EXERCISE 1.2

1. Write whether every positive integer can be of the form 4q + 2, where q is an integer. Justify your answer.

Solution:

No, every positive integer cannot be of the form 4q + 2, where q is an integer.

Justification:

All the numbers of the form 4q + 2, where 'q' is an integer, are even numbers which are not divisible by '4'.

For example,

When q=1,

4q+2 = 4(1) + 2= 6.

When q=2,

4q+2 = 4(2) + 2= 10

When q=0,

4q+2 = 4(0) + 2= 2 and so on.

So, any number which is of the form 4q+2 will give only even numbers which are not multiples of 4.

Hence, every positive integer **cannot** be written in the form 4q+2

2. "The product of two consecutive positive integers is divisible by 2". Is this statement true or false? Give reasons.

Solution:

Yes, the statement "the product of two consecutive positive integers is divisible by 2" is true.

Justification:

Let the two consecutive positive integers = a, a + 1

According to Euclid's division lemma,

We have,

$a = bq + r$, where $0 \le r < b$

For b = 2, we have $a = 2q + r$, where $0 \le r < 2$ ... (i)

Substituting r = 0 in equation (i),

We get,

a = 2q, is divisible by 2.

a + 1 = 2q + 1, is not divisible by 2.

Substituting r = 1 in equation (i),

We get,

a = 2q + 1, is not divisible by 2.

a + 1 = 2q + 1+1 = 2q + 2, is divisible by 2.

Thus, we can conclude that, for $0 \le r < 2$, one out of every two consecutive integers is divisible by 2. So, the product of the two consecutive positive numbers will also be even.

Hence, the statement "product of two consecutive positive integers is divisible by 2" is true.

3. "The product of three consecutive positive integers is divisible by 6". Is this statement true or false? Justify your answer.

Solution:

Yes, the statement "the product of three consecutive positive integers is divisible by 6" is true.

Justification:

Consider the 3 consecutive numbers 2, 3, 4

$(2 \times 3 \times 4)/6 = 24/6 = 4$

Now, consider another 3 consecutive numbers 4, 5, 6

$(4 \times 5 \times 6)/6 = 120/6 = 20$

Now, consider another 3 consecutive numbers 7, 8, 9

$(7 \times 8 \times 9)/6 = 504/6 = 84$

Hence, the statement "product of three consecutive positive integers is divisible by 6" is true.

4. Write whether the square of any positive integer can be of the form 3m + 2, where m is a natural number. Justify your answer.

Solution:

No, the square of any positive integer cannot be written in the form 3m + 2 where m is a natural number

Justification:

According to Euclid's division lemma,

A positive integer 'a' can be written in the form of bq + r

a = bq + r, where b, q and r are any integers,

For b = 3

a = 3(q) + r, where, r can be an integers,

For r = 0, 1, 2, 3..........

3q + 0, 3q + 1, 3q + 2, 3q + 3........are positive integers,

$(3q)^2 = 9q^2 = 3(3q^2) = 3m$ (where $3q^2 = m$)

$(3q+1)^2 = (3q+1)^2 = 9q^2+1+6q = 3(3q^2+2q) +1 = 3m + 1$ (Where, $m = 3q^2+2q$)

$(3q+2)^2 = (3q+2)^2 = 9q^2+4+12q = 3(3q^2+4q) +4 = 3m + 4$ (Where, $m = 3q^2+2q$)

$(3q+3)^2 = (3q+3)^2 = 9q^2+9+18q = 3(3q^2+6q) +9 = 3m + 9$ (Where, $m = 3q^2+2q$)

Hence, there is no positive integer whose square can be written in the form 3m + 2 where m is a natural number.

5. A positive integer is of the form 3q + 1, q being a natural number. Can you write its square in any form other than 3m + 1, i.e., 3m or 3m + 2 for some integer m? Justify your answer.

Solution:

No.

Justification:

Consider the positive integer 3q + 1, where q is a natural number.

$(3q + 1)2 = 9q^2 + 6q + 1$

$= 3(3q^2 + 2q) + 1$

= 3m + 1, (where m is an integer which is equal to 3q2 + 2q.

Thus $(3q + 1)^2$ cannot be expressed in any other form apart from 3m + 1.