

EXERCISE 19.13

$$1. \int \frac{x^2}{(a^2 - x^2)^{\frac{3}{2}}} dx$$

Solution:

Given

$$\int \frac{x^2}{(a^2 - x^2)^{\frac{3}{2}}} dx$$

Put $x = a \sin \theta$, so $dx = a \cos \theta d\theta$ and $\theta = \sin^{-1}(x/a)$

Above equation becomes,

$$= \int \frac{a^2 \sin^2 \theta}{(a^2 - a^2 \sin^2 \theta)^{\frac{3}{2}}} (a \cos \theta d\theta) = \int \frac{a^2 \sin^2 \theta}{(a^2)(a^2 - a^2 \sin^2 \theta)^{\frac{3}{2}}} (a \cos \theta d\theta)$$

By taking a^2 common we get

$$= \int \frac{a^2 \sin^2 \theta}{(a^2)^{\frac{3}{2}}(a^2 - a^2 \sin^2 \theta)^{\frac{3}{2}}} (a \cos \theta d\theta) = \int \sin^2 \theta * \frac{\cos \theta}{\cos^3 \theta} d\theta$$

$$= \int \frac{\sin^2 \theta}{\cos^2 \theta} d\theta = \int \tan^2 \theta d\theta = \int (\sec^2 \theta - 1) d\theta \quad (\sec^2 \theta - 1 = \tan^2 \theta)$$

$$= \int \sec^2 \theta d\theta - \int 1.d\theta$$

$$= \tan \theta - \theta + c$$

Put $\theta = \sin^{-1}(x/a)$

$$= \frac{x}{(\sqrt{a^2 - x^2})} - \sin^{-1} \frac{x}{a} + c$$

$$2. \int \frac{x^7}{(a^2 - x^2)^5} dx$$

Solution:

$$\text{Let } I = \int \frac{x^7}{(a^2 - x^2)^5} dx$$

$$\text{Let } x = a \sin \theta$$

On differentiating both sides we get

$$dx = a \cos \theta d\theta$$

$$\therefore I = \int \frac{a^8 \sin^7 \theta \cos \theta}{(a^2 - a^2 \sin^2 \theta)^5} d\theta$$

$$= \int \frac{a^8 \sin^7 \theta \cos \theta}{a^{10} (1 - \sin^2 \theta)^5} d\theta$$

$$= \int \frac{\sin^7 \theta}{a^2 \cos^9 \theta} d\theta$$

$$= \frac{1}{a^2} \int \tan^7 \theta \sec^2 \theta d\theta$$

Let

$$\tan \theta = t$$

Differentiating on both sides

$$\sec^2 \theta d\theta = dt$$

$$\therefore I = \frac{1}{a^2} \int t^7 dt$$

$$= \frac{1}{a^2} \frac{t^8}{8} + c$$

$$= \frac{1}{8a^2} (\tan^8 \theta) + c$$

$$= \frac{1}{8a^2} \left(\tan \left(\sin^{-1} \frac{x}{a} \right) \right)^8 + c$$

$$= \frac{1}{8a^2} \left(\tan \left(\tan^{-1} \frac{x}{\sqrt{a^2 - x^2}} \right) \right)^8 + c$$

$$= \frac{1}{8a^2} \left(\frac{x}{\sqrt{a^2 - x^2}} \right)^8 + c$$

$$= \frac{1}{8a^2} \frac{x^8}{(a^2 - x^2)^4} + c$$

$$\text{Hence, } \int \frac{x^7}{(a^2 - x^2)^5} dx = \frac{1}{8a^2} \frac{x^8}{(a^2 - x^2)^4} + c$$



Myclass24
Your Class. Your Pace.