EXERCISE 17.1

1. Evaluate the following:

(i) $^{14}C_3$

(ii) $^{12}C_{10}$

(iii) $^{35}C_{35}$

(iv) $^{n+1}C_n$

(v) $\sum_{r=1}^{5} {}^5C_r$

Solution:

(i) $^{14}C_3$

Let us use the formula,

$^nC_r = n!/r!(n - r)!$

So now, value of n = 14 and r = 3

$^nC_r = n!/r!(n - r)!$

$^{14}C_3 = 14! / 3!(14 - 3)!$

$= 14! / (3!\ 11!)$

$= [14\times13\times12\times11!] / (3!\ 11!)$

$= [14\times13\times12] / (3\times2)$

$= 14\times13\times2$

$= 364$

(ii) $^{12}C_{10}$

Let us use the formula,

$^nC_r = n!/r!(n - r)!$

So now, value of n = 12 and r = 10

$^nC_r = n!/r!(n - r)!$

$^{12}C_{10} = 12! / 10!(12 - 10)!$

$= 12! / (10!\ 2!)$

$= [12\times11\times10!] / (10!\ 2!)$

$= [12\times11] / (2)$

$= 6\times11$

$= 66$

(iii) $^{35}C_{35}$

Let us use the formula,

$^nC_r = n!/r!(n - r)!$

So now, value of n = 35 and r = 35

$^nC_r = n!/r!(n - r)!$
$^{35}C_{35} = 35! / 35!(35 - 35)!$
$= 35! / (35!\ 0!)$ [Since, 0! = 1]
$= 1$

(iv) $^{n+1}C_n$
Let us use the formula,
$^nC_r = n!/r!(n - r)!$
So now, value of n = n+1 and r = n
$^nC_r = n!/r!(n - r)!$
$^{n+1}C_n = (n+1)! / n!(n+1 - n)!$
$= (n+1)! / n!(1!)$
$= (n + 1) / 1$
$= n + 1$

(v) $\sum_{r=1}^{5} {}^5C_r$
Let us use the formula,
$^nC_r = n!/r!(n - r)!$

$$\sum_{r=1}^{5} {}^5C_r = {}^5C_1 + {}^5C_2 + {}^5C_3 + {}^5C_4 + {}^5C_5$$
$$= \frac{5!}{(5-1)!1!} + \frac{5!}{(5-2)!2!} + \frac{5!}{(5-3)!3!} + \frac{5!}{(5-4)!4!} + \frac{5!}{(5-5)!5!}$$
$$= \frac{5!}{4!1!} + \frac{5!}{3!2!} + \frac{5!}{2!3!} + \frac{5!}{1!4!} + \frac{5!}{0!5!}$$
$$= \frac{5}{1} + \frac{5 \times 4}{2 \times 1} + \frac{5 \times 4}{2 \times 1} + \frac{5}{1} + \frac{1}{1}$$
$$= 5 + 10 + 10 + 5 + 1$$
$$= 31$$

2. If $^nC_{12} = {}^nC_5$, find the value of n.
Solution:
We know that if $^nC_p = {}^nC_q$, then one of the following conditions need to be satisfied:
(i) p = q
(ii) n = p + q
So from the question $^nC_{12} = {}^nC_5$, we can say that
$12 \neq 5$
So, the condition (ii) must be satisfied,
n = 12 + 5

$n = 17$
∴ The value of n is 17.

3. If $^nC_4 = {^nC_6}$, find $^{12}C_n$.
Solution:
We know that if $^nC_p = {^nC_q}$, then one of the following conditions need to be satisfied:
(i) $p = q$
(ii) $n = p + q$
So from the question $^nC_4 = {^nC_6}$, we can say that
$4 \neq 6$
So, the condition (ii) must be satisfied,
$n = 4 + 6$
$n = 10$
Now, we need to find $^{12}C_n$,
We know the value of n so, $^{12}C_n = {^{12}C_{10}}$
Let us use the formula,
$^nC_r = n!/r!(n - r)!$
So now, value of $n = 12$ and $r = 10$
$^nC_r = n!/r!(n - r)!$
$^{12}C_{10} = 12! / 10!(12 - 10)!$
$= 12! / (10!\ 2!)$
$= [12\times11\times10!] / (10!\ 2!)$
$= [12\times11] / (2)$
$= 6\times11$
$= 66$
∴ The value of $^{12}C_{10} = 66$.

4. If $^nC_{10} = {^nC_{12}}$, find $^{23}C_n$.
Solution:
We know that if $^nC_p = {^nC_q}$, then one of the following conditions need to be satisfied:
(i) $p = q$
(ii) $n = p + q$
So from the question $^nC_{10} = {^nC_{12}}$, we can say that
$10 \neq 12$
So, the condition (ii) must be satisfied,
$n = 10 + 12$
$n = 22$
Now, we need to find $^{23}C_n$,
We know the value of n so, $^{23}C_n = {^{23}C_{22}}$

Let us use the formula,
$^nC_r = n!/r!(n - r)!$
So now, value of n = 23 and r = 22
$^nC_r = n!/r!(n - r)!$
$^{23}C_{22} = 23! / 22!(23 - 22)!$
$= 23! / (22!\ 1!)$
$= [23 \times 22!] / (22!)$
$= 23$
∴ The value of $^{23}C_{22} = 23$.

5. If $^{24}C_x = {}^{24}C_{2x + 3}$, find x.
Solution:
We know that if $^nC_p = {}^nC_q$, then one of the following conditions need to be satisfied:
(i) p = q
(ii) n = p + q
So from the question $^{24}C_x = {}^{24}C_{2x + 3}$, we can say that
Let us check for condition (i)
$x = 2x + 3$
$2x – x = -3$
$x = -3$
We know that for a combination ^nC_r, $r \geq 0$, r should be a positive integer which is not satisfied here,
So, the condition (ii) must be satisfied,
$24 = x + 2x + 3$
$3x = 21$
$x = 21/3$
$x = 7$
∴ The value of x is 7.

6. If $^{18}C_x = {}^{18}C_{x + 2}$, find x.
Solution:
We know that if $^nC_p = {}^nC_q$, then one of the following conditions need to be satisfied:
(i) p = q
(ii) n = p + q
So from the question $^{18}C_x = {}^{18}C_{x + 2}$, we can say that
$x \neq x + 2$
So, the condition (ii) must be satisfied,
$18 = x + x + 2$
$18 = 2x + 2$

$2x = 18 - 2$
$2x = 16$
$x = 16/2$
$= 8$
∴ The value of x is 8.

7. If $^{15}C_{3r} = {}^{15}C_{r+3}$, find r.
Solution:
We know that if $^nC_p = {}^nC_q$, then one of the following conditions need to be satisfied:
(i) $p = q$
(ii) $n = p + q$
So from the question $^{15}C_{3r} = {}^{15}C_{r+3}$, we can say that
Let us check for condition (i)
$3r = r + 3$
$3r - r = 3$
$2r = 3$
$r = 3/2$
We know that for a combination ^nC_r, $r \geq 0$, r should be a positive integer which is not satisfied here,
So, the condition (ii) must be satisfied,
$15 = 3r + r + 3$
$15 - 3 = 4r$
$4r = 12$
$r = 12/4$
$= 3$
∴ The value of r is 3.

8. If $^8C_r - {}^7C_3 = {}^7C_2$, find r.
Solution:
To find r, let us consider the given expression,
$^8C_r - {}^7C_3 = {}^7C_2$
$^8C_r = {}^7C_2 + {}^7C_3$
We know that $^nC_r + {}^nC_{r+1} = {}^{n+1}C_{r+1}$
$^8C_r = {}^{7+1}C_{2+1}$
$^8C_r = {}^8C_3$
Now, we know that if $^nC_p = {}^nC_q$, then one of the following conditions need to be satisfied:
(i) $p = q$
(ii) $n = p + q$
So from the question $^8C_r = {}^8C_3$, we can say that

Let us check for condition (i)
r = 3
Let us also check for condition (ii)
8 = 3 + r
r = 5
∴ The values of ‘r’ are 3 and 5.

9. If $^{15}C_r : {}^{15}C_{r-1} = 11 : 5$, find r.
Solution:
Given:
$^{15}C_r : {}^{15}C_{r-1} = 11 : 5$
$^{15}C_r / {}^{15}C_{r-1} = 11 / 5$
Let us use the formula,
$^nC_r = n!/r!(n - r)!$

$$\frac{\frac{15!}{(15-r)!r!}}{\frac{15!}{(15-(r-1))!(r-1)!}} = \frac{11}{5}$$

$$\frac{(16-r)!}{(15-r)!r} = \frac{11}{5}$$

$$\frac{16-r}{r} = \frac{11}{5}$$

5(16 - r) = 11r
80 – 5r = 11r
80 = 11r + 5r
16r = 80
r = 80/16
= 5
∴ The value of r is 5.

10. If $^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$, find n.
Solution:
Given:
$^{n+2}C_8 : {}^{n-2}P_4 = 57 : 16$
$^{n+2}C_8 / {}^{n-2}P_4 = 57 / 16$
Let us use the formula,
$^nC_r = n!/r!(n - r)!$

$$\frac{\frac{(n+2)!}{(n+2-8)!8!}}{\frac{(n-2)!}{(n-2-4)!}} = \frac{57}{16}$$

[(n+2)! (n-6)!] / [(n-6)! (n-2)! 8!] = 57/16

$(n+2)(n+1)(n)(n-1) / 8! = 57/16$

$(n+2)(n+1)(n)(n-1) = (57 \times 8!) / 16$

$(n+2)(n+1)(n)(n-1) = [19 \times 3 \times 8 \times 7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1]/16$

$(n + 2)(n + 1)(n)(n - 1) = 21 \times 20 \times 19 \times 18$

Equating the corresponding terms on both sides we get,

$n = 19$

∴ The value of n is 19.