

EXERCISE 32.6

Calculate the mean and S.D. for the following data:

Expenditure (in ₹):	0-10	10-20	20-30	30-40	40-50
Frequency:	14	13	27	21	15

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

Expenditure	Mid Point(X_i)	F_i	$F_i X_i$	$(x_i - \bar{X})$	$(x_i - \bar{X})^2$	$(x_i - \bar{X})^2 f$
0-10	5	14	70	-21.1	445.21	6233.94
10-20	15	13	195	-11.1	123.21	1601.1
20-30	25	27	675	-1.1	1.21	34.67
30-40	35	21	735	8.9	79.21	1663.41
40-50	45	15	675	18.9	357.21	53.58
		$\sum f_i = 90$	$\sum f_i x_i = 2350$			$\sum (x_i - \bar{X})^2 f = 1797.32$

Now,

$$\text{Mean, } \bar{X} = \frac{\sum f_i x_i}{\sum f_i}$$

$$\bar{X} = \frac{2350}{90} = 26.11$$

$$\text{Var}(X) = \frac{14891.9}{90}$$

$$\text{Variance} = 165.47$$

$$\text{Standard Deviation, } \sigma = \sqrt{165.47} = 12.86$$

∴ The standard deviation is 12.86

1. Calculate the standard deviation for the following data:

Class:	0-30	30-60	60-90	90-120	120-150	150-180	180-210
Frequency:	9	17	43	82	81	44	24

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

Class	F_i	x_i	$u_i = \frac{x_i - \text{mean}}{30}$	$f_i u_i$	U_i^2	$f_i u_i^2$
0-30	9	15	-3	-27	9	81
30-60	17	45	-2	-34	4	68
60-90	43	75	-1	-43	1	43
90-120	82	105	0	0	0	0
120-150	81	135	1	81	1	81
150-180	44	165	2	88	4	176
180-210	24	195	3	72	9	216
		$\sum f_i = 300$		$\sum u_i f_i = 137$		$\sum u_i^2 f_i = 665$

Now,

$$N = 300, \sum u_i f_i = 137, \sum u_i^2 f_i = 665$$

$$\text{Mean, } \bar{X} = A + h \left(\frac{\sum u_i f_i}{N} \right)$$

$$\begin{aligned} \bar{X} &= 105 + 30 \left(\frac{137}{300} \right) \\ &= 118.7 \end{aligned}$$

$$\text{Var}(X) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\text{Var}(X) = \frac{900}{90000} [300 \times 665 - 18769]$$

$$= \frac{1}{100} [199500 - 18769]$$

$$\text{Variance} = 1807.31$$

$$\begin{aligned} \text{Standard Deviation, } \sigma &= \sqrt{1807.31} \\ &= 42.51 \end{aligned}$$

\therefore The standard deviation is 42.51

2. Calculate the A.M. and S.D. for the following distribution:

Class:	0-10	10-20	20-30	30-40	40-50	50-60	60-70	70-80
Frequency:	18	16	15	12	10	5	2	1

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

Class	F_i	x_i	$u_i = \frac{x_i - \text{mean}}{10}$	$f_i u_i$	$f_i u_i^2$
0-10	18	5	-3	-54	162
10-20	16	15	-2	-32	64
20-30	15	25	-1	-15	15
30-40	12	35	0	0	0
40-50	10	45	1	10	10
50-60	5	55	2	10	20
60-70	2	65	3	6	18
70-80	1	75	4	4	16
	$\sum f_i = 79$			$\sum u_i f_i = -71$	$\sum u_i^2 f_i = 305$

Now,

$$N = 79, \sum u_i f_i = -71, \sum u_i^2 f_i = 305$$

$$\text{Mean, } \bar{X} = A + h \left(\frac{\sum u_i f_i}{N} \right)$$

$$\bar{X} = 35 + 10 \left(\frac{-71}{79} \right)$$

$$= 26.01$$

$$\text{Var}(X) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\text{Var}(X) = 100 \left[\frac{305}{79} - \frac{5041}{6241} \right]$$

$$\text{Variance} = 305.20$$

$$\text{Standard Deviation, } \sigma = \sqrt{305.20}$$

$$= 17.47$$

\therefore The standard deviation is 17.47

3. A student obtained the mean and standard deviation of 100 observations as 40 and 5.1 respectively. It was later found that one observation was wrongly copied as 50, the correct figure is 40. Find the correct mean and S.D.

Solution:

Given: Uncorrected mean is 40 and corrected SD is 5.1 and $N = 100$

Here, $\bar{x} = 40$, $\sigma = 5.1$ and $n = 100$

Then, $\sum x_o = 4000$

The corrected sum of observation, $\sum x_n = 4000 - 50 + 40$

$$\sum x_n = 3990$$

So,

$$\begin{aligned}\bar{x}_n &= \frac{\sum x_n}{n} \\ &= 3990/100 \\ &= 39.90\end{aligned}$$

Now,

Given Incorrect SD = 5.1

$\sigma = 5.1$

$$\sum (x_i - \bar{x}_o)^2 = 2601$$

$$\sum (x_i - \bar{x}_o)^2 = 2601 - 100 + 0.01 = 2501.1$$

Corrected SD, $\sigma_n = \sqrt{\frac{\sum (x_i - \bar{x}_o)^2}{n}}$

$$\begin{aligned}\sigma_n &= \sqrt{\frac{2501.01}{100}} \\ &= 5\end{aligned}$$

\therefore Correct mean is 39.9 and correct SD is 5

4. Calculate the mean, median and standard deviation of the following distribution

Class-interval	31-35	36-40	41-45	46-50	51-55	56-60	61-65	66-70
Frequency:	2	3	8	12	16	5	2	3

Solution:

By using the formula to find standard deviation:

$$SD = \sqrt{\text{Var}(X)}$$

Class	F_i	x_i	$u_i = \frac{x_i - \text{mean}}{4}$	$f_i u_i$	$f_i u_i^2$
31-35	2	33	-4	-8	32
36-40	3	38	-3	-9	27
41-45	8	43	-2	-16	32
46-50	12	48	-1	-12	12
51-55	16	53	0	0	0
56-60	5	58	1	5	5
61-65	2	63	2	4	8
66-70	2	68	3	6	18
	$\sum f_i = 50$			$\sum u_i f_i = -30$	$\sum u_i^2 f_i = 134$

Now,

$$N = 50, \sum u_i f_i = -30, \sum u_i^2 f_i = 134$$

$$\text{Mean, } \bar{X} = A + h \left(\frac{\sum u_i f_i}{N} \right)$$

$$\bar{X} = 53 + 5 \left(-\frac{30}{50} \right)$$

$$= 50$$

$$\text{Var}(X) = h^2 \left[\frac{1}{N} \sum_{i=1}^n f_i u_i^2 - \left(\frac{1}{N} \sum_{i=1}^n u_i f_i \right)^2 \right]$$

$$\text{Var}(X) = 25 \left[\frac{134}{50} - \frac{9}{25} \right]$$

$$\text{Variance} = 58$$

$$\text{Standard Deviation, } \sigma = \sqrt{58}$$

$$= 7.62$$

\therefore The standard deviation is 7.62