NCERT Solutions for Class 9 Maths Chapter 9 Circles Ex 9.5

Question 1.

In Fig. 10.36, A,B and C are three points on a circle with centre O such that ∠ BOC = 30° and∠ AOB = 60°. If D is a point on the circle other than the arc ABC, find ∠ADC.

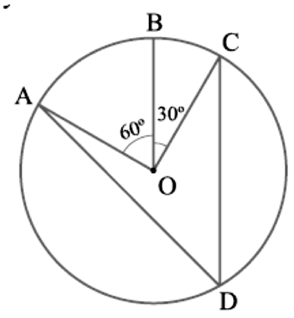


Fig. 10.36

Solution.

∠AOC = ∠AOB + ∠BOC

= 60° + 30°

= 90°

We know that angle subtended by an arc at the centre is double the angle subtended by it any point on the remaining part of the circle

∠ADC = $\frac{1}{2}$ ∠AOC

= $\frac{1}{2}$ * 90°

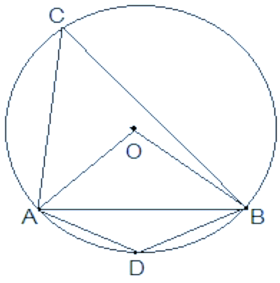
= 45°

Question 2.

A chord of a circle is equal to the radius of the circle. Find the angle subtended by the chord at a point on the minor arc and also at a point on the major arc

Solution.

In ΔOAB,



AB = OA = OB = Radius

ΔOAB is an equilateral triangle

Therefore, each interior angle of this triangle will be of 60°

∠AOB = 60°

∠ACB = $\frac{1}{2}$ * AOB

= $\frac{1}{2}$ * 60°

= 30°

In cyclic quadrilateral ACBD,

∠ACB + ∠ADB = 180° (Opposite angle in cyclic quadrilateral)

∠ADB = 180° − 30°

= 150°

Therefore, angle subtended by this chord at a point on the major arc and the minor arc are 30° and 150° respectively

Question 3.

In Fig. 10.37, ∠ PQR = 100°, where P, Q and R are points on a circle with centre O. Find ∠ OPR.

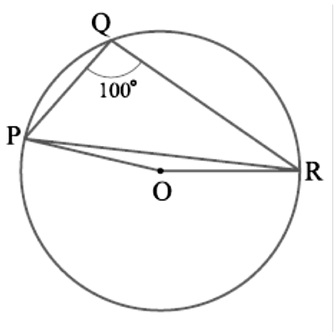
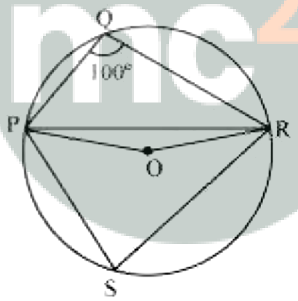


Fig. 10.37

Solution.

Consider PR as a chord of the circle. Take any point S on the major arc of the circle

PQRS is a cyclic quadrilateral



∠PQR + ∠PSR = 180° (Opposite angles of a cyclic quadrilateral)

∠PSR = 180° − 100° = 80°

We know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle

∠POR = 2 ∠PSR

= 2 (80°)

= 160°

In ΔPOR,

OP = OR (Radii of the same circle)

∠OPR = ∠ORP (Angles opposite to equal sides of a triangle)

∠OPR + ∠ORP + ∠POR = 180° (Angle sum property of a triangle)

∠OPR + 160° = 180°

2 ∠OPR = 180° − 160°

2 ∠OPR = 20º

∠OPR = 10°

Question 4.

In Fig. 10.38, ∠ ABC = 69°, ∠ ACB = 31°, find ∠ BDC.

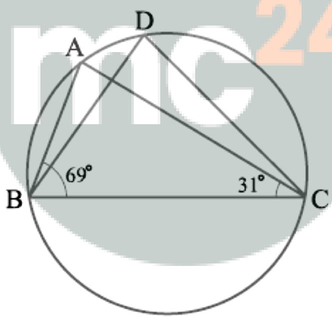


Fig. 10.38

Solution.

∠BAC = ∠BDC (Angles in the segment of the circle)

In triangle ABC,

∠BAC + ∠ABC + ∠ACB = 180° (Sum of all angles in a triangle)

∠BAC + 69° + 31° = 180°

∠BAC = 180° – 100°

∠BAC = 80°

Thus,

∠BDC = 80°

Question 5.

In Fig. 10.39, A, B, C and D are four points on a circle. AC and BD intersect at a point E such that ∠ BEC = 130° and ∠ ECD = 20°. Find∠ BAC.

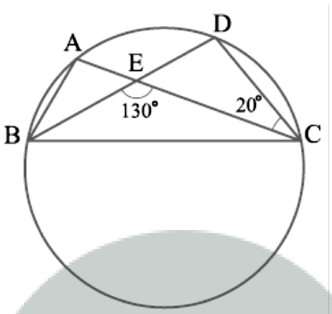


Fig. 10.39

Solution.

In ΔCDE,

∠CDE + ∠DCE = CEB (Exterior angle)

∠CDE + 20° = 130°

∠CDE = 110°

However, ∠BAC = ∠CDE (Angles in the same segment of a circle)

∠BAC = 110°

Question 6.

ABCD is a cyclic quadrilateral whose diagonals intersect at a point E. If ∠DBC = 70°, ∠BAC is 30°, find ∠BCD. Further, if AB = BC, find ∠ECD.

Solution.

For chord CD,

∠CBD = ∠CAD (Angles in the same segment)

∠CAD = 70°

∠BAD = ∠BAC + ∠CAD

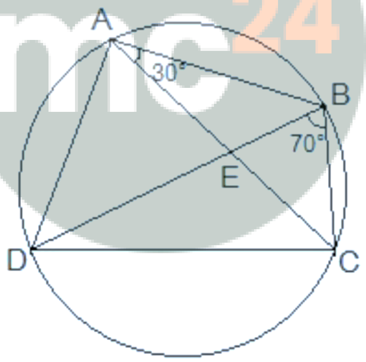
= 30° + 70°

= 100°

∠BCD + ∠BAD = 180° (Opposite angles of a cyclic quadrilateral)

∠BCD + 100° = 180°

∠BCD = 80°



In ΔABC,

AB = BC (Given)

BCA = CAB (Angles opposite to equal sides of a triangle)

∠BCA = 30°

We have,

∠BCD = 80°

$\angle BCA + \angle ACD = 80°$

$30° + \angle ACD = 80°$

$\angle ACD = 50°$

$\angle ECD = 50°$

Question 7.

If diagonals of a cyclic quadrilateral are diameters of the circle through the vertices of the quadrilateral, prove that it is a rectangle.

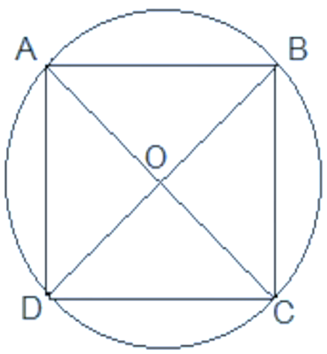
Solution.

Let ABCD be a cyclic quadrilateral having diagonals BD and AC, intersecting each other at point O

$\angle BAD = \frac{1}{2} \angle BOD$

$= \frac{180}{2}$

$= 90^o$



$\angle BCD + \angle BAD = 180°$ (Cyclic quadrilateral)

Consider BD as a chord

$\angle BCD = 180° - 90° = 90°$

$\angle ADC = \frac{1}{2} \angle AOC$

$= \frac{1}{2} * 180^o$

$= 90^o$

$\angle ADC + \angle ABC = 180°$ (Cyclic quadrilateral)

$90° + \angle ABC = 180°$

$\angle ABC = 90^o$

Each interior angle of a cyclic quadrilateral is of 90°. Hence, it is a rectangle

Question 8.

If the non-parallel sides of a trapezium are equal, prove that it is cyclic.

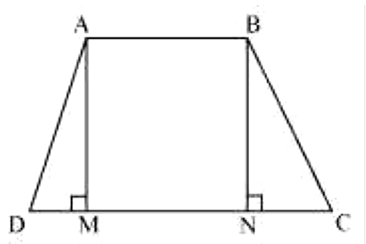
Solution.

Consider a trapezium ABCD with

AB | |CD and

BC = AD

Draw AM perpendicular to CD and BN perpendicular to CD



In ΔAMD and ΔBNC,

AD = BC (Given)

$\angle AMD = \angle BNC$ (By construction, each is 90°)

AM = BM (Perpendicular distance between two parallel lines is same)

ΔAMD ≅ ΔBNC (RHS congruence rule)

∠ADC = ∠BCD (CPCT) (i)

BAD and ADC are on the same side of transversal AD

∠BAD + ∠ADC = 180° (ii)

∠BAD + ∠BCD = 180° [Using equation (i)]

This equation shows that the opposite angles are supplementary

Therefore, ABCD is a cyclic quadrilateral

Question 9.

Two circles intersect at two points B and C. Through B, two line segments ABD and PBQ are drawn to intersect the circles at A, D and P,Q respectively (see Fig. 10.40). Prove that∠ ACP = ∠ QCD

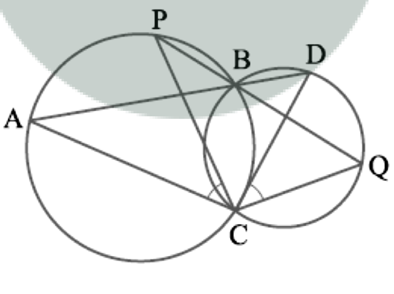


Fig. 10.40

Solution.

Join chords AP and DQ

For chord AP,

∠PBA = ∠ACP (Angles in the same segment) (i)

For chord DQ,

∠DBQ = ∠QCD (Angles in the same segment) (ii)

ABD and PBQ are line segments intersecting at B

∠PBA = ∠DBQ (Vertically opposite angles) (iii)

From (i), (ii), and (iii), we get

∠ACP = ∠QCD

Question 10.

If circles are drawn taking two sides of a triangle as diameters, prove that the point of intersection of these circles lie on the third side.

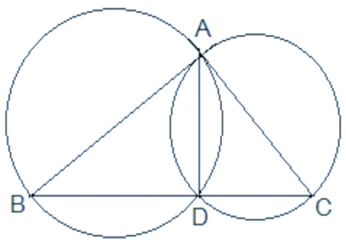
Solution.

Consider a ΔABC

Two circles are drawn while taking AB and AC as the diameter

Let they intersect each other at D and let D not lie on BC

Join AD



∠ADB = 90° (Angle subtended by semi-circle)

∠ADC = 90° (Angle subtended by semi-circle)

∠BDC =∠ ADB + ∠ADC = 90° + 90° = 180°

Therefore, BDC is a straight line and hence, our assumption was wrong

Thus, Point D lies on third side BC of ΔABC

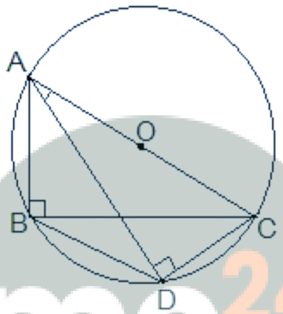
Question 11.

ABC and ADC are two right triangles with common hypotenuse AC. Prove that ∠CAD = ∠CBD

Solution.

It is given in the question that,

Ac is the common hypotenuse



So, B = D = 90^o

We have to prove that,

∠CAD = ∠CBD

Proof: We know that,

∠ABC and ∠ADC are 90^o as these angles are in the semi-circle

Thus, both the triangles are lying in the semi-circle and both have same diameter i.e. AC

Points A, B, C and D are concyclic

Therefore, CD is the chord

So,

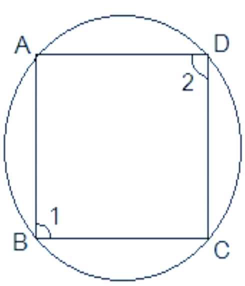
$\angle CAD = \angle CBD$ (As, angles in the same segment are equal)

Question 12.

Prove that a cyclic parallelogram is a rectangle.

Solution.

Given that: ABCD is a cyclic quadrilateral



We have to prove that: ABCD is a rectangle

Proof: $\angle 1 + \angle 2 = 180^o$ (Sum of opposite angles of a cyclic parallelogram)

We also know that, opposite angles of a cyclic parallelogram are equal

Therefore,

$\angle 1 = \angle 2$

$\angle 1 + \angle 1 = 180^o$

$\angle 1 = 90^o$

As,

One of the interior angle of the parallelogram is 90^o

Therefore, ABCD is a rectangle