33. Linear Programming

Exercise 33A

1. Question

Graph the solution sets of the following inequations:

$x + y \geq 4$

Answer

Given $x + y \geq 4$

$\Rightarrow y \geq 4 - x$

Consider the equation $y = 4 - x$.

Finding points on the coordinate axes:

If $x = 0$, the y value is 4 i.e, $y = 4$

$\Rightarrow$ the point on the Y axis is A(0,4)

If $y = 0$, $0 = 4 - x$

$\Rightarrow x = 4$

The point on the X axis is B(4,0)

Plotting the points on the graph: fig. 1a

Now consider the inequality $y \geq 4 - x$

Here we need the y value greater than or equal to $4 - x$

$\Rightarrow$ the required region is above point A.

Therefore the graph of the inequation $x + y \geq 4$ is fig. 1b

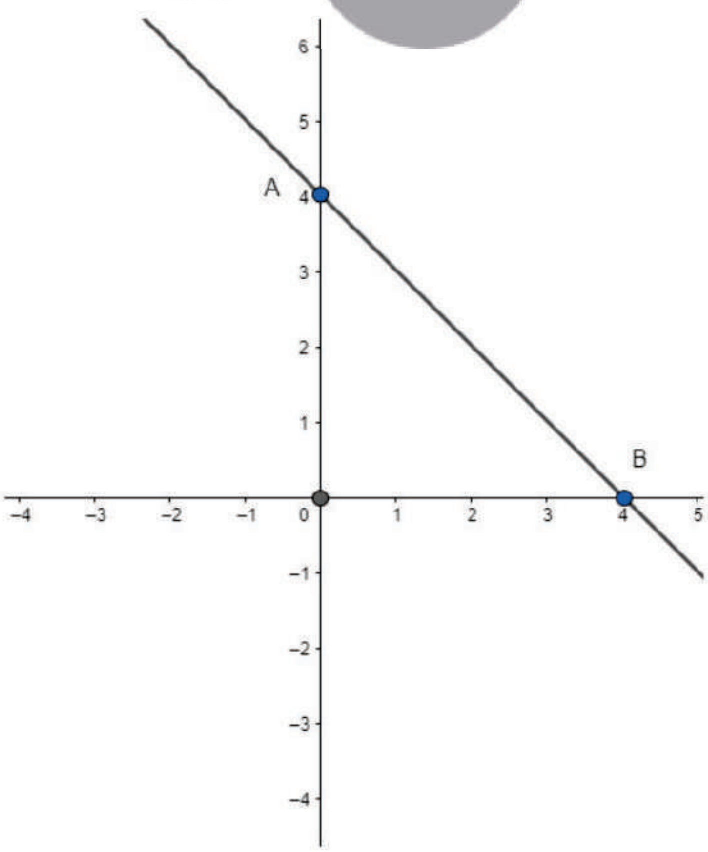


Fig 1a

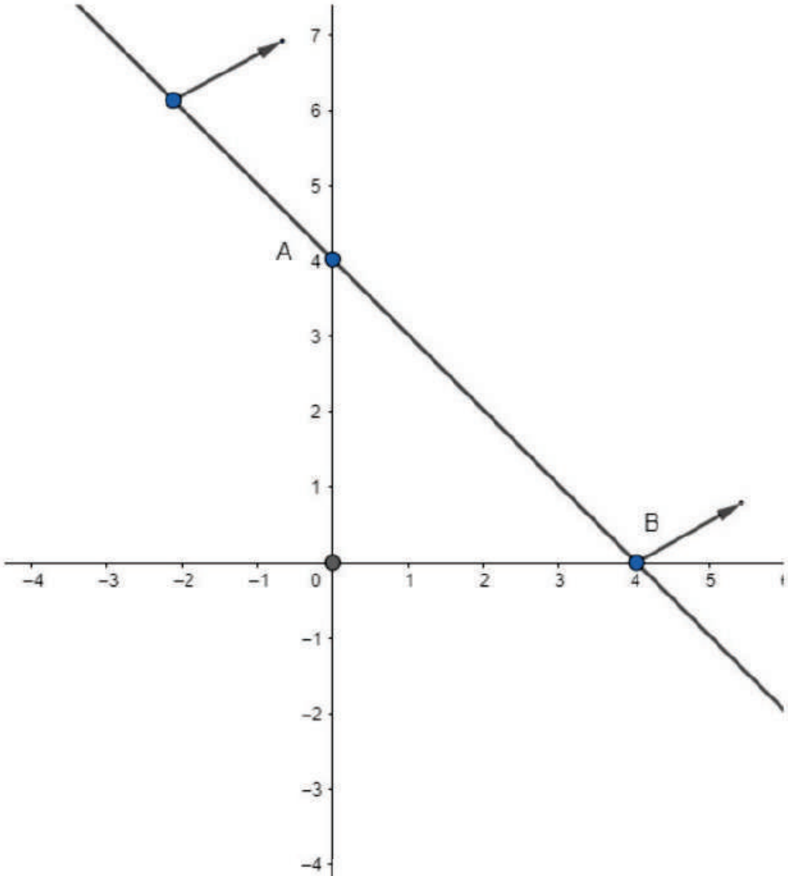


Fig 1b

2. Question

Graph the solution sets of the following inequations:

$x - y \leq 3$

Answer

Given $x - y \leq 3$

$\Rightarrow - y \leq 3 - x$

Multiplying by minus on both the sides, we'll get

$y \geq - 3 + x$

$y \geq x - 3$

Consider the equation $y = x - 3$.

Finding points on the coordinate axes:

If $x = 0$, the y value is $- 3$ i.e, $y = - 3$

$\Rightarrow$ the point on the Y axis is A(0, - 3)

If $y = 0$, $0 = x - 3$

$\Rightarrow x = 3$

The point on the X axis is B(3,0)

Plotting the points on the graph: fig. 2a

Now consider the inequality $y \geq x - 3$

Here we need the y value greater than or equal to $x - 3$

⇒ the required region is above point A.

Therefore the graph of the inequation x + y≥ 4 is fig. 2b

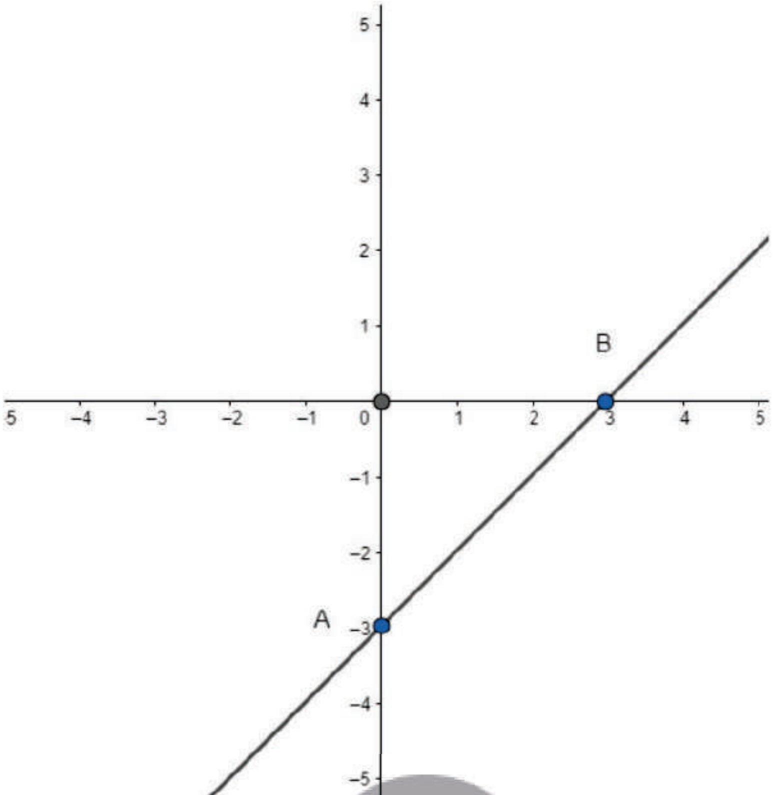


Fig 2a

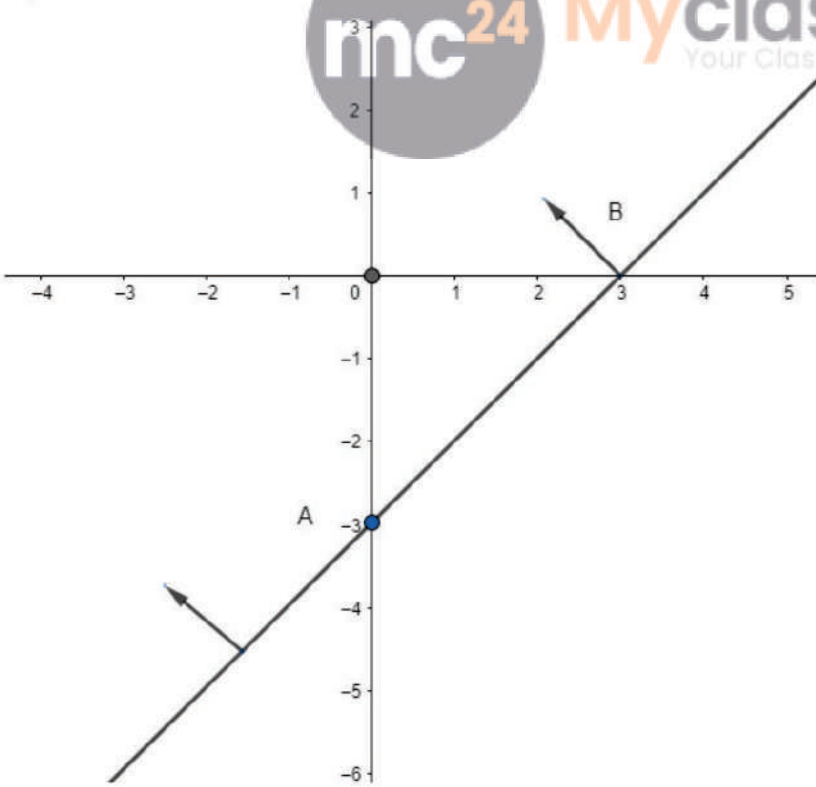


Fig 2b

3. Question

Graph the solution sets of the following inequations:

x + 2y>1

Answer

Given x + 2y>1

⇒ 2y>1 - x

⇒$y > \frac{1}{2} - \frac{x}{2}$

Consider the equation $y = \frac{1}{2} - \frac{x}{2}$

Finding points on the coordinate axes:

If x = 0, the y value is $\frac{1}{2}$ i.e., y = 4

⇒ the point on the Y axis is A(0,$\frac{1}{2}$)

If y = 0, x = 1

The point on the X axis is B(1,0)

Plotting the points on the graph: fig. 3a

Now consider the inequality $y > \frac{1}{2} - \frac{x}{2}$

Here we need the y value greater than $\frac{1}{2} - \frac{x}{2}$

⇒ the required region is above point A.

Also , the line AB is represented in dotted line. This is s done because $y \neq \frac{1}{2} - \frac{x}{2}$

Therefore the graph of the inequation $y > \frac{1}{2} - \frac{x}{2}$ is fig. 3b

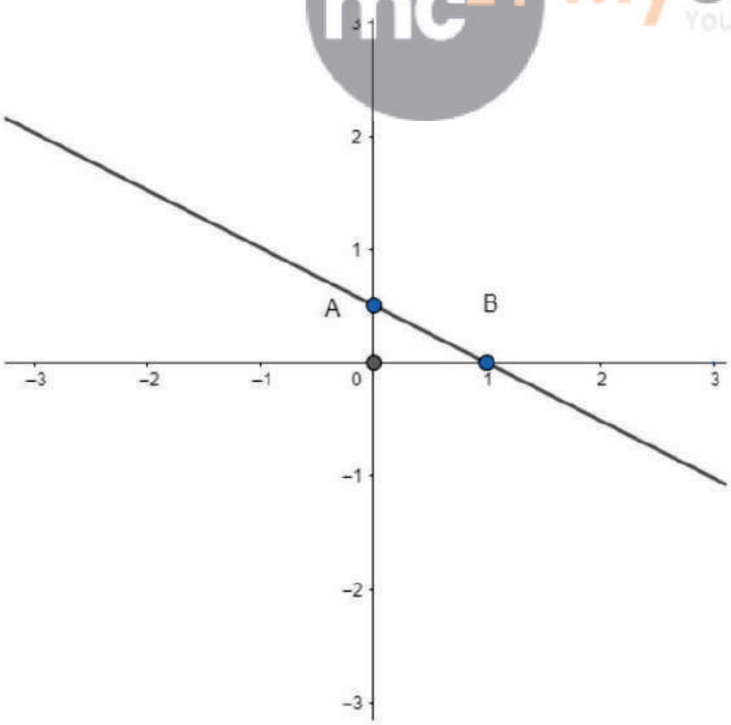


Fig 3a

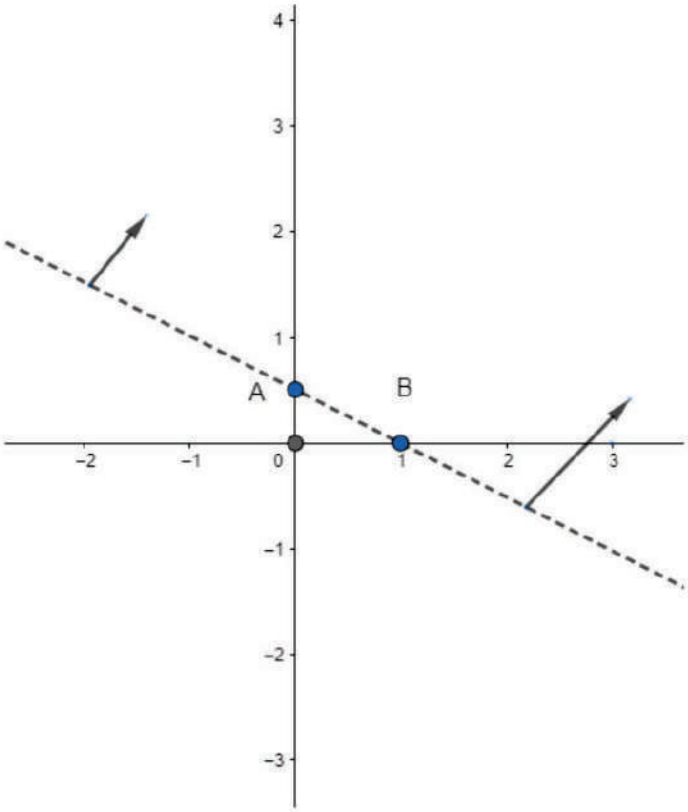


Fig 3b

4. Question

Graph the solution sets of the following inequations:

2x - 3y<4

Answer

Given 2x - 3y<4

⇒ 2x - 4<3y

⇒3y>2x - 4

⇒$y > \frac{2}{3}x - \frac{4}{3}$

Consider the equation $y = \frac{2}{3}x - \frac{4}{3}$

Finding points on the coordinate axes:

If x = 0, the y value is $\frac{1}{2}$ i.e., $y = -\frac{4}{3}$

⇒ the point on the Y axis is $A(0, -\frac{4}{3})$

If y = 0, x = 2

The point on the X axis is B(2,0)

Plotting the points on the graph: fig. 4a

Now consider the inequality $y > \frac{2}{3}x - \frac{4}{3}$

Here we need the y value greater than $\frac{2}{3}x - \frac{4}{3}$

⇒ the required region is above point A.

Also , the line AB is represented in dotted line. This is s done because $y \neq \frac{2}{3}x - \frac{4}{3}$

Therefore the graph of the inequation $y > \frac{2}{3}x - \frac{4}{3}$is fig. 4b

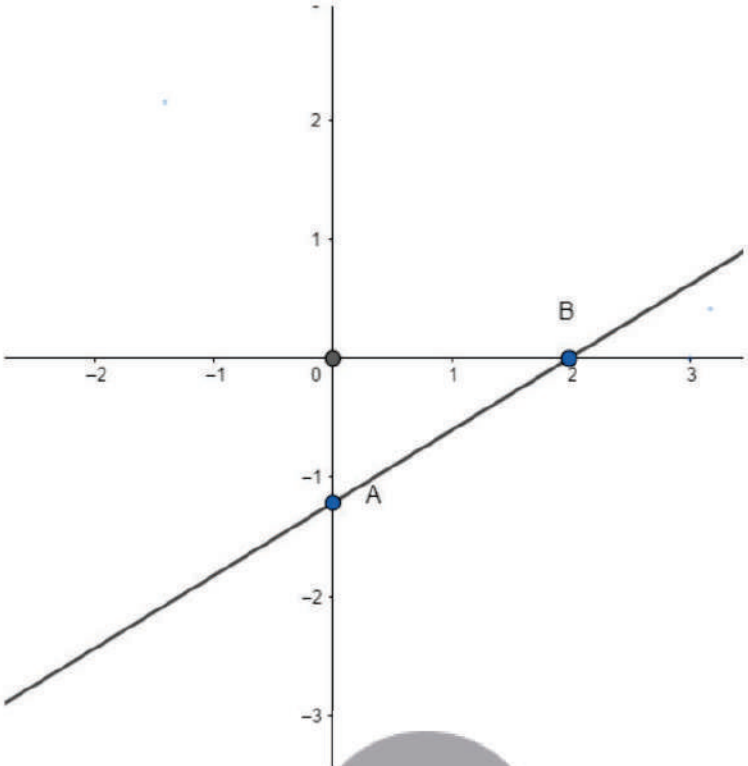


Fig 4a

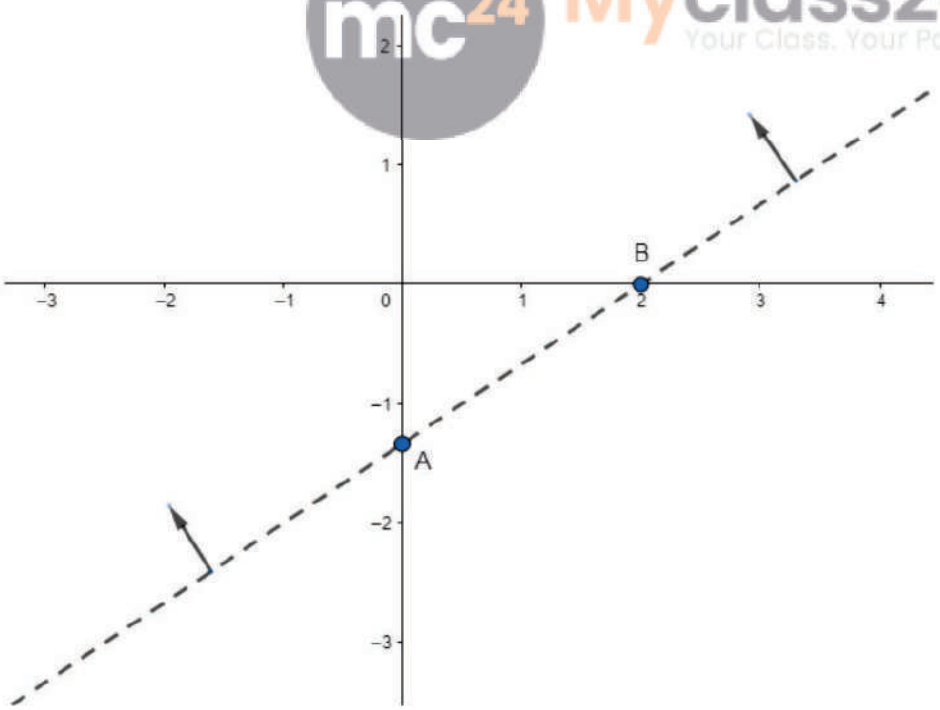


Fig 4b

5. Question

Graph the solution sets of the following inequations:

$x \geq y - 2$

Answer

Given x ≥ y - 2

⇒ y≤x + 2

Consider the equation y = x + 2

Finding points on the coordinate axes:

If x = 0, the y value is 2 i.e, y = 2

⇒ the point on the Y axis is A(0,2)

If y = 0, 0 = x + 2

⇒x = - 2

The point on the X axis is B(- 2,0)

Plotting the points on the graph: fig. 5a

Now consider the inequality y≤x + 2

Here we need the y value less than or equal to x + 2

⇒ the required region is below point A.

Therefore the graph of the inequation x ≥ y - 2 is fig. 5b

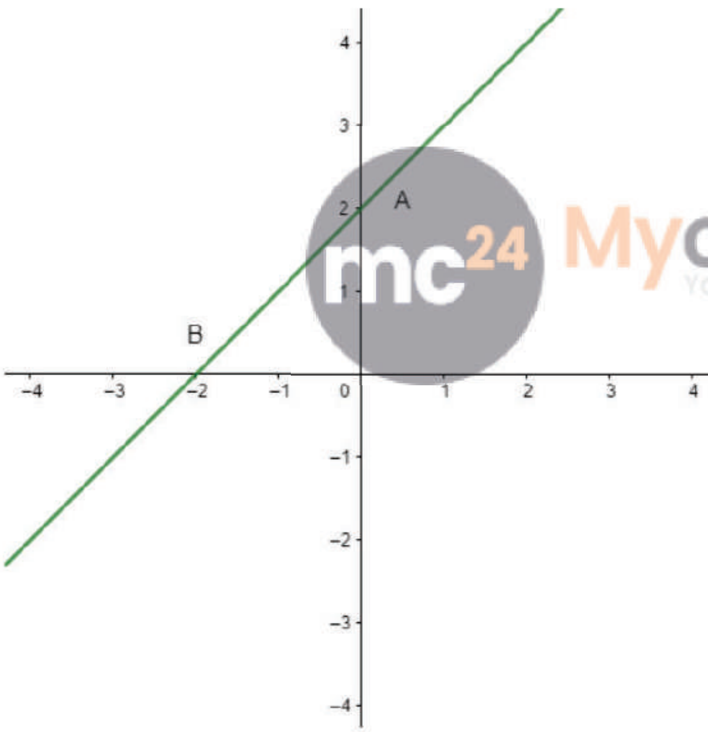


fig 5a

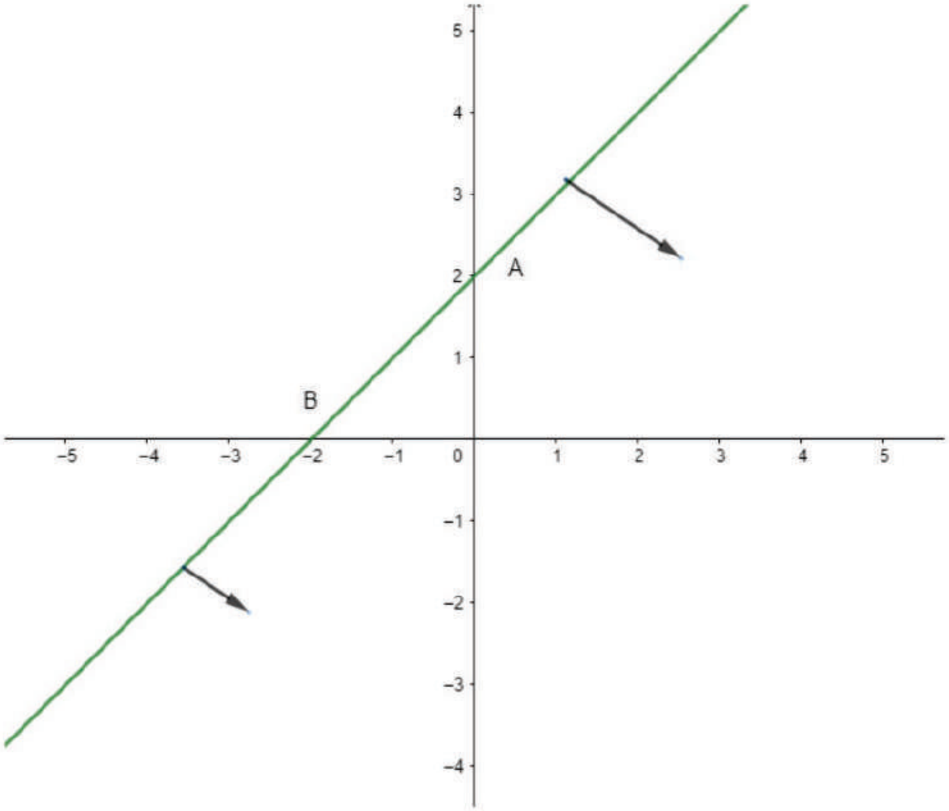


Fig 5b

6. Question

Graph the solution sets of the following inequations:

$y - 2 \leq 3x$

Answer

Given $y - 2 \leq 3x$

$\Rightarrow y \leq 3x + 2$

Consider the equation $y = 3x + 2$

Finding points on the coordinate axes:

If $x = 0$, the y value is 2 i.e, $y = 2$

$\Rightarrow$ the point on Y axis is A(0,2)

If $y = 0$, $0 = 3x + 2$

$\Rightarrow x = -\frac{3}{2}$

The point on the X axis is B($- -\frac{3}{2}$,0)

Plotting the points on the graph: fig. 6a

Now consider the inequality $y \leq 3x + 2$

Here we need the y value less than or equal to $3x + 2$

$\Rightarrow$ the required region is below point A.

Therefore the graph of the inequation $y \leq 3x + 2$ is fig. 5b

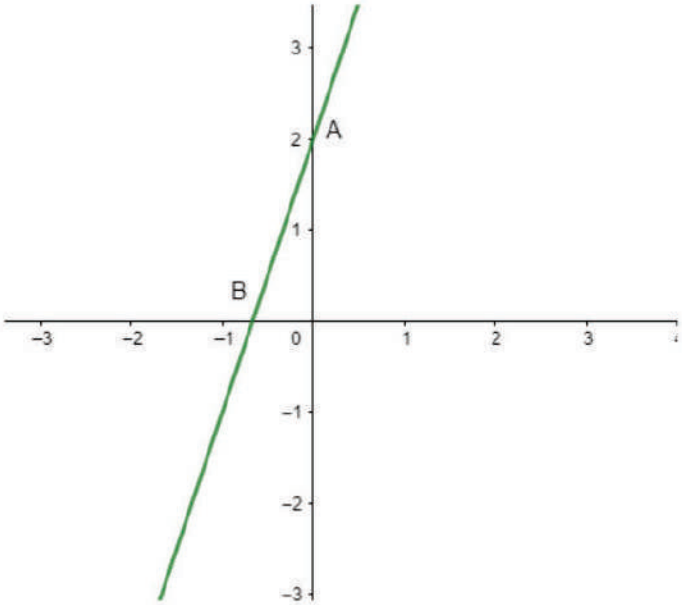


Fig 6a

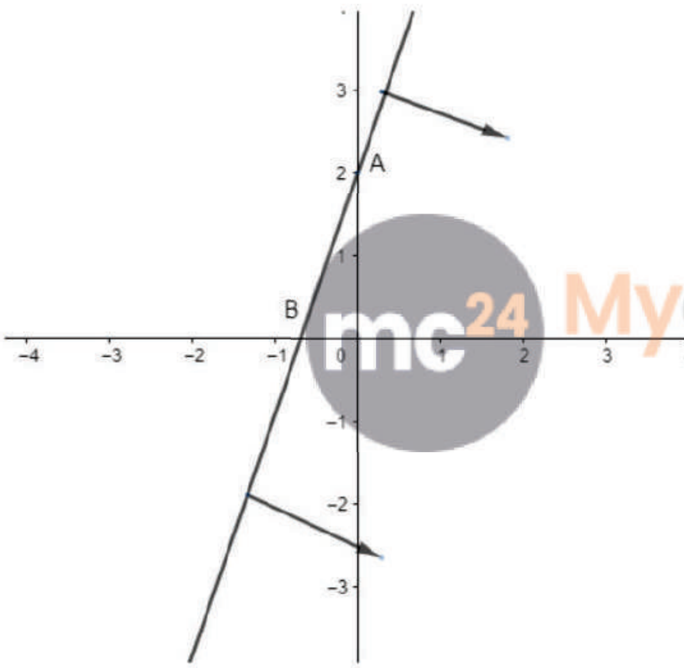


Fig 6b

7. Question

Solve each of the following systems of simultaneous inequations:

$2x + y>1$ and $2x - y \geq - 3$

Answer

Consider the inequation $2x + y>1$:

$\Rightarrow y>1 - 2x$

Consider the equation $y = 1 - 2x$

Finding points on the coordinate axes:

If $x = 0$, the y value is 1 i.e, $y = 1$

$\Rightarrow$ the point on Y axis is A(0,1)

If $y = 0$, $0 = x + 2$

$\Rightarrow x = \frac{1}{2}$

The point on the X axis is $B(\frac{1}{2},0)$

Plotting the points on the graph: fig. 7a

Now consider the inequality y>1 - 2x

Here we need the y value greater than x + 2

⇒ the required region is below point A.

Therefore the graph of the inequation y>1 - 2x is fig. 7b

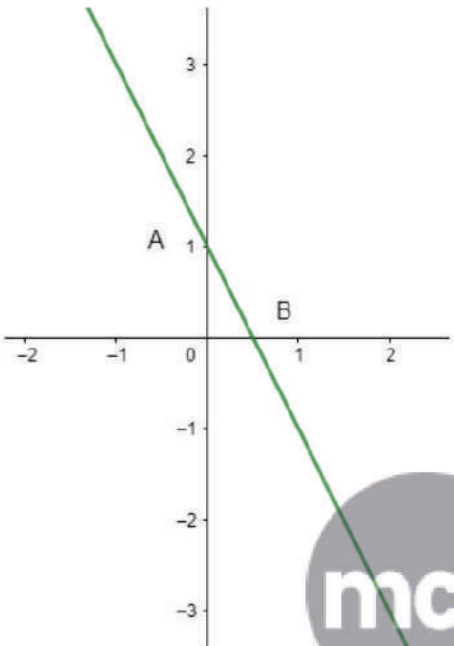


Fig 7a

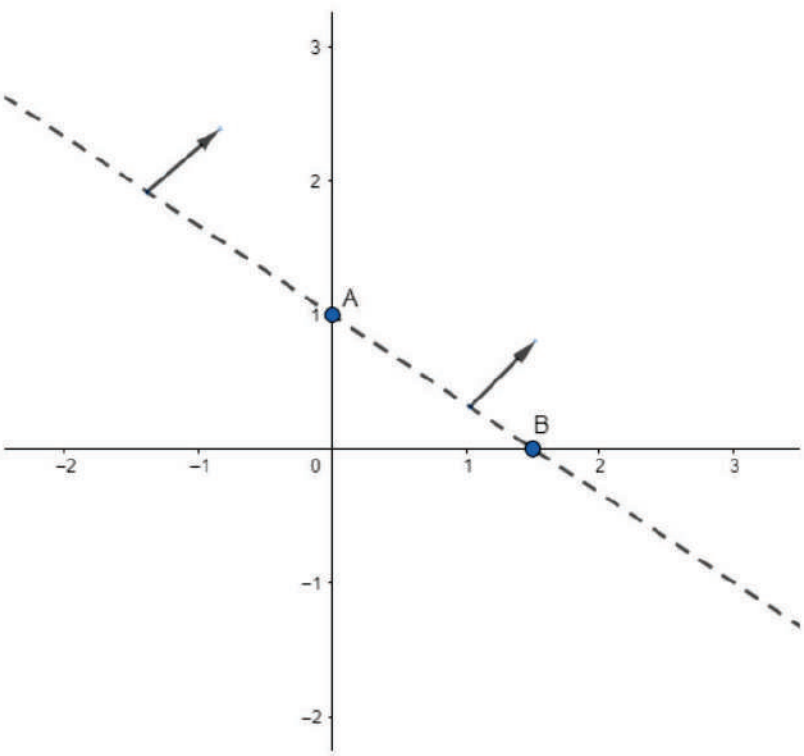


Fig 7b

Consider the inequation $2x - y \geq 3$

$\Rightarrow y \leq 2x - 3$

Consider the equation $y = 2x - 3$

Finding points on the coordinate axes:

If $x = 0$, the y value is -3 i.e, $y = -3$

$\Rightarrow$ the point on the Y axis is C(0, - 3)

If $y = 0$, $0 = 2x + 3$

$\Rightarrow x = \frac{3}{2}$

The point on the X axis is $D(\frac{3}{2}, 0)$

Plotting the points on the graph: fig. 7c

Now consider the inequality $y \leq 2x - 3$

Here we need the y value less than or equal to $2x - 3$

$\Rightarrow$ the required region is below point C.

Therefore the graph of the inequation $y \leq 2x - 3$ is fig. 7d

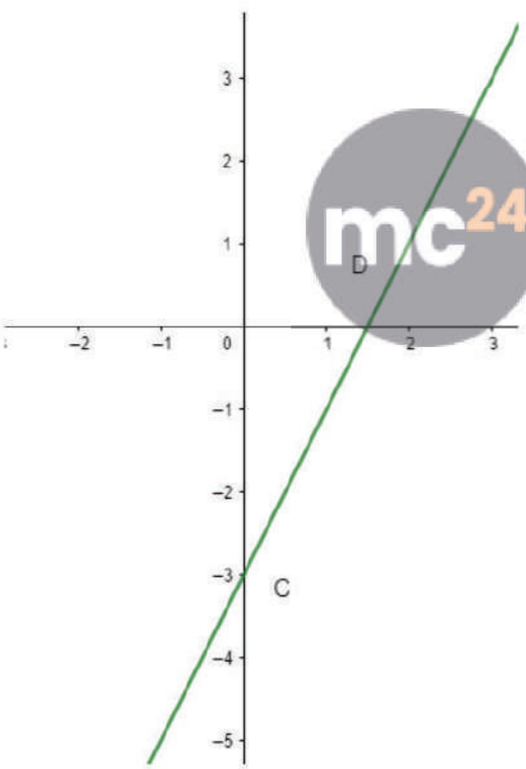


Fig 7c

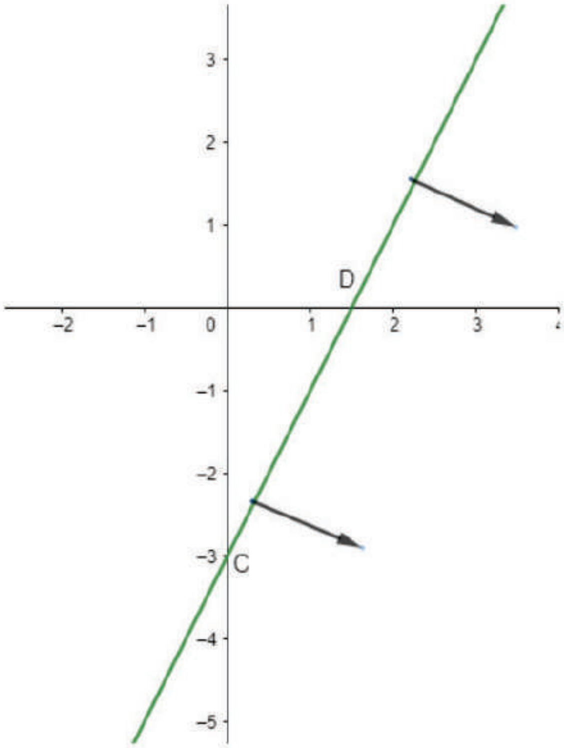
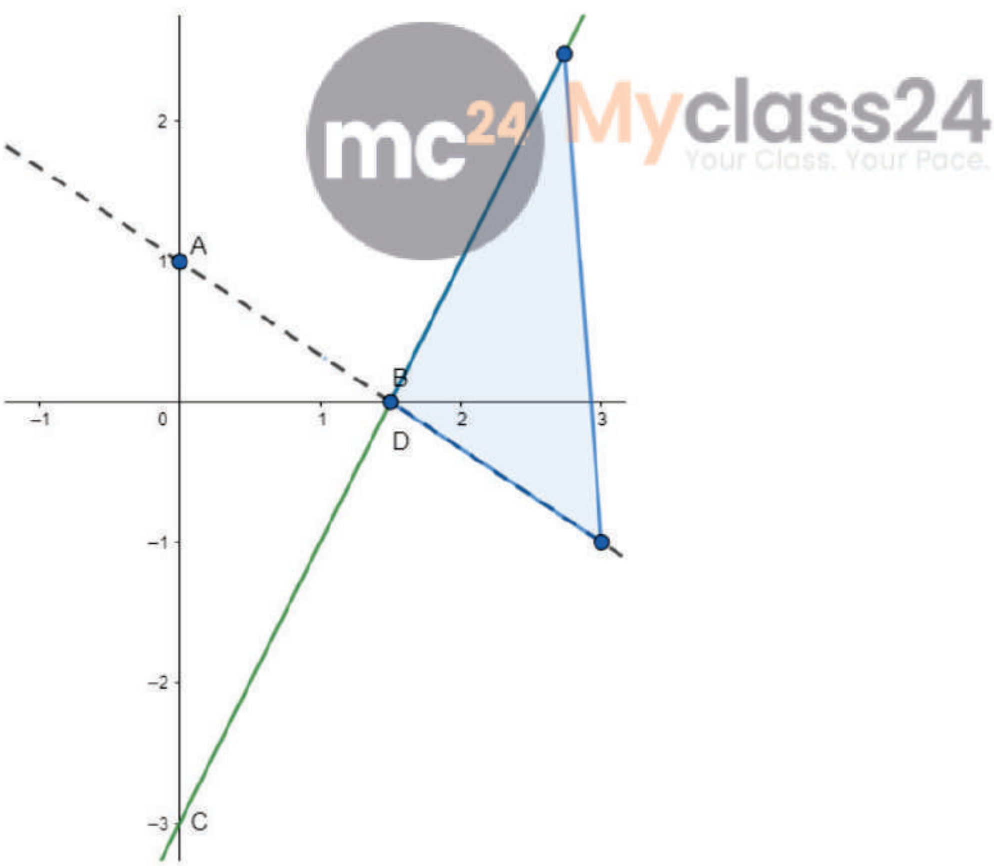


Fig 7d

Combining the graphs 7c and 7d, we'll get,



The solution of the system of simultaneous inequations is the intersection region of the solutions of the two given inequations.

8. Question

Solve each of the following systems of simultaneous inequations:

$x - 2y \geq 0,\ 2x - y \leq -2$

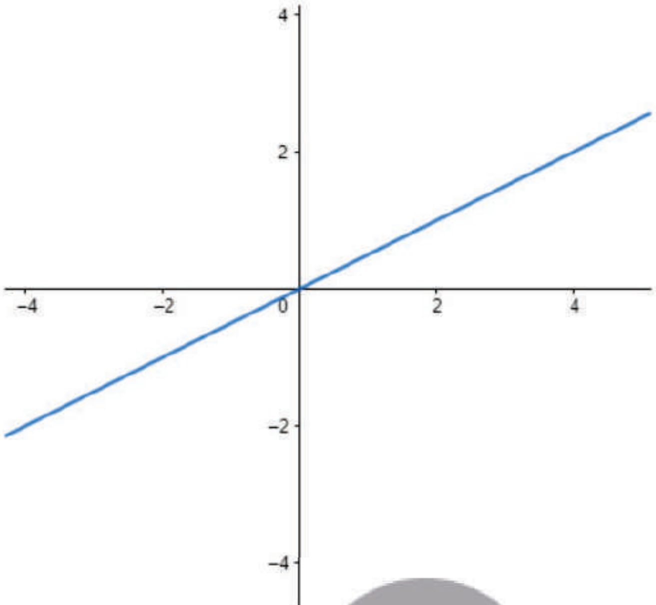
Answer

Consider the inequation x - 2y ≥ 0:

⇒x ≥ 2y

⇒$y \le \frac{x}{2}$

consider the equation $y = \frac{x}{2}$. This equation's graph is a straight line passing through origin.



Now consider the inequality $y \le \frac{x}{2}$

Here we need the y value less than or equal to $\frac{x}{2}$

⇒ the required region is below the origin.

Therefore the graph of the inequation $y \le \frac{x}{2}$ is fig.8a

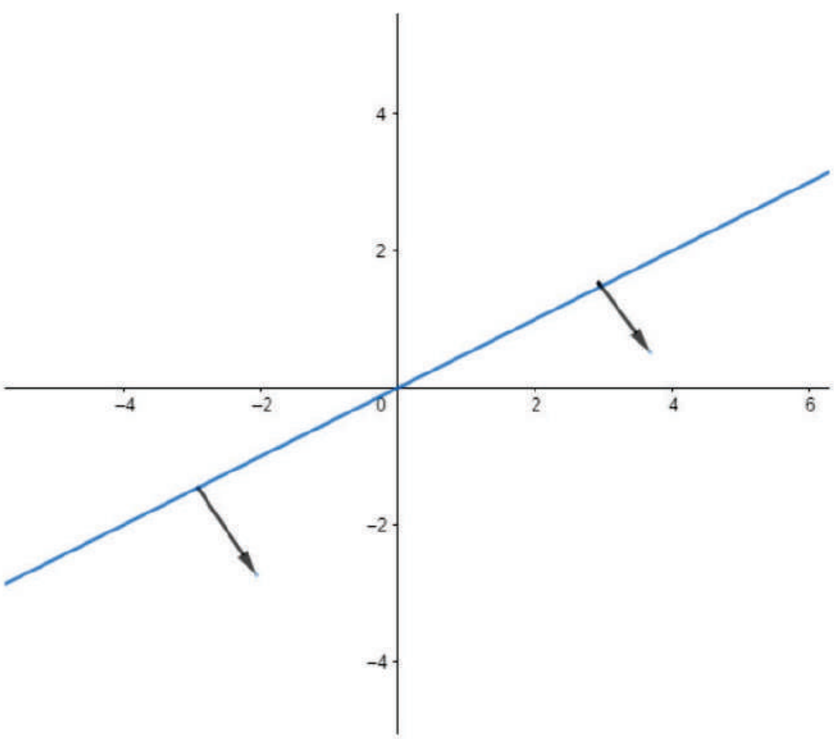


Fig 8a

Consider the inequation $2x - y \le -2$:

$\Rightarrow y \ge 2x + 2$

Consider the equation $y = 2x + 2$

Finding points on the coordinate axes:

If $x = 0$, the y value is 2 i.e, $y = 2$

$\Rightarrow$ the point on the Y axis is A(0,2)

If $y = 0$, $0 = 2x + 2$

$\Rightarrow x = -1$

The point on the X axis is B(- 1,0)

Plotting the points on the graph: fig. 8b.

Now consider the inequality $y \ge 2x + 2$

Here we need the y value greater than or equal to $2x + 2$

$\Rightarrow$ the required region is above point A.

Therefore the graph of the inequation $y \ge 2x + 2$ is fig. 8c

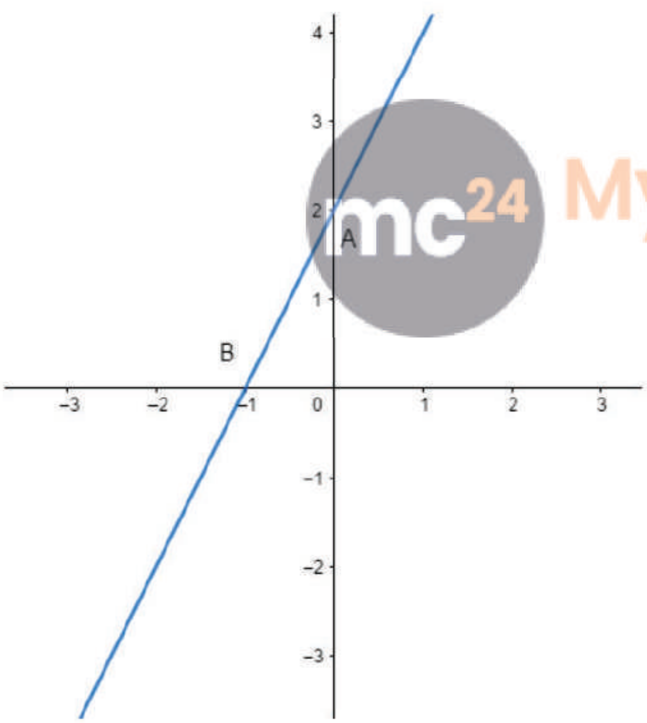


Fig 8b

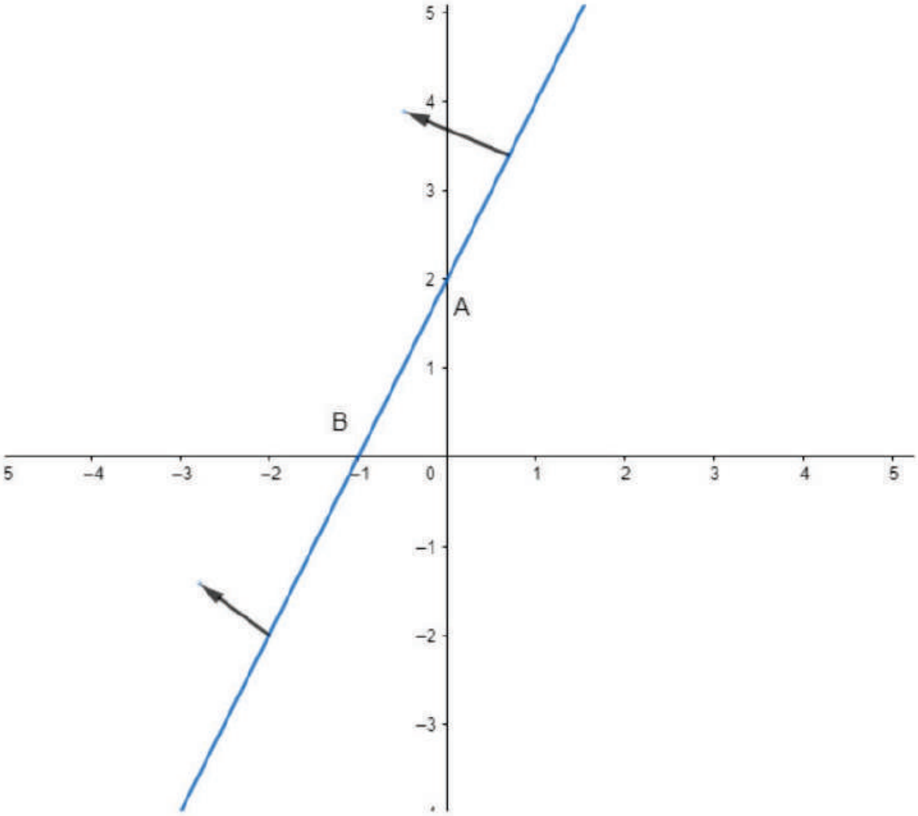
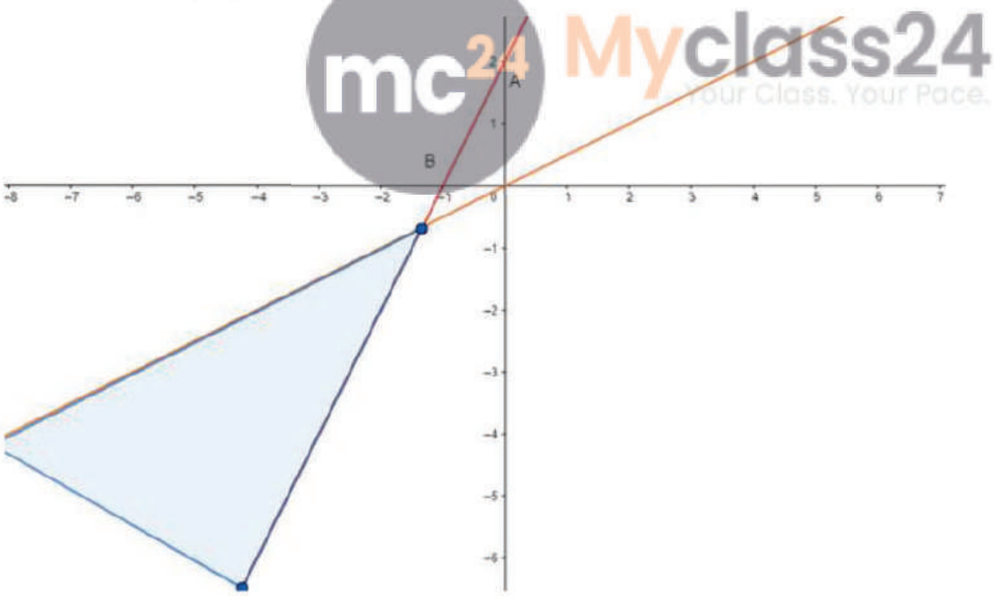


Fig 8c

Combining the graphs of 8a and 8c, we'll get



The solution of the system of simultaneous inequations is the intersection region of the solutions of the two given inequations.

9. Question

Solve each of the following systems of simultaneous inequations:

$3x + 4y \geq 12$, $x \geq 0$, $y \geq 1$ and $4x + 7y \leq 28$

Answer

Consider the inequation $3x + 4y \geq 12$:

$\Rightarrow 4y \geq 12 - 3x$

$\Rightarrow y \geq 3 - \frac{3}{4}x$

Consider the equation $y = 3 - \frac{3}{4}x$

Finding points on the coordinate axes:

If x = 0, the y value is 3 i.e, y = 3

⇒ the point on the Y axis is A(0,3)

If y = 0, $0 = 3 - \frac{3}{4}x$

⇒x = 4

The point on the X axis is B(4,0)

Now consider the inequality $y \geq 3 - \frac{3}{4}x$

Here we need the y value greater than or equal to $y \geq 3 - \frac{3}{4}x$

⇒ the required region is above point A.

Therefore the graph of the inequation $y \geq 3 - \frac{3}{4}x$ is fig. 9a

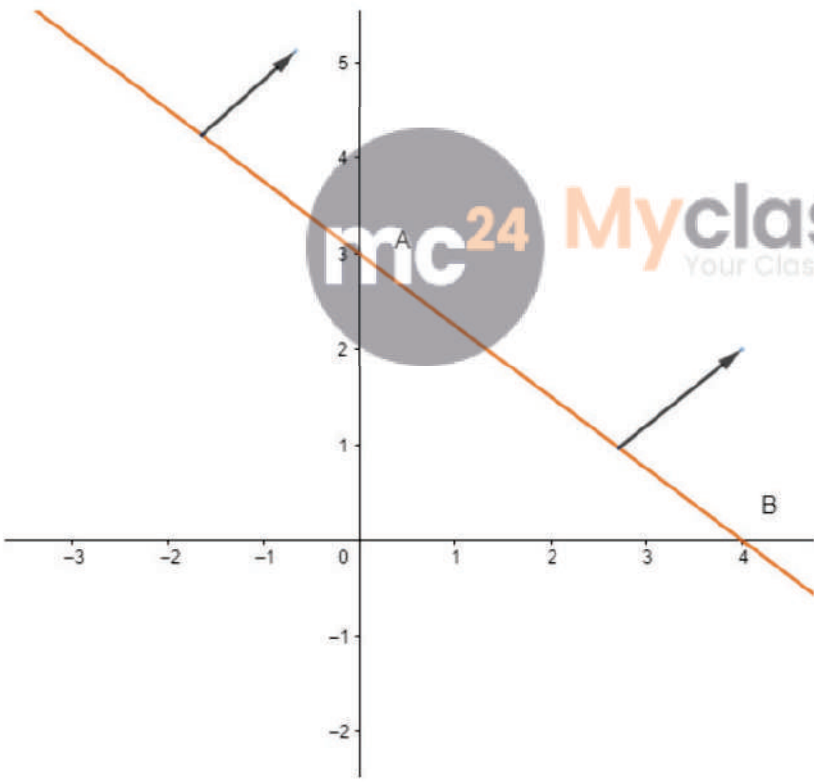


Fig 9a

Consider the inequation 4x + 7y≤28

⇒ 7y≤28 - 4x

$\Rightarrow y \leq 4 - \frac{4}{7}x$

Consider the equation $y = 4 - \frac{4}{7}x$

Finding points on the coordinate axes:

If x = 0, the y value is 4 i.e, y = 4

⇒ the point on the Y axis is C(0,4)

If y = 0, 0 = 4 - $\frac{4}{7}x$

⇒x = 7

The point on the X axis is D(7,0)

Now consider the inequality y≤4 - $\frac{4}{7}x$

Here we need the y value less than or equal to 4 - $\frac{4}{7}x$

⇒ the required region is below point C.

Therefore the graph of the inequation y≤4 - $\frac{4}{7}x$ is fig. 9b

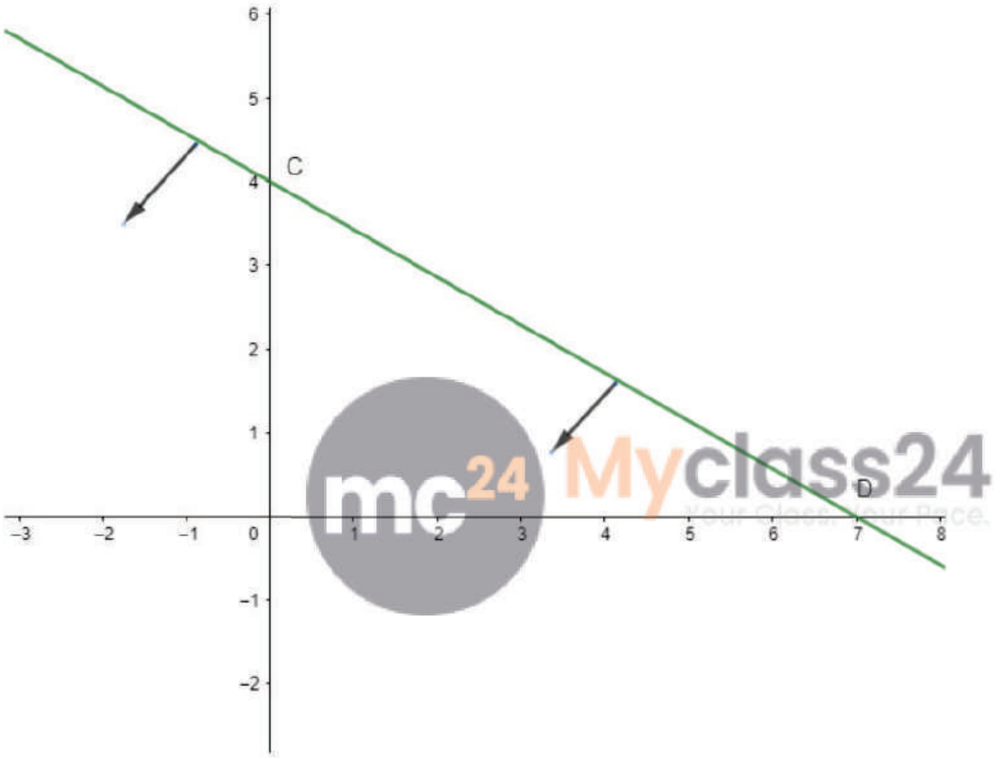
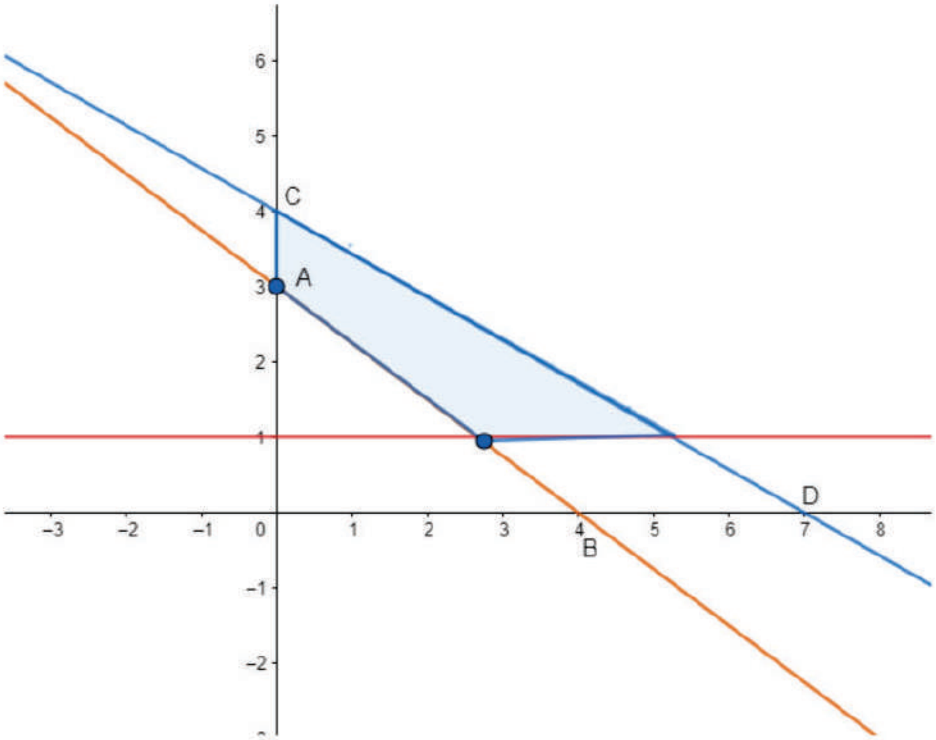


Fig 9b

x ≥ 0 is the region right side of Y - axis.

y ≥ 1 is the region above the line y = 1

Combining all the above results in a single graph , we'll get



The solution of the system of simultaneous inequations is the intersection region of the solutions of the two given inequations.

10. Question

Show that the solution set of the following linear constraints is empty:

$x - 2y \geq 0$, $2x - y \leq -2$, $x \geq 0$ and $y \geq 0$

Answer

Consider the inequation $x - 2y \geq 0$:

$\Rightarrow x \geq 2y$

$\Rightarrow y \leq \frac{x}{2}$

consider the equation $y = \frac{x}{2}$. This equation's graph is a straight line passing through origin.

Now consider the inequality $y \leq \frac{x}{2}$

Here we need the y value less than or equal to $\frac{x}{2}$

$\Rightarrow$ the required region is below origin.

Therefore the graph of the inequation $y \leq \frac{x}{2}$ is fig.10a

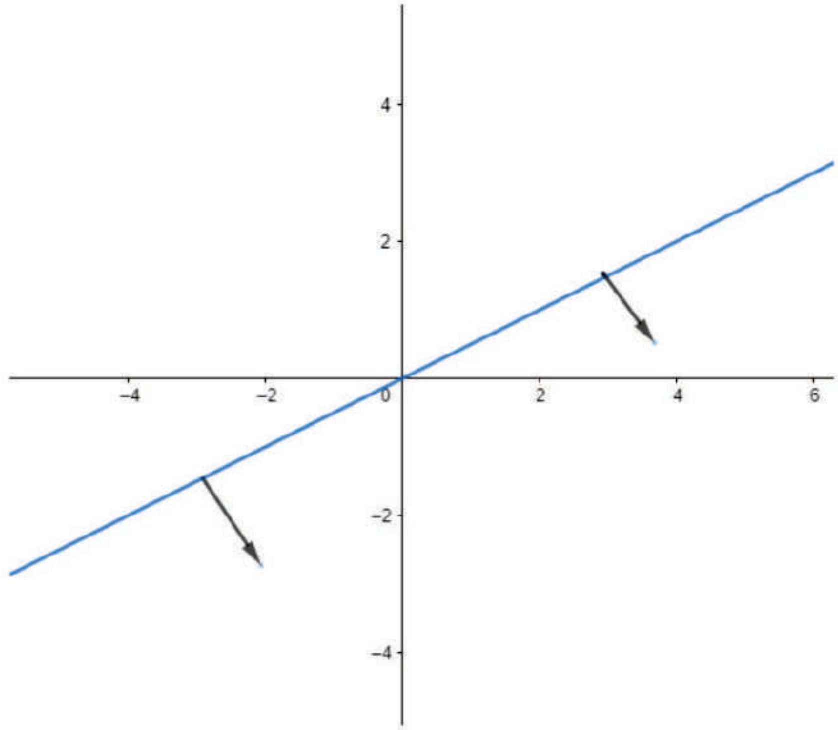


Fig 10a

Consider the inequation $2x - y \leq -2$:

$\Rightarrow y \geq 2x + 2$

Consider the equation $y = 2x + 2$

Finding points on the coordinate axes:

If $x = 0$, the y value is 2 i.e, $y = 2$

$\Rightarrow$ the point on Y axis is A(0,2)

If $y = 0$, $0 = 2x + 2$

$\Rightarrow x = -1$

The point on X axis is B(- 1,0)

Now consider the inequality $y \geq 2x + 2$

Here we need the y value greater than or equal to $2x + 2$

$\Rightarrow$ the required region is above point A.

Therefore the graph of the inequation $y \geq 2x + 2$ is fig. 10b

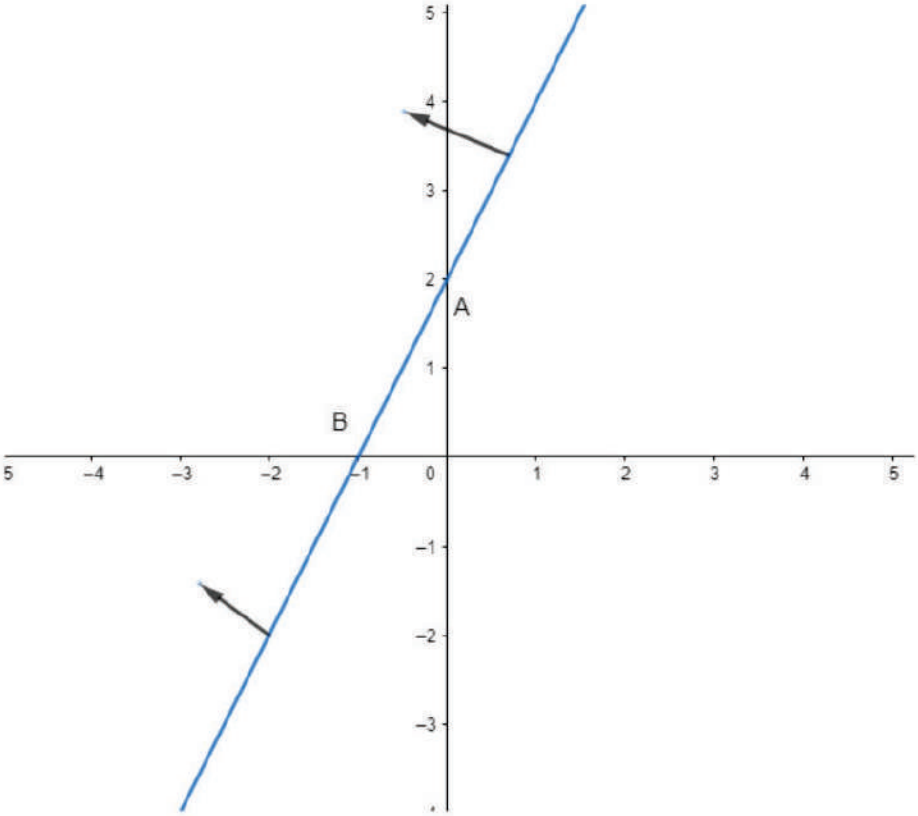
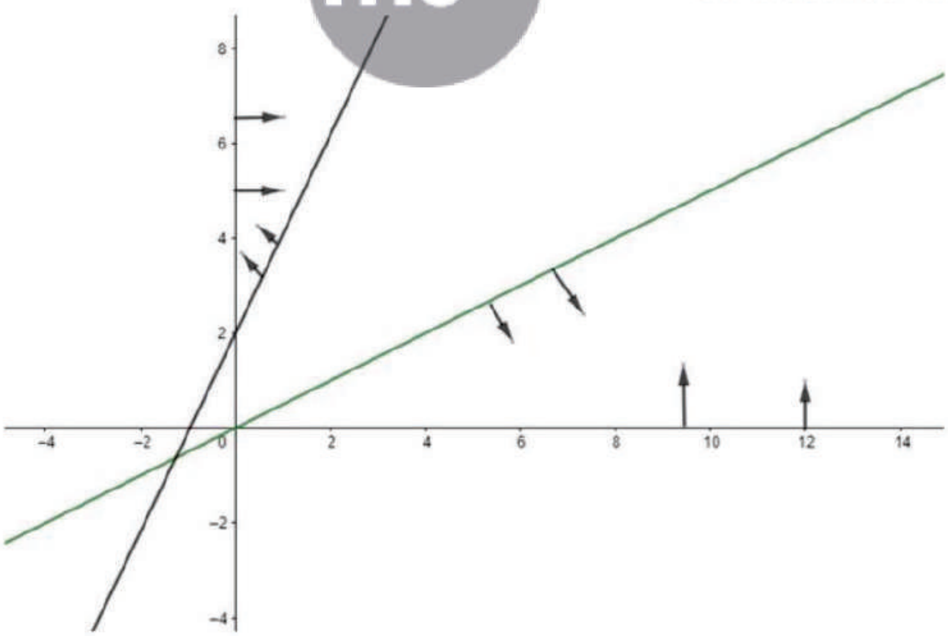


Fig 10b

y ≥ 0 is the region above X - axis

x ≥ 0 is the region right side of Y - axis

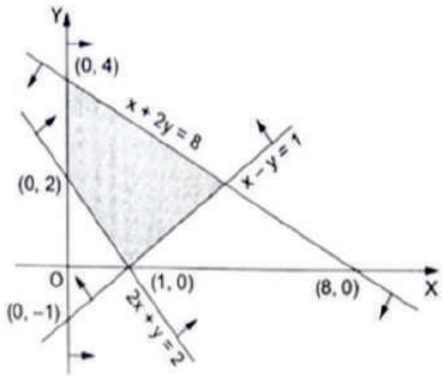
Combining the above results, we'll get



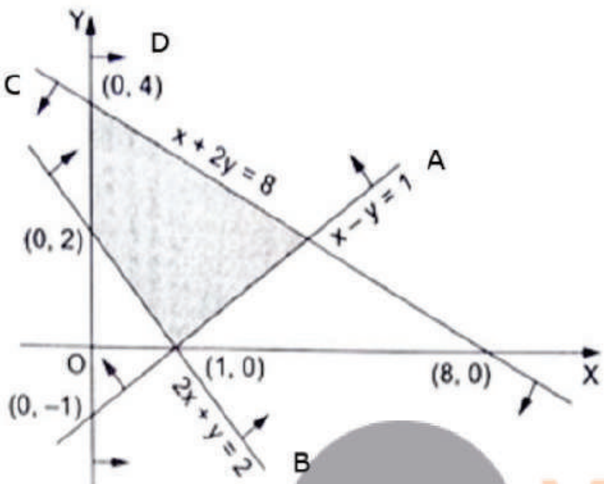
As they is no common area of intersection , there is no solution for the given set of simultaneous inequations.

11. Question

Find the linear constraints for which the shaded area in the figure given is the solution set.



Answer



Consider A:

Given line x - y = 1

⇒y = x - 1

As the region given in the figure is above the y - intercept's coordinates (0, - 1),

⇒y ≥ x - 1

⇒x - y≤1

Consider B:

Given line 2x + y = 2

⇒y = 2 - 2x

As the region given in the figure is above the y - intercept's coordinates (0,2),

⇒y ≥ 2 - 2x

⇒2x + y ≥ 2

Consider C:

Given line x + 2y = 8

⇒2y = 8 - x

⇒y = 4 - $\frac{x}{2}$

As the region given in the figure is below the y - intercept's coordinates (0,4),

⇒y≤4 - $\frac{x}{2}$

$\Rightarrow 2y \leq 8 - x$

$\Rightarrow x + 2y \leq 8$

Consider D:

It is the region right side of the Y - axis.

It is $x \geq 0$.

All the results derived:

$x - y \leq 1$

$2x + y \geq 2$

$x + 2y \leq 8$

$x \geq 0$

Exercise 33B

1. Question

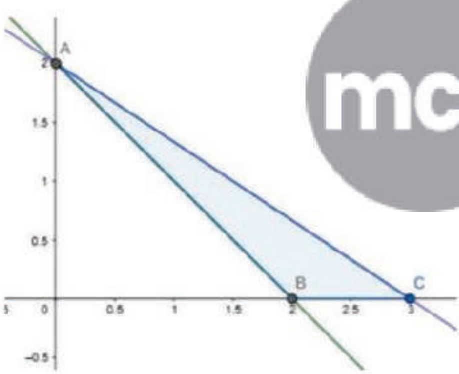
Find the maximum value of $Z = 7X + 7Y$, subject to the constraints.

$x \geq 0$, $y \geq 0$, $x + y \geq 2$ and $2x + 3y \leq 6$.

Answer

The feasible region determined by the constraints $x \geq 0$, $y \geq 0$,

$x + y \geq 2$, $2x + 3y \leq 6$ is given by



The corner points of the feasible region is A(0,2),B(2,0),C(3,0).

The values of Z at the following points is

Corner point	Z = 7x + 7y	
A(0,2)	14	
B(2,0)	14	
C(3,0)	21	Maximum

The maximum value of Z is 21 at point C(3,0) .

1. Question

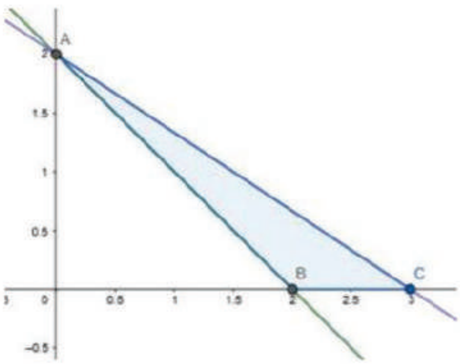
Find the maximum value of $Z = 7X + 7Y$, subject to the constraints.

$x \geq 0$, $y \geq 0$, $x + y \geq 2$ and $2x + 3y \leq 6$.

Answer

The feasible region determined by the constraints $x \geq 0$, $y \geq 0$,

$x + y \geq 2$, $2x + 3y \leq 6$ is given by



The corner points of the feasible region is A(0,2),B(2,0),C(3,0).

The values of Z at the following points is

Corner point	Z = 7x + 7y	
A(0,2)	14	
B(2,0)	14	
C(3,0)	21	Maximum

The maximum value of Z is 21 at point C(3,0) .

2. Question

Maximize $Z = 4x + 9y$, subject to the constraints

$x \geq 0$, $y \geq 0$, $x + 5y \leq 200$, $2x + 3y \leq 134$.

Answer

The feasible region determined by the constraints $x \geq 0$, $y \geq 0$, $x + 5y \leq 200$, $2x + 3y \leq 134$ is given by