EXERCISE 5.3

Solve each of the following Cryptarithms:

1.

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  3  7
+ A  B
------
  9  A
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Solution:

Firstly let us solve for unit's place,

7 + B = A

And for ten's place,

3 + A = 9

Which means that A = 6 and B = -1 which is not possible.

So, there should be one carry in ten's place which means 7 + B >9

Now solving for ten's place with one carry,

3 +A +1 = 9

A = 9-1-3 = 5

For unit's place subtracting 10 as one carry is given to ten's place,

7 + B - 10= 5

B = 5+10-7 = 8

∴A = 5 and B = 8

2.

```
  A  B
+ 3  7
------
  9  A
```

Solution:

Firstly let us solve for unit's place,

B + 7 = A

And for ten's place,

A + 3 = 9

Which means that A = 6 and B = -1 which is not possible.

So, there should be one carry in ten's place, which means B + 7 >9

Now solving for ten's place with one carry,

A +3 +1 = 9

A = 9-4 = 5

For unit's place subtracting 10 as one carry is given to ten's place,

B + 7 - 10= 5

B = 5+10-7 = 8

∴ A = 5 and B = 8

3.

$$\begin{array}{r} A\ \ 1 \\ +1\ \ B \\ \hline B\ \ 0 \end{array}$$

Solution:

Firstly let us solve for unit's place,

1 + B = 0

Which means that B = -1 which is not possible.

So, there should be one carry in ten's place,

A + 1 +1 = B ---- (1)

For unit's place, we need to subtract 10 as one carry is given in ten's place,

1 + B – 10 = 0

B = 10-1 = 9

Substituting B = 9 in (1),

A + 1 + 1 = 9

A = 9-1-1 = 7

∴ A = 7 and B = 9

4.

$$\begin{array}{r} 2\ \ A\ \ B \\ +A\ \ B\ \ 1 \\ \hline B\ \ 1\ \ 8 \end{array}$$

Solution:

Firstly let us solve for unit's place,

B + 1 = 8

B = 7

Now let us solve for ten's place,

A + B = 1

A + 7 = 1

A = -6 which is not possible.

Hence, A + B > 9

We know that now there should be one carry in hundred's place and so we need to subtract 10 from ten's place,

i.e., A + B – 10 = 1

A + 7 = 11

A = 11-7 = 4

Now to check whether our values of A and B are correct, we should solve for hundred's place.

2 + A + 1 = B
2 + 4 + 1 = 7
7 = 7
i.e., RHS = LHS
∴ A = 4 and B = 7

5.

$$\begin{array}{ccc} & 1 & 2 & A \\ + & 6 & A & B \\ \hline & A & 0 & 9 \end{array}$$

Solution:
Firstly let us solve for unit's place,
A + B = 9 -----(1)
With this condition we know that sum of 2 digits can be greater than 18.
So, there is no need to carry one from ten's place.
Now let us solve for ten's place,
2 + A = 0
Which means A = -2 which is never possible
Hence, 2 + A > 9
Now, there should be one carry in hundred's place and hence we need to subtract 10 from ten's place,
i.e., 2 + A – 10 = 0
A = 10-2 = 8
Now, substituting A=8 in 1,
A + B = 9
8 + B = 9
B = 9 – 8
B = 1
∴ A = 8 and B = 1

6.

$$\begin{array}{ccc} & A & B & 7 \\ + & 7 & A & B \\ \hline & 9 & 8 & A \end{array}$$

Solution:
Firstly let us solve for unit's place,
We have two conditions here, 7 + B ≤ 9 and 7 + B > 9
For 7 + B ≤ 9
7 + B = A
A – B = 7 ---- (1)

Now let us solve for ten's place,
B + A = 8 -----(2)
Solving 1 and 2 simultaneously,
2A = 15 which means A = 7.5 which is not possible
So, our condition $7 + B \leq 9$ is wrong.
∴ $7 + B > 9$ is correct condition
Hence, there should be one carry in ten's place and subtracting 10 from unit's place,
7 + B – 10 = A
B – A = 3 ---- (3)
For ten's place,
B + A + 1 = 8
B + A = 8-1
B + A = 7 -----(4)
Solving (3) and (4) simultaneously,
2B = 10
B = 10/2 = 5
Substituting the value of B in equation 4
B + A = 7
5 + A = 7
A = 7-5 = 2
∴ B = 5 and A = 2

7. Show that the Cryptarithm $4 \times \overline{AB} = \overline{CAB}$ does not have any solution.
Solution:
If B is multiplied by 4 then only 0 satisfies the above condition.
So, for unit place to satisfy the above condition, we should have B = 0.
Similarly for ten's place, only 0 satisfies the above condition.
But, AB cannot be 00 as 00 is not a two digit number.
So, A and B cannot be equal to 0
∴ there is no solution satisfying the condition $4 \times \overline{AB} = \overline{CAB}$.