Chapter 5

EXERCISE 4.4 **Chapter 5: Arithmetic Progressions**

EXERCISE 5.1:

1. In which of the following situations, does the list of numbers involved make an arithmeticprogression, and why?
   (i) The taxi fare after each km when the fare is Rs 15 for the first km and Rs 8 for each additional km.
   (ii) The amount of air present in a cylinder when a vacuum pump removes $\frac{1}{4}$ of the air remaining in the cylinder at a time.
   (iii) The cost of digging a well after every metre of digging, when it costs Rs 150 forthe first metre and rises by Rs 50 for each subsequent metre.
   (iv) The amount of money in the account every year, when Rs 10000 is deposited atcompound interest at 8 % per annum

   Solution: (i) Fare for 1st km = Rs. 15

Fare for 2ndkm = Rs. 15 + 8 = Rs 23

Fare for 3rdkm = Rs. 23 + 8 = 31

In this, each subsequent term is obtained by adding a fixed number (8) to the previous term.

Hence, it is an AP

(ii) Let us take initial quantity of air = 1

Hence, quantity removed in first step = ¼

Remaining quantity after 1st step

$$= 1 - \frac{1}{4} = \frac{3}{4}$$

Quantity removed after 2nd step

$$= \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$$

Remaining quantity after 2nd step would be:

$$= \frac{3}{4} - \frac{3}{16} = \frac{12 - 3}{16} = \frac{9}{16}$$

Here a fixed number is not added to each subsequent term.

Hence, it is not an AP

(iii) Cost of digging of 1st meter = 150

Cost of digging of 2nd meter = 150 + 50

= 200

Cost of digging of 3rd meter = 200 + 50

= 250

Here, each subsequent term is obtained by adding a fixed number (50) to the previous term.

Hence, it is an AP.

(iv) Amount in the beginning = Rs. 10000

Interest at the end of 1st year at rate of 8%

= 10000 x 8%

= 800

Hence, amount at the end of 1^{st}year

= 10000 + 800

= 10800

Interest at the end of 2^{nd} year at rate of 8%

= 10800 x 8%

= 864

Thus, amount at the end of 2^{nd}year

= 10800 + 864 = 11664

Since, each subsequent term is not obtained by adding a unique number to the previous term; hence, it is not an AP.

2. Write first four terms of the AP, when the first term a and the common difference d aregiven as follows:

(i) $a = 10, \quad d = 10$

(ii) $a = -2, \quad d = 0$

(iii) $a = 4, \quad d = -3$

(iv) $a = -1, \quad d = \frac{1}{2}$

(v) $a = -1.25, \quad d = -0.25$

Solution:

(i) Here, first term $a_1 = 10$ and common difference, d = 10

Hence,

2^{nd} term $a_2 = a_1 + d$

= 10 + 10

= 20

3^{rd} term $a_3 = a_1 + 2d$

= 10 + 2 x 10

= 30

4^{th} term $a_4 = a_1 + 3d$

= 10 + 30

= 40

Therefore,

first four terms of the AP are:

10, 20, 30, 40, ……

(ii) Here,

First term a = 1 and Common difference = 0

Therefore, first four terms of the given AP are:

$a_1 = - 2$, $a_2 = - 2$, $a_3 = - 2$ and $a_4 = - 2$

(iii) Here, first term $a_1 = 4$ and common difference d = - 3

We know that $a_n = a + (n – 1)d$, where n = number of terms

Thus, second term $a_2 = a + (2 - 1)d$

$a_2 = 4 + (2-1)\times(-3)$

$= 4 - 3 = 1$

3^{rd} term $a_3 = a + (3 - 1)d$

$= 4 + (3-1) \times (-3)$

$= 4 - 6$

$= -2$

4^{th} term $a_4 = a + (4-1)d$

$= 4 + (4 - 1) \times(-3)$

$= 4 - 9 = -5$

Therefore,

First four terms of given AP are:

4, 1, - 2, - 5

(iv) We have,

1^{st} term = - 1 and d = ½

Hence,

2^{nd} term $a_2 = a_1 + d$
$= -1 + ½$
$= - ½$
3^{rd}term $a_3 = a_1 + 2d$
$= -1 + 2 * ½$
$= 0$
4^{th} term $a_4 = a_1 + 3d$
$= -1 + 3 * ½$
$= ½$
Therefore,
The four terms of A.P. are -1, - ½, 0, ½

(v) We have

1st term = - 1.25 and d = - 0.25

2nd term $a_2 = a + d$

$= -1.25 - 0.25$

$= -1.5$

3rd term $a_3 = a + 2d$

$= -1.25 + 2 \times (-0.25)$

$= -1.25 - 0.5$

$= -1.75$

4th term $a_4 = a + 3d$

$= 1.25 + 3 \times (-0.25)$

$= -2.25$

Therefore, first four terms of the A.P. are: 1.25, -1.5, -1.75 and – 2.25

3. For the following APs, write the first term and the common difference:
   (i) $3, 1, -1, -3, \ldots.$
   (ii) $-5, -1, 3, 7, \ldots.$
   (iii) $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots.$
   (iv) $0.6, 1.7, 2.8, 3.9, \ldots\ldots$

Solution:

(i) Here, first term a = 3

Now,

The Common difference of the A.P. can be calculated as:

$a_4 - a_3$

$= -3 - (-1)$

$= -3 + 1$

$= -2$

$a_3 - a_2$

$= -1 - 1$

$= -2$

$a_2 - a_1$

$= 1 - 3$

$= -2$

Now, here$a_{k+1} - a_k = -2$ for all values of k

Hence, first term = 3 and common difference = - 2

(ii) $a_4 - a_3$

$= 7 - 3$

$= 4$

$a_3 - a_2$

$= 3 - (-1)$

$= 3 + 1$

$= 4$

$a_2 - a_1$

$= -1 - (-5)$

$= -1 + 5$

$= 5$

Now, here, $a_{k+1} - a_k = -2$ for all values of k

Therefore, first term = - 5 and common difference = 4

(iii) From the question,

$a_4 - a_3$

$= \frac{13}{3} - \frac{9}{3}$

$= \frac{4}{3}$

$a_3 - a_2$

$= \frac{9}{3} - \frac{5}{3}$

$$= \frac{4}{3}$$

$a_2 - a_1$

$= \frac{4}{3}$

Now, $a_{k+1} - a_k = -2$ for all values of k

Therefore,

First term = 1/3 and common difference = 4/3

(iv) $a_4 - a_3$

$= 3.9 - 2.8$

$= 1.1$

$a_3 - a_2$

$= 2.8 - 1.7$

$= 1.1$

$a_2 - a_1$

$= 1.7 - 0.6$

$= 1.1$

Now, here, $a_{k+1} - a_k = -2$ for all values of k

Therefore,

First term = 0.6 and common difference = 1.1

4. Which of the following are APs ? If they form an AP, find the common difference d andwrite three more terms.

(i) $2, 4, 8, 16$

(ii) $2, \frac{5}{2}, 3, \frac{7}{2}, ...$

(iii) $-1.2, -3.2, -5.2, -7.2, ...$

(iv) $-10, -6, -2, 2, ...$

(v) $3, 3+\sqrt{2}, 3+2\sqrt{2}, 3+3\sqrt{2}, ...$

(vi) $0.2, 0.22, 0.222, 0.2222, ...$

(vii) $0, -4, -8, -12, ...$

(viii) $-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, ...$

(ix) 1, 3, 9, 27,...

(x) $a, 2a, 3a, 4a, ...$

(xi) $a, a^2, a^3, a^4, ...$

(xii) $\sqrt{2}, \sqrt{8}, \sqrt{18}, \sqrt{32}, ...$

(xiii) $\sqrt{3}, \sqrt{6}, \sqrt{9}, \sqrt{12}, ...$

(xiv) $1^2, 3^2, 5^2, 7^2, ...$

Solution: (i) if $a_{k+1} - a_k$ is same for different values of k then the series is an AP.

We have, $a_1 = 2$, $a_2 = 4$, $a_3 = 8$ and $a_4 = 16$

$a_4 - a_3 = 16 - 8 = 8$

$a_3 - a_2$

$= 8 - 4$

$= 5$

$a_2 - a_1$

$= 4 - 2$

$= 2$

Here, $a_{k+1} - a_k$ is not same for all values of k.

Hence, the given series is not an AP.

(ii) As per the question:
a1 = 2,
a2 = 5/2,
a3 = 3
And

$a4 = 7/2$

$a_4 - a_3 = \frac{7}{2} - 3 = ½$

$a_3 - a_2 = 3 - \frac{5}{2} = ½$

$a_2 - a_1 = \frac{5}{2} - 2 = ½$

Now, we can observe that $a_{k+1} - a_k$ is same for all values of k.

Hence, it is an AP.

And, the common difference = ½

Next three terms of this series are:

$a_5 = a + 4d$

$= 2 + 4 * ½$

$= 4$

$a_6 = a + 5d$

$= 2 + 5 * ½$

$= \frac{9}{2}$

$a_7 = a + 6d$

$= 2 + 6 * ½$

$= 5$

Hence,

The next three terms of the AP are: 4, 9/2 and 5

(iii) $a_4 - a_3 = - 7.2 + 5.2 = - 2$

$a_3 - a_2 = - 5.2 + 3.2 = - 2$

$a_2 - a_1 = - 3.2 + 1.2 = - 2$

In this, $a_{k+1} - a_k$ is same for all values of k.

Hence, the given series is an AP.

Common difference = - 2

Next three terms of the series are:

$a_5 = a + 4d$

$= -1.2 + 4 \times (-2)$

$= -1.2 - 8 = -9.2$

$a_6 = a + 5d$

$= -1.2 + 5 \times (-2)$

$= -1.2 - 10 = -11.2$

$a_7 = a + 6d$

$= -1.2 + 6 \times (-2)$

$= -1.2 - 12 = -13.2$

Next three terms of AP are: - 9.2, - 11.2 and – 13.2

(iv)$a_4 – a_3 = 2 + 2 = 4$

$a_3 – a_2 = - 2 + 6 = 4$

$a_2 – a_1 = - 6 + 10 = 5$

Here, $a_{k+1} – a_k$ is same for all values of k

Hence, the given series is an AP.

Common difference = 4

Next three terms of the AP are:

$a_5 = a + 4d$

$= -10 + 4 \times 4$

$= -10 + 16 = 6$

$a_6 = a + 5d$

$= -10 + 5 \times 4$

$= -10 + 20 = 10$

$a_7 = a + 6d$

$= -10 + 6 \times 4$

$= -10 + 24 = 14$

Next three terms of AP are: 6, 10 and 14.

(v) $a_4 - a_3 = 3 + 3\sqrt{2} - 3 - 2\sqrt{2} = \sqrt{2}$

$a_3 - a_2 = 3 + 2\sqrt{2} - 3 - \sqrt{2} = \sqrt{2}$

$a_2 - a_1 = 3 + \sqrt{2} - 3 = \sqrt{2}$

Here, $a_{k+1} - a_k$ is same for all values of k

Hence, the given series is an AP.

Common difference = $\sqrt{2}$

Next three terms of the AP are

$a_6 = a + 5d = 3 + 5\sqrt{2}$

$a_7 = a + 6d = 3 + 6\sqrt{2}$

Next three terms of AP are: $3 + 4\sqrt{2}$, $3 + 5\sqrt{2}$ and $3 + 6\sqrt{2}$

$a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

Here, $a_{k+1} - a_k$ is not same for all values of k

Hence, the given series is not an AP.

(vi) $a_4 - a_3 = 0.2222 - 0.222 = 0.0002$

$a_3 - a_2 = 0.222 - 0.22 = 0.002$

$a_2 - a_1 = 0.22 - 0.2 = 0.02$

Here, $a_{k+1} - a_k$ is not same for all values of k

Hence, the given series is not an AP.

(vii) Here; $a_4 - a_3 = -12 + 8 = -4$

$a_3 - a_2 = -8 + 4 = -4$

$a_2 - a_1 = -4 - 0 = -4$

Since $a_{k+1} - a_k$ is same for all values of k.

Hence, this is an AP.

The next three terms can be calculated as follows:

$a_5 = a + 4d = 0 + 4(-4) = -16$

$a_6 = a + 5d = 0 + 5(-4) = -20$

$a_7 = a + 6d = 0 + 6(-4) = -24$

Thus, next three terms are; - 16, - 20 and – 24

(viii) Here, it is clear that d = 0

Since $a_{k+1} - a_k$ is same for all values of k.

Hence, it is an AP.

The next three terms will be same, i.e. – ½

(ix) $a_4 - a_3 = 27 - 9 = 18$

$a_3 - a_2 = 9 - 3 = 6$

$a_2 - a_1 = 3 - 1 = 2$

Since $a_{k+1} - a_k$ is not same for all values of k.

Hence, it is not an AP.

(x) $a_4 - a_3 = 4a - 3a = a$

$a_3 - a_2 = 3a - 2a = a$

$a_2 - a_1 = 2a - a = a$

Since $a_{k+1} – a_k$ is same for all values of k.

Hence, it is an AP.

Next three terms are:

$a_5 = a + 4d = a + 4a = 5a$

$a_6 = a + 5d = a + 5a = 6a$

$a_7 = a + 6d = a + 6a = 7a$

Next three terms are; 5a, 6a and 7a.

(xi) Here, exponent is increasing in each subsequent term.

Since $a_{k+1} – a_k$ is not same for all values of k.

Hence, it is not an AP.

(xii) Different terms of this AP can also be written as follows:

$\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}, 4\sqrt{2}, ...$

$a4 – a3 = 4\sqrt{2} - 3\sqrt{2} = \sqrt{2}$

$a3 – a2 = 3\sqrt{2} - 2\sqrt{2} = \sqrt{2}$

$a2 – a1 = 2\sqrt{2} - \sqrt{2} = \sqrt{2}$

Since ak+1 – ak is same for all values of k.

Hence, it is an AP.

Next three terms can be calculated as follows:

$a5 = a + 4d = \sqrt{2} + 4\sqrt{2} = 5\sqrt{2}$

$a6 = a + 5d = \sqrt{2} + 5\sqrt{2} = 6\sqrt{2}$

$a7 = a + 6d = \sqrt{2} + 6\sqrt{2} = 7\sqrt{2}$

Next three terms are; $5\sqrt{2}$, $6\sqrt{2}$ and $7\sqrt{2}$

(xiii) $a_4 – a_3 = \sqrt{12} - \sqrt{9} = 2\sqrt{3} – 3$

$a_3 – a_2 = \sqrt{9} - \sqrt{6} = 3 - \sqrt{6}$

$a_2 - a_1 = \sqrt{6} - \sqrt{3}$

Since $a_{k+1} - a_k$ is not same for all values of k.

Hence, it is not an AP

(xiv) The given terms can be written as follows:

1, 9, 25, 49, …

Here, $a_4 - a_3 = 49 - 25 = 24$

$a_3 - a_2 = 25 - 9 = 16$

$a_2 - a_1 = 9 - 1 = 8$

Since $a_{k+1} - a_k$ is not same for all values of k.

Hence, it is not an AP