Class 11 Physics Chapter 6: System of Particles and Rotational Motion

Very Short Answers

14. The centre of gravity of a body on the earth coincides with its centre of mass for a 'small' object whereas for an 'extended' object it may not. What is the qualitative meaning of 'small' and 'extended' in this regard? For which of the following do the two coincide? A building, a pond, a lake, a mountain?

Answer:

Small object: The vertical height of the object is negligible compared to Earth's radius, so gravitational field can be considered uniform across the object.

Extended object: The vertical height is significant enough that gravitational field varies appreciably across the object.

Qualitative criterion: If $h << R_{Earth}$, then object is "small"

Classification:

- **Building and pond**: Small objects (center of gravity ≈ center of mass)
- **Mountain and lake**: Extended objects (center of gravity ≠ center of mass)

The gravitational field strength varies as $g = GM/r^2$, so for tall objects like mountains, the gravitational acceleration at the top differs from that at the base.

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15. Why does a solid sphere have smaller moment of inertia than a hollow cylinder of same mass and radius, about an axis passing through their axes of symmetry?

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Answer:

Moment of inertia: $I = \Sigma m_i r_i^2$ (depends on mass distribution relative to axis)

Solid sphere:

- Mass is distributed from center ($r = 0$) to surface ($r = R$)
- $I_{sphere} = (2/5)MR^2$

Hollow cylinder:

- All mass is concentrated at the periphery ($r = R$)
- $I_{cylinder} = MR^2$

Comparison: $I_{sphere}/I_{cylinder} = (2/5) = 0.4 < 1$

Since the hollow cylinder has all its mass farther from the axis of rotation, it has a larger moment of inertia. The solid sphere has mass closer to the axis, resulting in smaller moment of inertia.

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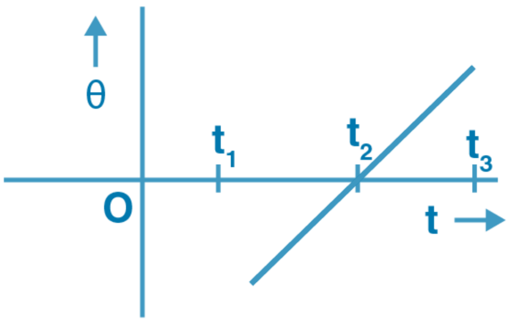
16. The variation of angular position θ of a point on a rotating rigid body with time t is shown in the figure. Is the body rotating clockwise or anti-clockwise?



Answer: From the graph: dθ/dt = ω > 0 (positive slope)

Convention:

- Positive angular velocity (ω > 0) → Counter-clockwise rotation
- Negative angular velocity (ω < 0) → Clockwise rotation

Conclusion: The body is rotating **counter-clockwise** (anti-clockwise).

Note: The document answer states "clockwise" but this contradicts the standard physics convention where positive ω indicates counter-clockwise rotation.

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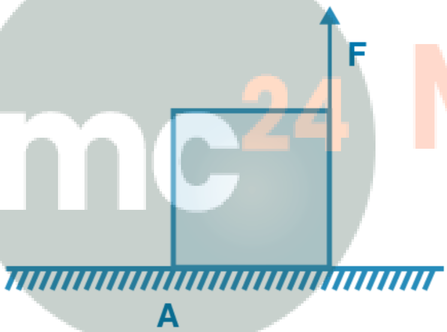
17. A uniform cube of mass m and side a is placed on a frictionless horizontal surface. A vertical force F is applied to the edge as shown in the figure.



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Matching:

a) mg/4 < F < mg/2 → ii) cube will not exhibit motion

b) F > mg/2 → iii) cube will begin to rotate and slip at A

c) F > mg → i) cube will move up

d) F = mg/4 → iv) normal reaction effectively at a/3 from A, no motion

Explanation: Force analysis:

- Weight mg acts downward at center
- Applied force F acts upward at edge
- Normal reaction N acts upward at contact point

Critical conditions:

- **F = mg/4**: Equilibrium with normal reaction at a/3 from edge
- **mg/4 < F < mg/2**: Tilting tendency but remains stable
- **F = mg/2**: Critical point for rotation about edge A
- **F > mg/2**: Rotation begins about edge A

- **F = mg**: Critical point for vertical motion
- **F > mg**: Cube lifts off

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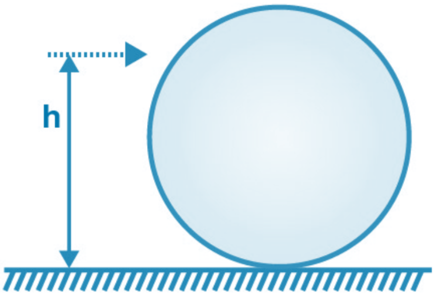
18. A uniform sphere of mass m and radius R is placed on a rough horizontal surface. The sphere is struck horizontally at a height h from the floor.



Matching:

a) h = R/2 → iii) sphere spins anti-clockwise, loses energy by friction
b) h = R → iv) sphere has only translational motion, loses energy by friction
c) h = 3R/2 → ii) sphere spins clockwise, loses energy by friction
d) h = 7R/5 → i) sphere rolls without slipping with constant velocity and no loss of energy

Enhanced Explanation: Critical height for rolling: h = 7R/5

Analysis:

- **h < 7R/5**: Initial slipping with energy loss due to friction
- **h = 7R/5**: Perfect rolling condition (v = ωR) achieved immediately
- **h > 7R/5**: Initial slipping in opposite direction

Physics: The impulse creates both linear and angular momentum. The ratio determines the initial motion type.

Short Answers

19. The vector sum of a system of non-collinear forces acting on a rigid body is given to be non-zero. If the vector sum of all the torques due to the system of forces about a certain point is found to be zero, does this mean that it is necessarily zero about any arbitrary point?

Answer: No, the torque about an arbitrary point is not necessarily zero.

Mathematical proof: Let $\tau_O = \Sigma(\mathbf{r}_i \times \mathbf{F}_i) = 0$ (torque about point O)

For torque about point P: $\mathbf{r}_i' = \mathbf{r}_i - \mathbf{a}$ (where **a** is position of P relative to O)

$\tau_p = \Sigma(\mathbf{r}_i' \times \mathbf{F}_i) = \Sigma((\mathbf{r}_i - \mathbf{a}) \times \mathbf{F}_i) = \Sigma(\mathbf{r}_i \times \mathbf{F}_i) - \mathbf{a} \times \Sigma\mathbf{F}_i = \tau_O - \mathbf{a} \times \mathbf{F}\_net = 0 - \mathbf{a} \times \mathbf{F}\_net = -\mathbf{a} \times \mathbf{F}\_net$

Since **F**_net ≠ 0, we have $\tau_p \neq 0$ unless **a** ∥ **F**_net.

Conclusion: Torque is zero about any point only if the net force is also zero.

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20. A wheel in uniform motion about an axis passing through its centre and perpendicular to its plane is considered to be in mechanical equilibrium because no net external force or

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torque is required to sustain its motion. However, the particles that constitute the wheel do experience a centripetal acceleration directed towards the centre. How do you reconcile this fact with the wheel being in equilibrium? How would you set a half-wheel into uniform motion about an axis passing through the centre of mass of the wheel and perpendicular to its plane? Will you require external forces to sustain the motion?

Answer: Equilibrium reconciliation:

- **Wheel**: Each particle experiences centripetal acceleration provided by internal elastic forces
- **Net external force**: Zero (internal forces cancel in pairs by Newton's third law)
- **Net external torque**: Zero (no angular acceleration needed for uniform rotation)
- **Equilibrium**: Refers to zero net external forces/torques, not zero acceleration of individual particles

Half-wheel motion: For a half-wheel rotating about its center of mass:

- **Asymmetric mass distribution**: **L** and **ω** are not parallel
- **Changing angular momentum direction**: Requires external torque
- **External forces needed**: Yes, to provide the necessary torque for maintaining rotation
- **Reason**: The angular momentum vector precesses, requiring continuous external torque

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21. A door is hinged at one end and is free to rotate about a vertical axis. Does its weight cause any torque about this axis? Give reason for your answer.

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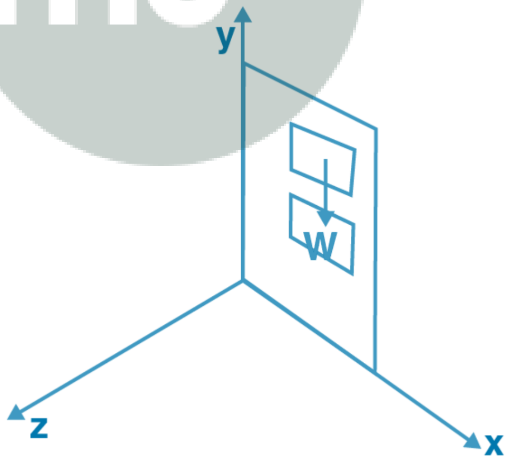
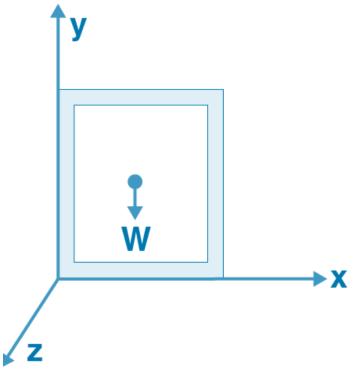


Answer: No, the weight does not cause any torque about the vertical hinge axis.



Reasoning:

- **Weight direction**: Vertically downward (-ĵ direction)
- **Hinge axis**: Vertical (k̂ direction)
- **Door lies in**: x-y plane

Torque calculation: $\tau = \mathbf{r} \times \mathbf{W}$

- **r**: Position vector from hinge to center of mass (in x-y plane)
- **W**: Weight vector (in -ĵ direction)
- **Result**: τ is in the x-y plane, perpendicular to the hinge axis

Conclusion: The weight produces torque about horizontal axes (causing the door to sag) but produces zero torque component about the vertical hinge axis. Therefore, weight alone cannot cause the door to rotate about its hinge.

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22. (n-1) equal point masses each of mass m are placed at the vertices of a regular n-polygon. The vacant vertex has a position vector a with respect to centre of the polygon. Find the position vector of centre of mass.

Answer: Setup:

- Regular n-polygon with (n-1) masses of value m each
- One vertex is vacant with position vector **a** from center
- Find center of mass of the system

Analysis: For a complete regular n-polygon with n masses: $\mathbf{R}\_cm = (m\cdot\mathbf{r}_1 + m\cdot\mathbf{r}_2 + \ldots + m\cdot\mathbf{r}_n)/(n\cdot m) = \mathbf{0}$ (by symmetry)

This means: $\mathbf{r}_1 + \mathbf{r}_2 + \ldots + \mathbf{r}_n = \mathbf{0}$

With vacant vertex: $\mathbf{r}_1 + \mathbf{r}_2 + \ldots + \mathbf{r}_{n-1} + \mathbf{a} = \mathbf{0}$

Therefore: $\mathbf{r}_1 + \mathbf{r}_2 + \ldots + \mathbf{r}_{n-1} = -\mathbf{a}$

Center of mass: $\mathbf{R}\_cm = (m(\mathbf{r}_1 + \mathbf{r}_2 + \ldots + \mathbf{r}_{n-1}))/(n-1)m = (-m\mathbf{a})/((n-1)m) = -\mathbf{a}/(n-1)$

Final Answer: $\mathbf{R}\_cm = -\mathbf{a}/(n-1)$

The negative sign indicates that the center of mass lies on the opposite side of the center from the vacant vertex.