

EXERCISE 20.2

Find three numbers in G.P. whose sum is 65 and whose product is 3375. Solution:

Let the three numbers be a/r , a , ar

So, according to the question

$$a/r + a + ar = 65 \dots \text{equation (1)}$$

$$a/r \times a \times ar = 3375 \dots \text{equation (2)}$$

From equation (2) we get,

$$a^3 = 3375$$

$$a = 15.$$

From equation (1) we get,

$$(a + ar + ar^2)/r = 65$$

$$a + ar + ar^2 = 65r \dots \text{equation (3)}$$

Substituting $a = 15$ in equation (3) we get

$$15 + 15r + 15r^2 = 65r$$

$$15r^2 - 50r + 15 = 0 \dots \text{equation (4)}$$

Dividing equation (4) by 5 we get

$$3r^2 - 10r + 3 = 0$$

$$3r^2 - 9r - r + 3 = 0$$

$$3r(r - 3) - 1(r - 3) = 0$$

$$r = 3 \text{ or } r = 1/3$$

Now, the equation will be

$$15/3, 15, 15 \times 3 \text{ or}$$

$$15/(1/3), 15, 15 \times 1/3$$

So the terms are 5, 15, 45 or 45, 15, 5

\therefore The three numbers are 5, 15, 45.

1. Find three number in G.P. whose sum is 38 and their product is 1728.

Solution:

Let the three numbers be a/r , a , ar

So, according to the question

$$a/r + a + ar = 38 \dots \text{equation (1)}$$

$$a/r \times a \times ar = 1728 \dots \text{equation (2)}$$

From equation (2) we get,

$$a^3 = 1728$$

$$a = 12.$$

From equation (1) we get,

$$(a + ar + ar^2)/r = 38$$

$$a + ar + ar^2 = 38r \dots \text{equation (3)}$$

Substituting $a = 12$ in equation (3) we get

$$12 + 12r + 12r^2 = 38r$$

$$12r^2 - 26r + 12 = 0 \dots \text{equation (4)}$$

Dividing equation (4) by 2 we get

$$6r^2 - 13r + 6 = 0$$

$$6r^2 - 9r - 4r + 6 = 0$$

$$3r(3r - 3) - 2(3r - 3) = 0$$

$$r = 3/2 \text{ or } r = 2/3$$

Now the equation will be

$$12/(3/2) = 8 \text{ or}$$

$$12/(2/3) = 18$$

So the terms are 8, 12, 18

\therefore The three numbers are 8, 12, 18

2. The sum of first three terms of a G.P. is $13/12$, and their product is -1 . Find the G.P.

Solution:

Let the three numbers be a/r , a , ar

So, according to the question

$$a/r + a + ar = 13/12 \dots \text{equation (1)}$$

$$a/r \times a \times ar = -1 \dots \text{equation (2)}$$

From equation (2) we get,

$$a^3 = -1$$

$$a = -1$$

From equation (1) we get,

$$(a + ar + ar^2)/r = 13/12$$

$$12a + 12ar + 12ar^2 = 13r \dots \text{equation (3)}$$

Substituting $a = -1$ in equation (3) we get

$$12(-1) + 12(-1)r + 12(-1)r^2 = 13r$$

$$12r^2 + 25r + 12 = 0$$

$$12r^2 + 16r + 9r + 12 = 0 \dots \text{equation (4)}$$

$$4r(3r + 4) + 3(3r + 4) = 0$$

$$r = -3/4 \text{ or } r = -4/3$$

Now the equation will be

$$-1/(-3/4), -1, -1 \times -3/4 \text{ or } -1/(-4/3), -1, -1 \times -4/3$$

$$4/3, -1, 3/4 \text{ or } 3/4, -1, 4/3$$

\therefore The three numbers are $4/3, -1, 3/4$ or $3/4, -1, 4/3$

3. The product of three numbers in G.P. is 125 and the sum of their products taken in pairs is $87 \frac{1}{2}$. Find them.

Solution:

Let the three numbers be $a/r, a, ar$

So, according to the question

$$a/r \times a \times ar = 125 \dots \text{equation (1)}$$

From equation (1) we get,

$$a^3 = 125$$

$$a = 5$$

$$a/r \times a + a \times ar + ar \times a/r = 87 \frac{1}{2}$$

$$a/r \times a + a \times ar + ar \times a/r = 195/2$$

$$a^2/r + a^2r + a^2 = 195/2$$

$$a^2 (1/r + r + 1) = 195/2$$

Substituting $a = 5$ in above equation we get,

$$5^2 [(1+r^2+r)/r] = 195/2$$

$$1+r^2+r = (195r/2 \times 25)$$

$$2(1+r^2+r) = 39r/5$$

$$10 + 10r^2 + 10r = 39r$$

$$10r^2 - 29r + 10 = 0$$

$$10r^2 - 25r - 4r + 10 = 0$$

$$5r(2r-5) - 2(2r-5) = 0$$

$$r = 5/2, 2/5$$

So G.P is $10, 5, 5/2$ or $5/2, 5, 10$

\therefore The three numbers are $10, 5, 5/2$ or $5/2, 5, 10$

4. The sum of the first three terms of a G.P. is $39/10$, and their product is 1. Find the common ratio and the terms.

Solution:

Let the three numbers be $a/r, a, ar$

So, according to the question

$$a/r + a + ar = 39/10 \dots \text{equation (1)}$$

$$a/r \times a \times ar = 1 \dots \text{equation (2)}$$

From equation (2) we get,

$$a^3 = 1$$

$$a = 1$$

From equation (1) we get,

$$(a + ar + ar^2)/r = 39/10$$

$$10a + 10ar + 10ar^2 = 39r \dots \text{equation (3)}$$

Substituting $a = 1$ in 3 we get

$$10(1) + 10(1)r + 10(1)r^2 = 39r$$

$$10r^2 - 29r + 10 = 0$$

$$10r^2 - 25r - 4r + 10 = 0 \dots \text{equation (4)}$$

$$5r(2r - 5) - 2(2r - 5) = 0$$

$$r = 2/5 \text{ or } 5/2$$

so now the equation will be,

$$1/(2/5), 1, 1 \times 2/5 \text{ or } 1/(5/2), 1, 1 \times 5/2$$

$$5/2, 1, 2/5 \text{ or } 2/5, 1, 5/2$$

\therefore The three numbers are $2/5, 1, 5/2$



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