

EXERCISE 6.5

Solve each of the following system of homogeneous linear equations:

$$1. \ x + y - 2z = 0$$

$$2x + y - 3z = 0$$

$$5x + 4y - 9z = 0$$

Solution:

$$\text{Given } x + y - 2z = 0$$

$$2x + y - 3z = 0$$

$$5x + 4y - 9z = 0$$

Any system of equation can be written in matrix form as $AX = B$

Now finding the Determinant of these set of equations,

$$D = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$$

$$|A| = 1 \begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1 \begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} - 2 \begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

$$= 1(1 \times (-9) - 4 \times (-3)) - 1(2 \times (-9) - 5 \times (-3)) - 2(4 \times 2 - 5 \times 1)$$

$$= 1(-9 + 12) - 1(-18 + 15) - 2(8 - 5)$$

$$= 1 \times 3 - 1 \times (-3) - 2 \times 3$$

$$= 3 + 3 - 6$$

Since $D = 0$, so the system of equation has infinite solution.

Now let $z = k$

$$\Rightarrow x + y = 2k$$

$$\text{And } 2x + y = 3k$$

Now using the Cramer's rule

$$x = \frac{D_1}{D}$$

$$x = \frac{\begin{vmatrix} 2k & 1 \\ 3k & 1 \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}}$$

$$x = \frac{-k}{-1}$$

$$x = k$$

Similarly,

$$y = \frac{D_2}{D}$$

$$y = \frac{\begin{vmatrix} 1 & 2k \\ 2 & 3k \end{vmatrix}}{\begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix}}$$

$$y = \frac{-k}{-1}$$

$$y = k$$

Hence, $x = y = z = k$.

2. $2x + 3y + 4z = 0$

$x + y + z = 0$

$2x + 5y - 2z = 0$

Solution:

Given

$$2x + 3y + 4z = 0$$

$$x + y + z = 0$$

$$2x + 5y - 2z = 0$$

Any system of equation can be written in matrix form as $AX = B$

Now finding the Determinant of these set of equations,

$$D = \begin{vmatrix} 2 & 3 & 4 \\ 1 & 1 & 1 \\ 2 & 5 & -2 \end{vmatrix}$$



$$\begin{aligned} |A| &= 2 \begin{vmatrix} 1 & 1 \\ 5 & -2 \end{vmatrix} - 3 \begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} + 4 \begin{vmatrix} 1 & 1 \\ 2 & 5 \end{vmatrix} \\ &= 2(1 \times (-2) - 1 \times 5) - 3(1 \times (-2) - 2 \times 1) + 4(1 \times 5 - 2 \times 1) \\ &= 2(-2 - 5) - 3(-2 - 2) + 4(5 - 2) \\ &= 1 \times (-7) - 3 \times (-4) + 4 \times 3 \\ &= -7 + 12 + 12 \\ &= 17 \end{aligned}$$

Since $D \neq 0$, so the system of equation has infinite solution.

Therefore the system of equation has only solution as $x = y = z = 0$.



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