NCERT Solutions for Class-XI Maths

Chapter-13 Exercise-13.2

NCERT Math Class 11

1. Find the derivative of x^2-2 at $x=10$.

1. Let $f(x)=x^2-2$. Accordingly,

$$f'(10)=\lim_{h\to 0}\frac{f(10+h)-f(10)}{h}$$

$$=\lim_{h\to 0}\frac{\left[(10+h)^2-2\right]-\left(10^2-2\right)}{h}$$

$$=\lim_{h\to 0}\frac{10^2+2\cdot 10\cdot h+h^2-2-10^2+2}{h}$$

$$=\lim_{h\to 0}\frac{20h+h^2}{h}$$

$$=\lim_{h\to 0}(20+h)=(20+0)=20$$

Thus, the derivative of x^2-2 at $x=10$ is 20.

2. Find the derivative of 99x at x = 100.

2. Let f(x) = 99x,

From first principle

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(10)}{h}$$

$$f'(100)=\lim_{h\to 0}\frac{f(100+h)-f(100)}{h}$$

$$=\lim_{h\to 0}\frac{99(100+h)-99\times 100}{h}=\lim_{h\to 0}\frac{99\times 100+99h-99\times 100}{h}$$

$$=\lim_{h\to 0}\frac{99\times h}{h}=\lim_{h\to 0}99$$

$$=99$$

Thus, the derivative of 99x at x = 100 is 99.

3. Find the derivative of x at $x=1$.

3. Let $f(x)=x$. Accordingly,

$$f'(1)=\lim_{h\to 0}\frac{f(1+h)-f(1)}{h}$$

$= \lim_{h\to 0}\frac{(1+h)-1}{h}$

$= \lim_{h\to 0}\frac{h}{h}$

$= \lim_{h\to 0}(1)=1$

Thus, the derivative of x at $x=1$ is 1.

4. Find the derivative of the following functions from first principle.

(i) x^3-27

(ii) $(x-1)(x-2)$

(iii) $\frac{1}{x^2}$

(iv) $\frac{x+1}{x-1}$

4. (i) x^3-27

Let $f(x) = x^3 - 27$

Accordingly, from first principle

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{\left[(x+h)^3-27\right]-(x^3-27)}{h} = \lim_{h\to 0}\frac{x^3+h^3+3x^2h+3xh^2-x^3}{h}$$

$$= \lim_{h\to 0}\frac{h^3+3x^2h+3xh^2}{h} = \lim_{h\to 0}\left(h^2+3x^2+3xh\right)$$

$$= 0 + 3x^2 = 3x^2$$

(ii) $(x-1)(x-2)$

Let $f(x) = (x-1)(x-2)$

Accordingly, from first principle

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$= \lim_{h\to 0}\frac{(x+h-1)(x+h-2)-(x-1)(x-2)}{h}$$

$$= \lim_{h\to 0}\frac{\left(x^2+hx-2x+hx+h^2-2h-x-h+2\right)-\left(x^2-2x-x+2\right)}{h}$$

$$= \lim_{h\to 0}\frac{hx+hx+h^2-2h-h}{h} = \lim_{h\to 0}(h+2x-3)$$

$$= 0 + 2x - 3 = 2x - 3$$

(iii) $\dfrac{1}{x^2}$

Let $f(x) =$

Accordingly, from the first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{1}{(x+h)^2} - \frac{1}{x^2}}{h} = \lim_{h \to 0} \frac{x^2 - (x+h)^2}{hx^2(x+h)^2} = \lim_{h \to 0} \frac{1}{h}\left[\frac{x^2 - x^2 - h^2 - 2hx}{x^2(x+h)^2}\right]$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{-h^2 - 2hx}{x^2(x+h)^2}\right] = \lim_{h \to 0}\left[\frac{-h - 2x}{x^2(x+h)^2}\right]$$

$$= \frac{0 - 2x}{x^2(x+0)^2} = -\frac{2}{x^3}$$

(iv) $\dfrac{x+1}{x-1}$

Let $f(x) =$

From first principle,

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h+1}{x+h-1} - \frac{x+1}{x-1}}{h} = \lim_{h \to 0} \frac{(x-1)(x+h+1) - (x+1)(x+h-1)}{h(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{1}{h}\left[\frac{(x^2 + hx + x - x - h - 1) - (x^2 + hx + x - x + h - 1)}{(x-1)(x+h-1)}\right] = \lim_{h \to 0} \frac{-2h}{h(x-1)(x+h-1)}$$

$$= \lim_{h \to 0} \frac{-2}{(x-1)(x+h-1)} = -\frac{2}{(x-1)(x-1)}$$

$$= -\frac{2}{(x-1)^2}$$

5. For the function

$$f(x) = \frac{x^{100}}{100} + \frac{x^{99}}{99} + \ldots + \frac{x^2}{2} + x + 1$$

Prove that $f'(1) = 100f'(0)$

5. The given function is

$$f(x)=\frac{x^{100}}{100}+\frac{x^{99}}{99}+\ldots+\frac{x^2}{2}+x+1$$

$$\frac{d}{dx}f(x)=\frac{d}{dx}\left[\frac{x^{100}}{100}+\frac{x^{99}}{99}+\ldots+\frac{x^2}{2}+x+1\right]$$

$$\frac{d}{dx}f(x)=\frac{d}{dx}\left(\frac{x^{100}}{100}\right)+\frac{d}{dx}\left(\frac{x^{99}}{99}\right)+\ldots+\frac{d}{dx}\left(\frac{x^2}{2}\right)+\frac{d}{dx}(x)+\frac{d}{dx}(1)$$

On using theorem $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$, we obtain

$$\frac{d}{dx}f(x)=\frac{100x^{99}}{100}+\frac{99x^{98}}{99}+\ldots+\frac{2x}{2}+1+0$$

$$=x^{99}+x^{98}+\ldots+x+1$$

$$\therefore f'(x)=x^{99}+x^{98}+\ldots+x+1$$

At $x=0$,

$$f'(0)=1$$

At $x=1$,

$$f'(1)=1^{99}+1^{98}+\ldots+1+1=[1+1+\ldots+1+1]_{100\text{ terms}}=1\times100=100$$

Thus, $f'(1)=100\times f'(0)$

6. Find the derivative of $x^n+ax^{n-1}+a^2x^{n-2}+\ldots+a^{n-1}x+a^n$ for some fixed real number a.

6. Given f(x) = $x^n+ax^{n-1}+a^2x^{n-2}+\ldots+a^{n-1}x+a^n$

$$f'(x)=\frac{d}{dx}(\ )$$

$$=\frac{d}{dx}\left(x^n\right)+a\frac{d}{dx}\left(x^{n-1}\right)+a^2\frac{d}{dx}\left(x^{n-2}\right)+\ldots+a^{n-1}\frac{d}{dx}(x)+a^n\frac{d}{dx}(1)$$

Since, $\frac{d}{dx}\left(x^n\right)=nx^{n-1}$

$f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + \ldots + a^{n-1} + a^n(0)$

$f'(x) = nx^{n-1} + a(n-1)x^{n-2} + a^2(n-2)x^{n-3} + \ldots + a^{n-1}$

7. For some constants a and b, find the derivative of

(i) $(x-a)(x-b)$

(ii) $\left(ax^2+b\right)^2$

(iii) $\dfrac{x-a}{x-b}$

7. (i) Let $f(x)=(x-a)(x-b)$

$\Rightarrow f(x)=x^2-(a+b)x+ab$

$\therefore f'(x)=\dfrac{d}{dx}\left(x^2-(a+b)x+ab\right)$

$=\dfrac{d}{dx}\left(x^2\right)-(a+b)\dfrac{d}{dx}(x)+\dfrac{d}{dx}(ab)$

On using theorem $\dfrac{d}{dx}\left(x^n\right)=nx^{n-1}$, we obtain

$f'(x)=2x-(a+b)+0=2x-a-b$

(ii) Let $f(x)=\left(ax^2+b\right)^2$

$\Rightarrow f(x)=a^2x^4+2abx^2+b^2$

$\therefore f'(x)=\dfrac{d}{dx}\left(a^2x^4+2abx^2+b^2\right)=a^2\dfrac{d}{dx}\left(x^4\right)+2ab\dfrac{d}{dx}\left(x^2\right)+\dfrac{d}{dx}\left(b^2\right)$

On using theorem $\dfrac{d}{dx}x^n=nx^{n-1}$, we obtain

$$\begin{aligned}f'(x)&=a^2\left(4x^3\right)+2ab(2x)+b^2(0)\\&=4a^2x^3+4abx\\&=4ax\left(ax^2+b\right)\end{aligned}$$

(iii) Let $f(x)=\dfrac{(x-a)}{(x-b)}$

$\Rightarrow f'(x)=\dfrac{d}{dx}\left(\dfrac{x-a}{x-b}\right)$

By quotient rule,

$$f'(x)=\frac{(x-b)\frac{d}{dx}(x-a)-(x-a)\frac{d}{dx}(x-b)}{(x-b)^2}$$

$$=\frac{(x-b)(1)-(x-a)(1)}{(x-b)^2}$$

$$= \frac{x-b-x+a}{(x-b)^2}$$

$$= \frac{a-b}{(x-b)^2}$$

8. Find the derivative of $\frac{x^n - a^n}{x-a}$ for some constant a.

8. Let $f(x) =$

$\Rightarrow f'(x) = \frac{d}{dx}(\ )$

By quotient rule,

$$f'(x) = \frac{(x-a)\frac{d}{dx}(x^n - a^n) - (x^n - a^n)\frac{d}{dx}(x-a)}{(x-a)^2}$$

$$= \frac{(x-a)(nx^{n-1} - 0) - (x^n - a^n)}{(x-a)^2}$$

$$\therefore f'(x) = \frac{(x-a)(nx^{n-1} - 0) - (x^n - a^n)}{(x-a)^2}$$

9. Find the derivative of

(i) $2x - \frac{3}{4}$

(ii) $(5x^3 + 3x - 1)(x-1)$

(iii) $x^{-3}(5+3x)$

(iv) $x^5(3-6x^{-9})$

(v) $x^{-4}(3-4x^{-5})$

(vi) $\frac{2}{x+1} - \frac{x^2}{3x-1}$

9. (i) $2x - \frac{3}{4}$

Let f(x) = $2x - \frac{3}{4}$

$f'(x) = \frac{d}{dx}(\ )$

$= 2\frac{d}{dx}(x) - \frac{d}{dx}\left(\frac{3}{4}\right)$

$= 2 - 0$

$\therefore f'(x) = 2$

(ii) $\left(5x^3 + 3x - 1\right)(x - 1)$

Let $f(x) = \left(5x^3 + 3x - 1\right)(x - 1)$

By product rule,

$f'(x) = \left(5x^3 + 3x - 1\right)\frac{d}{dx}(x - 1) + (x - 1)\frac{d}{dx}\left(5x^3 + 3x + 1\right)$

$= \left(5x^3 + 3x - 1\right) \times 1 + (x - 1) \times \left(15x^2 + 3\right)$

$= (5x^3 + 3x - 1) + (x - 1)(15x^2 + 3)$

$= 5x^3 + 3x - 1 + 15x^3 + 3x - 15x^2 - 3$

$= 20x^3 - 15x^2 + 6x - 4$

(iii) $x^{-3}(5 + 3x)$

Let $f(x) = x^{-3}(5 + 3x)$

By product rule,

$f'(x) = \frac{d}{dx}\left(x^{-3}(5 + 3x)\right)$

$= x^{-3}\frac{d}{dx}(0 + 3) + (5 + 3x)\frac{d}{dx}\left(x^{-3}\right)$

$= x^{-3}(0 + 3) + (5 + 3x)\left(-3x^{-3-1}\right)$

$= 3x^{-3} + (5 + 3x)\left(-3x^{-4}\right)$

$= 3x^{-3} - 15x^{-4} - 9x^{-3}$

$= -6x^{-3} - 15x^{-4}$

(iv) $x^5(3 - 6x^{-9})$

Let $f(x) = x^5(3 - 6x^{-9})$

By product rule,

$f'(x) = x^5\frac{d}{dx}\left(3 - 6x^{-9}\right) + \left(3 - 6x^{-9}\right)\frac{d}{dx}\left(x^5\right)$

$= x^5 (0 - 6 \times (-9)\, x^{-9-1}) + (3 - 6x^{-9})(5x^4)$

$= x^5 (54x^{-10}) + 15x^4 - 30x^{-5}$

$\therefore f'(x) = 24x^{-5} + 15x^4$

(v) $x^{-4}(3 - 4x^{-5})$

Let f(x) = $x^{-4}(3-4x^{-5})$

By product rule,

$$f'(x)=\frac{d}{dx}\left(x^{-4}\left(3-4x^{-5}\right)\right)$$

$$=x^{-4}\frac{d}{dx}\left(3-4x^{-5}\right)+\left(3-4x^{-5}\right)\frac{d}{dx}\left(x^{-4}\right)$$

$$=x^{-4}\left\{0-4(-5)x^{-5-1}\right\}+\left(3-4x^{-5}\right)(-4)x^{-4-1}$$

$$= x^{-4}(20x^{-6}) + (3-4x^{-5})(-4x^{-5})$$

$$= 20x^{-10} - 12x^{-5} + 16x^{-10}$$

$$f'(x) = 36x^{-10} - 12x^{-5}$$

(vi) $\frac{2}{x+1}-\frac{x^2}{3x-1}$

Let f(x) = $\frac{2}{x+1}-\frac{x^2}{3x-1}$

$$f'(x)=\frac{d}{dx}\left(\frac{2}{x+1}-\frac{x^2}{3x-1}\right)$$

By quotient rule,

$$f'(x)=\left[\frac{(x+1)\frac{d}{dx}(2)-2\frac{d}{dx}(x+1)}{(x+1)^2}\right]-\left[\frac{(3x-1)\frac{d}{dx}(x^2)-x^2\frac{d}{dx}(3x-1)}{(3x-1)^2}\right]$$

$$=\left[\frac{(x+1)(0)-2(1)}{(x+1)^2}\right]-\left[\frac{(3x-1)(2x)-(x^2)\times 3}{(3x-1)^2}\right]$$

$$=-\frac{2}{(x+1)^2}-\left[\frac{6x^2-2x-3x^2}{(3x-1)^2}\right]$$

$$f'(x)=-\frac{2}{(x+1)^2}-\frac{x(3x-2)}{(3x-1)^2}$$

10. Find the derivative of $\cos x$ from first principle.

10. Let $f(x)=\cos x$. Accordingly, from the first principle,

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\cos(x+h)-\cos x}{h}$$

$$=\lim_{h\to 0}\left[\frac{\cos x\cos h-\sin x\sin h-\cos x}{h}\right]$$

$$=\lim_{h\to 0}\left[\frac{-\cos x(1-\cos h)-\sin x\sin h}{h}\right]$$

$$=\lim_{h\to 0}\left[\frac{-\cos x(1-\cos h)}{h}-\frac{\sin x\sin h}{h}\right]$$

$$=-\cos x\left(\lim_{h\to 0}\frac{1-\cos h}{h}\right)-\sin x\lim_{h\to 0}\left(\frac{\sin h}{h}\right)$$

$$=-\cos x(0)-\sin x(1)$$

$$=-\sin x$$

$$\therefore f'(x)=-\sin x\ \lim_{h\to 0}\frac{1-\cos h}{h}=0 \text{ and } \lim_{h\to 0}\frac{\sin h}{h}=1$$

11. Find the derivative of the following functions:

(i) $\sin x\cos x$

(ii) $\sec x$

(iii) $5\sec x+4\cos x$

(iv) $\operatorname{cosec} x$

(v) $3\cot x+5\operatorname{cosec} x$

(vi) $5\sin x-6\cos x+7$

(vii) $2\tan x-7\sec x$

11. (i) sin x cos x

Let f(x) = sin x cos x

By product rule,

$$f'(x)=\sin x\frac{d}{dx}(\cos x)+\cos x\frac{d}{dx}(\sin x)$$

$=\sin x \times (-\sin x) + \cos x \times \cos x$

$= -\sin^2 x + \cos^2 x$

$\therefore f'(x) = \cos^2 x - \sin^2 x$

(ii) sec x

Let f(x) = sec x = 1/cos x

$$f'(x)=\frac{d}{dx}\left(\frac{1}{\cos x}\right)$$

By quotient rule,

$$f'(x)=\frac{\cos x\frac{d}{dx}(1)-1\frac{d}{dx}(\cos x)}{\cos^2 x}$$

$$=\frac{\cos x\times 0-(-\sin x)}{\cos^2 x}$$

$$=\frac{\sin x}{\cos^2 x}=\frac{\sin x}{\cos x}\times\frac{1}{\cos x}$$

∴ f'(x) = tan x sec x

(iii) 5sec x + 4cos x

Let f(x) = 5sec x + 4cos x

$$f'(x)=\frac{d}{dx}(5\sec x+4\cos x)$$

$$=5\frac{d}{dx}(\sec x)+4\frac{d}{dx}(\cos x)$$

$$=5\sec x\tan x+4\times(-\sin x)$$

∴ f'(x) = 5 sec x tan x – 4 sin x

(iv) cosec x

Let f(x) = cosec x, accordingly f(x + h) = cosec (x + h)

By first principle

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{\operatorname{cosec}(x+h)-\operatorname{cosec}x}{h}=\lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sin(x+h)}-\frac{1}{\sin x}\right)$$

$$=\lim_{h\to 0}\frac{1}{h}\left[\frac{\sin x-\sin(x+h)}{\sin x\sin(x+h)}\right]$$

$$=\frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)}\right]=\frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{-h}{2}\right)}{\sin(x+h)}\right]$$

$$=\frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{-\sin\left(\frac{h}{2}\right)\cos\left(\frac{2x+h}{2}\right)}{\left(\frac{h}{2}\right)\sin(x+h)}\right]=-\frac{1}{\sin x}\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\lim_{h\to 0}\frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)}$$

$$=-\frac{1}{\sin x}\times 1\times\frac{\cos\left(\frac{2x+0}{2}\right)}{\sin(x+0)}=-\frac{1}{\sin x}\times\frac{\cos x}{\sin x}$$

∴ f'(x) = -cosec x cot x

(v) 3 cot x + 5 cosec x

Let f(x) = 3 cot x + 5 cosec x

f'(x) = 3 (cot x)' + 5 (cosec x)'

Let $f_1(x)$ = cot x, accordingly $f_1(x + h)$ = cot (x + h)

By first principle

$$f_1'(x) = \lim_{x\to 0}\frac{f_1(x+h)-f_1(x)}{h}$$

$$= \lim_{h\to 0}\frac{\cot(x+h)-\cot x}{h} = \lim_{h\to 0}\frac{1}{h}\left(\frac{\cos(x+h)}{\sin(x+h)}-\frac{\cos x}{\sin x}\right)$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{\sin x\cos(x+h)-\cos x\sin(x+h)}{\sin x\cos(x+h)}\right)$$

$$= \lim_{h\to 0}\frac{1}{h}\left(\frac{\sin(x-x-h)}{\sin x\sin(x+h)}\right) = \lim_{h\to 0}\frac{1}{h}\left[\frac{\sin(-h)}{\sin(x+h)}\right]$$

$$= -\frac{1}{\sin x}\left(\lim_{h\to 0}\frac{\sin h}{h}\right)\left(\lim_{h\to 0}\frac{1}{\sin(x+h)}\right) = -\frac{1}{\sin x}\times 1\times\frac{1}{\sin(x+0)}$$

$$= -\frac{1}{\sin^2 x}$$

$\therefore$ (cot x)' = - cosec^2 x ………………..(2)

Let $f_2(x)$ = cosec x, accordingly $f_2(x + h)$ = cosec (x + h)

By first principle

$$f_2'(x) = \lim_{h\to 0}\frac{f_2(x+h)-f_2(x)}{h}$$

$$= \lim_{h\to 0}\frac{\text{cosec}(x+h)-\text{cosec}x}{h} = \lim_{h\to 0}\frac{1}{h}\left(\frac{1}{\sin(x+h)}-\frac{1}{\sin x}\right)$$

$$= \lim_{h\to 0}\frac{1}{h}\left[\frac{\sin x-\sin(x+h)}{\sin x\sin(x+h)}\right]$$

$$= \frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{x+x+h}{2}\right)\sin\left(\frac{x-x-h}{2}\right)}{\sin(x+h)}\right] = \frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{2\cos\left(\frac{2x+h}{2}\right)\sin\left(\frac{-h}{2}\right)}{\sin(x+h)}\right]$$

$$= \frac{1}{\sin x}\lim_{h\to 0}\frac{1}{h}\left[\frac{-\sin\left(\frac{h}{2}\right)\cos\left(\frac{2x+h}{2}\right)}{\left(\frac{h}{2}\right)\sin(x+h)}\right] = -\frac{1}{\sin x}\lim_{h\to 0}\frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}}\times\lim_{h\to 0}\frac{\cos\left(\frac{2x+h}{2}\right)}{\sin(x+h)}$$

$$= -\frac{1}{\sin x} \times 1 \times \frac{\cos\left(\frac{2x+0}{2}\right)}{\sin(x+0)} = -\frac{1}{\sin x} \times \frac{\cos x}{\sin x}$$

= -cosec x cot x

∴ (cosec x)’ = -cosec x cot x

So, f’(x) = 3 (cot x)’ + 5 (cosec x)’

Putting (cot x)’ and (cosec x)’ in f’(x)

f’(x) = 3 × (-cosec^2 x) + 5 × (-cosec x cot x)

f’(x) = -3cosec^2 x – 5cosec x cot x

(vi) 5 sin x – 6 cos x + 7

Let f(x) = 5 sin x – 6 cos x + 7

$$f'(x) = \frac{d}{dx}(5\sin x - 6\cos x + 7)$$

$$= 5\frac{d}{dx}(\sin x) - 6\frac{d}{dx}(\cos x) + \frac{d}{dx}(7)$$

= 5 × cos x – 6 × (- sin x) + 0

∴ f’(x) = 5cos x + 6sin x

(vii) 2tan x – 7sec x

Let f(x) = 2 tan x – 7 sec x

$$f'(x) = \frac{d}{dx}(2\tan x - 7\sec x)$$

$$= 2\frac{d}{dx}(\tan x) - 7\frac{d}{dx}(\sec x)$$

f’(x) = 2 × (sec^2 x) – 7 × (sec x tan x)

f’(x) = 2sec^2 x – 7sec x tan x