EXERCISE 7.6

Factorize each of the following algebraic expressions:

1. $4x^2 + 12xy + 9y^2$

Solution:

We have,

$4x^2 + 12xy + 9y^2$

By using the formula $(x + y)^2 = x^2 + y^2 + 2xy$

$(2x)^2 + (3y)^2 + 2 (2x) (3y)$

$(2x + 3y)^2$

$(2x + 3y) (2x + 3y)$

2. $9a^2 – 24ab + 16b^2$

Solution:

We have,

$9a^2 – 24ab + 16b^2$

By using the formula $(x - y)^2 = x^2 + y^2 - 2xy$

Here $x = 3a$, $y = 4b$ So,

$(3a)^2 + (4b)^2 – 2 (3a) (4b)$

$(3a – 4b)^2$

$(3a – 4b) (3a – 4b)$

3. $p^2q^2 – 6pqr + 9r^2$

Solution:

We have,

$p^2q^2 – 6pqr + 9r^2$

By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$

$(pq)^2 + (3r)^2 – 2 (pq) (3r)$

$(pq – 3r)^2$

$(pq – 3r) (pq – 3r)$

4. $36a^2 + 36a + 9$

Solution:

We have,

$36a^2 + 36a + 9$

$(6a)^2 + 2 \times 6a \times 3 + 3^2$

$(6a + 3)^2$

5. $a^2 + 2ab + b^2 - 16$

Solution:
We have,
$a^2 + 2ab + b^2 – 16$
By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$
$(a + b)^2 - 4^2$
By using the formula $(a^2 – b^2) = (a+b)(a-b)$
$(a + b + 4)(a + b – 4)$

6. $9z^2 – x^2 + 4xy – 4y^2$
Solution:
We have,
$9z^2 – x^2 + 4xy – 4y^2$
$(3z)^2 – [x^2 – 2(x)(2y) + (2y)^2]$
By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$
$(3z)^2 – (x – 2y)^2$
By using the formula $(a^2 – b^2) = (a+b)(a-b)$
$[(x – 2y) + 3z][–x + 2y + 3z)]$

7. $9a^4 – 24a^2b^2 + 16b^4 – 256$
Solution:
We have,
$9a^4 – 24a^2b^2 + 16b^4 – 256$
$(3a^2)^2 – 2(4a^2)(3b^2) + (4b^2)^2 – (16)^2$
By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$
$(3a^2 – 4b^2)^2 – (16)^2$
By using the formula $(a^2 – b^2) = (a+b)(a-b)$
$(3a^2 – 4b^2 + 16)(3a^2 – 4b^2 – 16)$

8. $16 – a^6 + 4a^3b^3 - 4b^6$
Solution:
We have,
$16 – a^6 + 4a^3b^3 – 4b^6$
$4^2 – [(a^3)^2 – 2(a^3)(2b^3) + (2b^3)^2]$
By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$
$4^2 – (a^3 – 2b^3)^2$
By using the formula $(a^2 – b^2) = (a+b)(a-b)$
$[4 + (a^3 – 2b^3)][4 – (a^3 – 2b^3)]$

9. $a^2 - 2ab + b^2 – c^2$

Solution:
We have,
$a^2 - 2ab + b^2 – c^2$
By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$
$(a – b)^2 – c^2$
By using the formula $(a^2 – b^2) = (a+b)(a-b)$
$(a – b + c)(a – b – c)$

10. $x^2 + 2x + 1 – 9y^2$
Solution:
We have,
$x^2 + 2x + 1 – 9y^2$
By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$
$(x + 1)^2 – (3y)^2$
By using the formula $(a^2 – b^2) = (a+b)(a-b)$
$(x + 3y + 1)(x – 3y + 1)$

11. $a^2 + 4ab + 3b^2$
Solution:
We have,
$a^2 + 4ab + 3b^2$
By using factors for 3 i.e., 3 and 1
$a^2 + ab + 3ab + 3b^2$
By grouping we get,
$a(a + b) + 3b(a + b)$
$(a + 3b)(a + b)$

12. $96 – 4x - x^2$
Solution:
We have,
$96 – 4x - x^2$
$-x^2 – 4x + 96$
By using factors for 96 i.e., 12 and 8
$-x^2 – 12x + 8x + 96$
By grouping we get,
$-x(x + 12) + 8(x + 12)$
$(x + 12)(-x + 8)$

13. $a^4 + 3a^2 + 4$

Solution:
We have,
$a^4 + 3a^2 + 4$
$(a^2)^2 + (a^2)^2 + 2 (2a^2) + 4 – a^2$
$(a^2 + 2)^2 + (-a^2)$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(a^2 + 2 + a) (a^2 + 2 – a)$
$(a^2 + a + 2) (a^2 – a + 2)$

14. $4x^4 + 1$
Solution:
We have,
$4x^4 + 1$
$(2x^2)^2 + 1 + 4x^2 – 4x^2$
$(2x^2 + 1)^2 – 4x^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(2x^2 + 1 + 2x) (2x^2 + 1 – 2x)$
$(2x^2 + 2x + 1) (2x^2 – 2x + 1)$

15. $4x^4 + y^4$
Solution:
We have,
$4x^4 + y^4$
$(2x^2)^2 + (y^2)^2 + 4x^2y^2 – 4x^2y^2$
$(2x^2 + y^2)^2 – 4x^2y^2$
By using the formula $(a^2 – b^2) = (a+b) (a-b)$
$(2x^2 + y^2 + 2xy) (2x^2 + y^2 – 2xy)$

16. $(x + 2)^4 – 6(x + 2) + 9$
Solution:
We have,
$(x + 2)^4 – 6(x + 2) + 9$
$(x^2 + 2^2)^2 – 6x – 12 + 9$
$(x^2 + 2^2 + 2(2)(x)) – 6x – 12 + 9$
$x^2 + 4 + 4x – 6x – 12 + 9$
$x^2 – 2x + 1$
By using the formula $(a – b)^2 = a^2 + b^2 – 2ab$
$(x – 1)^2$

17. $25 - p^2 - q^2 - 2pq$

Solution:

We have,

$25 - p^2 - q^2 - 2pq$

$25 - (p^2 + q^2 + 2pq)$

$(5)^2 - (p + q)^2$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$(5 + p + q)(5 - p - q)$

$-(p + q + 5)(p + q - 5)$

18. $x^2 + 9y^2 - 6xy - 25a^2$

Solution:

We have,

$x^2 + 9y^2 - 6xy - 25a^2$

$(x - 3y)^2 - (5a)^2$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$(x - 3y + 5a)(x - 3y - 5a)$

19. $49 - a^2 + 8ab - 16b^2$

Solution:

We have,

$49 - a^2 + 8ab - 16b^2$

$49 - (a^2 - 8ab + 16b^2)$

$49 - (a - 4b)^2$

By using the formula $(a^2 - b^2) = (a + b)(a - b)$

$(7 + a - 4b)(7 - a + 4b)$

$-(a - 4b + 7)(a - 4b - 7)$

20. $a^2 - 8ab + 16b^2 - 25c^2$

Solution:

We have,

$a^2 - 8ab + 16b^2 - 25c^2$

$(a - 4b)^2 - (5c)^2$

By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$(a - 4b + 5c)(a - 4b - 5c)$

21. $x^2 - y^2 + 6y - 9$

Solution:

We have,

$x^2 - y^2 + 6y - 9$
$x^2 + 6y - (y^2 - 6y + 9)$
$x^2 - (y - 3)^2$
By using the formula $(a^2 - b^2) = (a+b)(a-b)$
$(x + y - 3)(x - y + 3)$

22. $25x^2 - 10x + 1 - 36y^2$
Solution:
We have,
$25x^2 - 10x + 1 - 36y^2$
$(5x)^2 - 2(5x) + 1 - (6y)^2$
$(5x - 1)^2 - (6y)^2$
By using the formula $(a^2 - b^2) = (a+b)(a-b)$
$(5x - 6y - 1)(5x + 6y - 1)$

23. $a^2 - b^2 + 2bc - c^2$
Solution:
We have,
$a^2 - b^2 + 2bc - c^2$
$a^2 - (b^2 - 2bc + c^2)$
$a^2 - (b - c)^2$
By using the formula $(a^2 - b^2) = (a+b)(a-b)$
$(a + b - c)(a - b + c)$

24. $a^2 + 2ab + b^2 - c^2$
Solution:
We have,
$a^2 + 2ab + b^2 - c^2$
$(a + b)^2 - c^2$
By using the formula $(a^2 - b^2) = (a+b)(a-b)$
$(a + b + c)(a + b - c)$

25. $49 - x^2 - y^2 + 2xy$
Solution:
We have,
$49 - x^2 - y^2 + 2xy$
$49 - (x^2 + y^2 - 2xy)$
$7^2 - (x - y)^2$
By using the formula $(a^2 - b^2) = (a+b)(a-b)$

$[7 + (x - y)] [7 - x + y]$
$(x - y + 7) (y - x + 7)$

26. $a^2 + 4b^2 - 4ab - 4c^2$
Solution:
We have,
$a^2 + 4b^2 - 4ab - 4c^2$
$a^2 - 2 (a) (2b) + (2b)^2 - (2c)^2$
$(a - 2b)^2 - (2c)^2$
By using the formula $(a^2 - b^2) = (a+b) (a-b)$
$(a - 2b + 2c) (a - 2b - 2c)$

27. $x^2 - y^2 - 4xz + 4z^2$
Solution:
We have,
$x^2 - y^2 - 4xz + 4z^2$
$x^2 - 2 (x) (2z) + (2z)^2 - y^2$
As $(a-b)^2 = a^2 + b^2 - 2ab$
$(x - 2z)^2 - y^2$
By using the formula $(a^2 - b^2) = (a+b) (a-b)$
$(x + y - 2z) (x - y - 2z)$