

## EXERCISE 4.10

1. Evaluate:

(i)  $\text{Cot} (\sin^{-1} (3/4) + \sec^{-1} (4/3))$

(ii)  $\text{Sin} (\tan^{-1} x + \tan^{-1} 1/x)$  for  $x < 0$

(iii)  $\text{Sin} (\tan^{-1} x + \tan^{-1} 1/x)$  for  $x > 0$

(iv)  $\text{Cot} (\tan^{-1} a + \cot^{-1} a)$

(v)  $\text{Cos} (\sec^{-1} x + \text{cosec}^{-1} x)$ ,  $|x| \geq 1$

**Solution:**

(i) Given  $\text{Cot} (\sin^{-1} (3/4) + \sec^{-1} (4/3))$

$$= \cot \left( \sin^{-1} \frac{3}{4} + \cos^{-1} \frac{3}{4} \right)$$

$$\left( \because \sec^{-1} x = \cos^{-1} \frac{1}{x} \right)$$

We have

$$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$$

By substituting these values in given question, we get

$$= \cot \frac{\pi}{2}$$

$$= 0$$

(ii) Given  $\text{Sin} (\tan^{-1} x + \tan^{-1} 1/x)$  for  $x < 0$

$$= \sin \left( \tan^{-1} x + (\cot^{-1} x - \pi) \right) \left( \because \tan^{-1} \theta = \cot^{-1} \frac{1}{\theta} - \pi \quad \text{for } x < 0 \right)$$

$$= \sin\left(\frac{\pi}{2} - \pi\right) \left(\because \tan^{-1} \theta + \cot^{-1} \theta = \frac{\pi}{2}\right)$$

On simplifying, we get

$$= \sin\left(-\frac{\pi}{2}\right)$$

We know that  $\sin(-\theta) = -\sin \theta$

$$= -\sin \frac{\pi}{2} = -1$$

(iii) Given  $\sin(\tan^{-1} x + \tan^{-1} 1/x)$  for  $x > 0$

$$= \sin(\tan^{-1} x + \cot^{-1} x) \left(\because \tan^{-1} \theta = \cot^{-1} \frac{1}{\theta} \text{ for } x > 0\right)$$

Again we know that,

$$\tan^{-1} \theta + \cot^{-1} \theta = \frac{\pi}{2}$$

Now by substituting above identity in given question we get,

$$= \sin \frac{\pi}{2}$$

$$= 1$$

(iv) Given  $\cot(\tan^{-1} a + \cot^{-1} a)$

We know that,

$$\tan^{-1} \theta + \cot^{-1} \theta = \frac{\pi}{2}$$

Now by substituting above identity in given question we get,

$$= \cot\left(\frac{\pi}{2}\right)$$

$$= 0$$

(v) Given  $\cos(\sec^{-1} x + \operatorname{cosec}^{-1} x)$ ,  $|x| \geq 1$

We know that

$$\sec^{-1} \theta = \cos^{-1} \frac{1}{\theta}$$

Again we have

$$\operatorname{cosec}^{-1} \theta = \sin^{-1} \frac{1}{\theta}$$

By substituting these values in given question we get,

$$= \cos\left(\cos^{-1} \frac{1}{x} + \sin^{-1} \frac{1}{x}\right)$$

We know that from the identities,

$$\sin^{-1} \theta + \cos^{-1} \theta = \frac{\pi}{2}$$

Now by substituting we get,

$$= \cos \frac{\pi}{2}$$

$$= 0$$

2. If  $\cos^{-1} x + \cos^{-1} y = \pi/4$ , find the value of  $\sin^{-1} x + \sin^{-1} y$ .

**Solution:**

Given  $\cos^{-1} x + \cos^{-1} y = \pi/4$

We know that

$$\sin^{-1} \theta + \cos^{-1} \theta = \frac{\pi}{2}$$

Now substituting above identity in given question we get,

$$\Rightarrow \left( \frac{\pi}{2} - \sin^{-1} x \right) + \left( \frac{\pi}{2} - \sin^{-1} y \right) = \frac{\pi}{4}$$

Adding and simplifying we get,

$$\Rightarrow \pi - (\sin^{-1} x + \sin^{-1} y) = \frac{\pi}{4}$$

On rearranging,

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \pi - \frac{\pi}{4}$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{3\pi}{4}$$

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**3. If  $\sin^{-1} x + \sin^{-1} y = \pi/3$  and  $\cos^{-1} x - \cos^{-1} y = \pi/6$ , find the values of x and y.**

**Solution:**

Given  $\sin^{-1} x + \sin^{-1} y = \pi/3$  ..... Equation (i)

And  $\cos^{-1} x - \cos^{-1} y = \pi/6$  ..... Equation (ii)

Subtracting Equation (ii) from Equation (i), we get

$$\Rightarrow (\sin^{-1} x - \cos^{-1} x) + (\sin^{-1} y + \cos^{-1} y) = \frac{\pi}{3} - \frac{\pi}{6}$$

We know that,

$$\sin^{-1} \theta + \cos^{-1} \theta = \frac{\pi}{2}$$

By substituting above identity, we get

$$\Rightarrow (\sin^{-1} x - \cos^{-1} x) + \left(\frac{\pi}{2}\right) = \frac{\pi}{6}$$

Replacing  $\sin^{-1} x$  by  $\pi/2 - \cos^{-1} x$  and rearranging we get,

$$\Rightarrow \left(\frac{\pi}{2} - \cos^{-1} x\right) - \cos^{-1} x = -\frac{\pi}{3}$$

Now by adding,

$$\Rightarrow 2 \cos^{-1} x = \frac{5\pi}{6}$$

$$\Rightarrow \cos^{-1} x = \frac{5\pi}{12}$$

$$\Rightarrow x = \cos\left(\frac{5\pi}{12}\right)$$

$$\Rightarrow x = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$$

We know that  $\cos(A + B) = \cos A \cdot \cos B - \sin A \cdot \sin B$ , substituting this we get,

$$\Rightarrow x = \cos \frac{\pi}{4} \cdot \cos \frac{\pi}{6} - \sin \frac{\pi}{4} \cdot \sin \frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \cdot \frac{1}{2}$$



$$\Rightarrow x = \frac{\sqrt{3}-1}{2\sqrt{2}}$$

Now, putting the value of  $\cos^{-1} x$  in equation (ii)

$$\Rightarrow \frac{5\pi}{12} - \cos^{-1} y = \frac{\pi}{6}$$

$$\Rightarrow \cos^{-1} y = \frac{\pi}{4}$$

$$\Rightarrow y = \frac{1}{\sqrt{2}}$$

$$\Rightarrow x = \frac{\sqrt{3}-1}{2\sqrt{2}} \text{ And } y = \frac{1}{\sqrt{2}}$$

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4. If  $\cot(\cos^{-1} 3/5 + \sin^{-1} x) = 0$ , find the value of  $x$ .

**Solution:**

Given  $\cot(\cos^{-1} 3/5 + \sin^{-1} x) = 0$

On rearranging we get,

$$(\cos^{-1} 3/5 + \sin^{-1} x) = \cot^{-1}(0)$$

$$(\cos^{-1} 3/5 + \sin^{-1} x) = \pi/2$$

We know that  $\cos^{-1} x + \sin^{-1} x = \pi/2$

$$\text{Then } \sin^{-1} x = \pi/2 - \cos^{-1} x$$

Substituting the above in  $(\cos^{-1} 3/5 + \sin^{-1} x) = \pi/2$  we get,

$$(\cos^{-1} 3/5 + \pi/2 - \cos^{-1} x) = \pi/2$$

Now on rearranging we get,

$$(\cos^{-1} 3/5 - \cos^{-1} x) = \pi/2 - \pi/2$$

$$(\cos^{-1} 3/5 - \cos^{-1} x) = 0$$

$$\text{Therefore } \cos^{-1} 3/5 = \cos^{-1} x$$

On comparing the above equation we get,

$$x = 3/5$$

5. If  $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36$ , find  $x$ .

**Solution:**

$$\text{Given } (\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36$$

$$\text{We know that } \cos^{-1} x + \sin^{-1} x = \pi/2$$

$$\text{Then } \cos^{-1} x = \pi/2 - \sin^{-1} x$$

$$\text{Substituting this in } (\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36 \text{ we get}$$

$$(\sin^{-1} x)^2 + (\pi/2 - \sin^{-1} x)^2 = 17 \pi^2/36$$

$$\text{Let } y = \sin^{-1} x$$

$$y^2 + ((\pi/2) - y)^2 = 17 \pi^2/36$$

$$y^2 + \pi^2/4 - y^2 - 2y((\pi/2) - y) = 17 \pi^2/36$$

$$\pi^2/4 - \pi y + 2y^2 = 17 \pi^2/36$$

On rearranging and simplifying, we get

$$2y^2 - \pi y + 2/9 \pi^2 = 0$$

$$18y^2 - 9 \pi y + 2 \pi^2 = 0$$

$$18y^2 - 12 \pi y + 3 \pi y + 2 \pi^2 = 0$$

$$6y(3y - 2\pi) + \pi(3y - 2\pi) = 0$$

$$\text{Now, } (3y - 2\pi) = 0 \text{ and } (6y + \pi) = 0$$

$$\text{Therefore } y = 2\pi/3 \text{ and } y = -\pi/6$$

Now substituting  $y = -\pi/6$  in  $y = \sin^{-1} x$  we get

$$\sin^{-1} x = -\pi/6$$

$$x = \sin(-\pi/6)$$

$$x = -1/2$$

Now substituting  $y = 2\pi/3$  in  $y = \sin^{-1} x$  we get

$$x = \sin(2\pi/3)$$

$$x = \sqrt{3}/2$$

Now substituting  $x = \sqrt{3}/2$  in  $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36$  we get,

$$= \pi^2/36 + \pi^2/36$$

$$= \pi^2/18 \text{ which is not equal to } 17 \pi^2/36$$

So we have to neglect this root.

Now substituting  $x = -1/2$  in  $(\sin^{-1} x)^2 + (\cos^{-1} x)^2 = 17 \pi^2/36$  we get,

$$= \pi^2/36 + 4 \pi^2/9$$

$$= 17 \pi^2/36$$

$$\text{Hence } x = -1/2.$$