec^2(7 - 4x)dx = \frac{-1}{4} \int sec^2 tdt$

$= \frac{-1}{4}(tant) + C$

$= \frac{-1}{4} \tan(7 - 4x) + C$

23. $\dfrac{\sin^{-1}x}{\sqrt{1-x^2}}$

23. Let $\sin^{-1}x = t$

$\dfrac{1}{\sqrt{1-x^2}}dx = dt$

$\Rightarrow \int \dfrac{\sin^{-1}x}{\sqrt{1-x^2}}dx = \int t\,dt$

$= \dfrac{t^2}{2} + C$

$= \dfrac{\left(\sin^{-1}x\right)^2}{2} + C$

24. $\dfrac{2\cos x - 3\sin x}{6\cos x + 4\sin x}$

24. $\frac{2cosx-3sinx}{6cosx+4sinx} = \frac{2cosx-3sinx}{2(3cosx+2sinx)}$

let 3cosx + 2sinx = t

(-3sinx + 2cosx)dx = dt

$\int \frac{2cosx-3sinx}{6cosx+4sinx}dx = \int \frac{dt}{2t}$

$= \frac{1}{2}\int \frac{1}{t}dt$

$= \frac{1}{2}\log|t| + C$

$= \dfrac{1}{2}\log|2sinx + 3cosx| + C$

25. $\dfrac{1}{\cos^2 x(1-\tan x)^2}$

25. $\dfrac{1}{\cos^2 x(1-\tan x)^2} = \dfrac{\sec^2 x}{(1-\tan x)^2}$

Let (1 – tan x) = t

$-\sec^2 x dx = dt$

$\Rightarrow \int \dfrac{\sec^2 x}{(1-\tan x)^2}dx = \int \dfrac{-dt}{t^2}$

$= -\int t^{-2}dt$

$= +\dfrac{1}{t} + C$

$= \dfrac{1}{(1-\tan x)} + C$

26. $\frac{\cos\sqrt{x}}{\sqrt{x}}$

26. Let $\sqrt{x} = t$

$\Longrightarrow \frac{1}{2\sqrt{x}}dx = dt$

$\Longrightarrow \int \frac{cos\sqrt{x}}{\sqrt{x}}dx = 2\int costdt$

= 2sint + C

= $2\sin\sqrt{x}$ + C

27. $\sqrt{\sin 2x}\cos 2x$

27. Let sin 2x = t

So, 2 cos 2x dx = dt

$\Rightarrow \int \sqrt{\sin 2x}\cos 2x dx = \frac{1}{2}\int \sqrt{t}dt$

$= \frac{1}{2}\left(\frac{t^{\frac{3}{2}}}{\frac{3}{2}}\right) + C$

$= \frac{1}{3}t^{\frac{3}{2}} + C$

$= \frac{1}{3}(\sin 2x)^{\frac{3}{2}} + C$

28. $\frac{\cos x}{\sqrt{1+\sin x}}$

28. Let 1 + sinx = t

=> cosx dx = dt

$\Longrightarrow \int \frac{cosx}{\sqrt{1+sinx}}dx = \int \frac{dt}{\sqrt{t}}$

$= \frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= 2\sqrt{t} + C$

$= 2\sqrt{1 + sinx} + C$

29. cot x log sin x

29. Let log sin x = t

$\Rightarrow \frac{1}{\sin x}\cos x dx = dt$

$\therefore \cot x dx = dt$

$\Rightarrow \int \cot x \log \sin dx = \int 1dt$

$= \frac{t^2}{2} + C$

$= \frac{1}{2}(\log \sin x)^2 + C$

30. $\frac{\sin x}{1+\cos x}$

30. Let 1+ cosx = t

=> -sinx dx = dt

$\Rightarrow \int \frac{sinx}{1+cosx} = \int -\frac{dt}{t}$

= - log|t| + C

= -log|1 + cosx| + C

31. $\frac{\sin x}{(1+\cos x)^2}$

31. Let 1 + cos x = t

$\therefore -\sin x\, dx = dt$

$\Rightarrow \int \frac{\sin x}{(1+\cos x)^2} dx = \int -\frac{dt}{t^2}$

$= -\int t^{-2} dt$

$= \frac{1}{t} + C$

$= \frac{1}{1+\cos x} + C$

32. $\frac{1}{1+\cot x}$

32. Let I = $\int \frac{1}{1+cotx} dx$

$= \int \frac{1}{1+\frac{cosx}{sinx}} dx$

$= \int \frac{sinx}{sinx+cosx} dx$

$= \frac{1}{2} \int \frac{2sinx}{sinx+cosx} dx$

$= \frac{1}{2} \int \frac{(sinx+cosx)+(sinx-cosx)}{sinx+cosx} dx$

$= \frac{1}{2} \int 1.\, dx + \frac{1}{2} \int \frac{(sinx-cosx)}{sinx+cosx} dx$

$= \frac{1}{2} x + \frac{1}{2} \int \frac{(sinx-cosx)}{sinx+cosx} dx$

Let sinx + cosx = t

⇨ (cosx-sinx)dx = dt

Therefore, I = $\frac{x}{2} + \frac{1}{2} \int \frac{-dt}{t}$

$= \frac{x}{2} - \frac{1}{2}\log|t| + C$

$= \frac{x}{2} - \frac{1}{2}\log|sinx + cosx| + C$

33. $\frac{1}{1-\tan x}$

33. Let $I = \int \frac{1}{1-\tan x}dx$

$= \int \frac{1}{1-\frac{\sin x}{\cos x}}dx$

$= \int \frac{\cos x}{\cos x - \sin x}dx$

$= \frac{1}{2}\int \frac{2\cos x}{\cos x - \sin x}dx$

$= \frac{1}{2}\int \frac{(\cos x - \sin x) + (\cos x + \sin x)}{(\cos x - \sin x)}dx$

$= \frac{1}{2}\int 1dx + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$

$= \frac{x}{2} + \frac{1}{2}\int \frac{\cos x + \sin x}{\cos x - \sin x}dx$

Put $\cos x - \sin x = t \Rightarrow (-\sin x - \cos x)\, dx = dt$

$\therefore I = \frac{x}{2} + \frac{1}{2}\int \frac{-(dt)}{t}$

$= \frac{x}{2} - \frac{1}{2}\log|t| + C$

$= \frac{x}{2} - \frac{1}{2}\log|\cos x - \sin x| + C$

34. $\frac{\sqrt{\tan x}}{\sin x \cos x}$

34. Let I = $\int \frac{\sqrt{tanx}}{sinxcosx}$

$= \int \frac{\sqrt{tanx}.cosx}{sinxcosx.cosx}dx$

$= \int \frac{\sqrt{tanx}}{tanxcos^2x}dx$

$= \int \frac{sec^2xdx}{\sqrt{tanx}}$

Let tanx = t

⇨ $sec^2x\, dx = dt$

⇨ I = $\int \frac{dt}{\sqrt{t}}$

$=2\sqrt{t}+C$

$= 2\sqrt{tanx} + C$

35. $\dfrac{(1+\log x)^2}{x}$

35. Let 1 + log x = t

$\therefore \frac{1}{x}dx = dt$

$\Rightarrow \int \frac{(1+\log x)^2}{x}dx = \int t^2 dt$

$= \frac{t^3}{3} + C$

$= \frac{(1+\log x)^3}{3} + C$

36. $\dfrac{(x+1)(x+\log x)^2}{x}$

36. : $\frac{(x+1)(x+logx)^2}{x} = \left(\frac{x+1}{x}\right)(x+logx)^2 = \left(1+\frac{1}{x}\right)(x+logx)^2$

Let (x+logx) = t

=> $\left(1+\frac{1}{x}\right)dx = dt$

$\Rightarrow \int \left(1+\frac{1}{x}\right)(x+logx)^2 dx = \int t^2 dt$

$= \frac{t^3}{3} + C$

$= \frac{1}{3}(x+logx)^3 + C$

37. $\dfrac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}$

37. Let $x^4 = t$

$\therefore\ 4x^3\ dx = dt$

$\Rightarrow \int \frac{x^3 \sin(\tan^{-1} x^4)}{1+x^8}dx = \frac{1}{4}\int \frac{\sin(\tan^{-1} t)}{1+t^2}dt \qquad \ldots(1)$

Let $\tan^{-1} t = u$

$\therefore \frac{1}{1+t^2}dt = du$

From (1), we obtain

$\int \frac{x^3 \sin(\tan^{-1} x^4)dx}{1+x^8} = \frac{1}{4}\int \sin u du$

$= \frac{1}{4}(-\cos u) + C$

$= \frac{-1}{4}\cos\left(\tan^{-1} t\right) + C$

$= \frac{-1}{4}\cos\left(\tan^{-1} x^4\right) + C$

38. $\int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx$ **equals**

(a) $10^x - x^{10} + C$ (b) $10^x + x^{10} + C$

(c) $\left(10^x - x^{10}\right)^{-1} + C$ (d) $\log\left(10^x + x^{10}\right) + C$

38. **:** Let $x^{10} + 10^x = t$

$\Rightarrow (10x^9 + 10^x \log_e 10)dx = dt$

$\Rightarrow \int \frac{10x^9 + 10^x \log_e 10}{x^{10} + 10^x} dx = \int \frac{dt}{t}$

$= \log t + C$

$= \log(x^{10} + 10^x) + C$

39. $\int \frac{dx}{\sin^2 x \cos^2 x}$ equals

(a) tan x = cot x + c (b) tan x – cot x + C

(c) tan x cot x + C (d) tan x – cot 2x + C

39. Let $I = \int \frac{dx}{\sin^2 x \cos^2 x}$

$= \int \frac{1}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \frac{\sin^2 x}{\sin^2 x \cos^2 x} dx + \int \frac{\cos^2 x}{\sin^2 x \cos^2 x} dx$

$= \int \sec^2 x dx + \int \operatorname{cosec}^2 x dx$

$= \tan x - \cot x + C$

Hence, the correct Answer is B.