

## THE SPECIAL THEORY OF RELATIVITY

### CHAPTER - 47

1.  $S = 1000 \text{ km} = 10^6 \text{ m}$

The process requires minimum possible time if the velocity is maximum.

We know that maximum velocity can be that of light i.e.  $= 3 \times 10^8 \text{ m/s}$ .

$$\text{So, time} = \frac{\text{Distance}}{\text{Speed}} = \frac{10^6}{3 \times 10^8} = \frac{1}{300} \text{ s.}$$

2.  $l = 50 \text{ cm}$ ,  $b = 25 \text{ cm}$ ,  $h = 10 \text{ cm}$ ,  $v = 0.6 c$

a) The observer in the train notices the same value of  $l$ ,  $b$ ,  $h$  because relativity are in due to difference in frames.

b) In 2 different frames, the component of length parallel to the velocity undergoes contraction but the perpendicular components remain the same. So length which is parallel to the x-axis changes and breadth and height remain the same.

$$e' = e \sqrt{1 - \frac{v^2}{c^2}} = 50 \sqrt{1 - \frac{(0.6)^2 C^2}{C^2}}$$

$$= 50 \sqrt{1 - 0.36} = 50 \times 0.8 = 40 \text{ cm.}$$

The lengths observed are  $40 \text{ cm} \times 25 \text{ cm} \times 10 \text{ cm}$ .

3.  $L = 1 \text{ m}$

a)  $v = 3 \times 10^5 \text{ m/s}$

$$L' = 1 \sqrt{1 - \frac{9 \times 10^{10}}{9 \times 10^{16}}} = \sqrt{1 - 10^{-6}} = 0.9999995 \text{ m}$$

b)  $v = 3 \times 10^6 \text{ m/s}$

$$L' = 1 \sqrt{1 - \frac{9 \times 10^{12}}{9 \times 10^{16}}} = \sqrt{1 - 10^{-4}} = 0.99995 \text{ m.}$$

c)  $v = 3 \times 10^7 \text{ m/s}$

$$L' = 1 \sqrt{1 - \frac{9 \times 10^{14}}{9 \times 10^{16}}} = \sqrt{1 - 10^{-2}} = 0.9949 = 0.995 \text{ m.}$$

4.  $v = 0.6 \text{ cm/sec}$  ;  $t = 1 \text{ sec}$

a) length observed by the observer  $= vt \Rightarrow 0.6 \times 3 \times 10^8 \Rightarrow 1.8 \times 10^8 \text{ m}$

b)  $l = l_0 \sqrt{1 - v^2/c^2} \Rightarrow 1.8 \times 10^8 = l_0 \sqrt{1 - \frac{(0.6)^2 C^2}{C^2}}$

$$l_0 = \frac{1.8 \times 10^8}{0.8} = 2.25 \times 10^8 \text{ m/s.}$$

5. The rectangular field appears to be a square when the length becomes equal to the breadth i.e.  $50 \text{ m}$ .

i.e.  $L' = 50$  ;  $L = 100$  ;  $v = ?$

$C = 3 \times 10^8 \text{ m/s}$

We know,  $L' = L \sqrt{1 - v^2/c^2}$

$$\Rightarrow 50 = 100 \sqrt{1 - v^2/c^2} \Rightarrow v = \sqrt{3/2} C = 0.866 C.$$

6.  $L_0 = 1000 \text{ km} = 10^6 \text{ m}$

$v = 360 \text{ km/h} = (360 \times 5) / 18 = 100 \text{ m/sec.}$

a)  $h' = h_0 \sqrt{1 - v^2/c^2} = 10^6 \sqrt{1 - \left(\frac{100}{3 \times 10^8}\right)^2} = 10^6 \sqrt{1 - \frac{10^4}{9 \times 10^6}} = 10^9.$

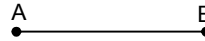
Solving change in length =  $56 \text{ nm}$ .

b)  $\Delta t = \Delta L/v = 56 \text{ nm} / 100 \text{ m} = 0.56 \text{ ns.}$

7.  $v = 180 \text{ km/hr} = 50 \text{ m/s}$

$t = 10 \text{ hours}$

let the rest dist. be  $L$ .



$$L' = L\sqrt{1 - v^2/c^2} \Rightarrow L' = 10 \times 180 = 1800 \text{ k.m.}$$

$$1800 = L\sqrt{1 - \frac{180^2}{(3 \times 10^5)^2}}$$

or,  $1800 \times 1800 = L(1 - 36 \times 10^{-14})$

or,  $L = \frac{3.24 \times 10^6}{1 - 36 \times 10^{-14}} = 1800 + 25 \times 10^{-12}$

or 25 nm more than 1800 km.

b) Time taken in road frame by Car to cover the dist =  $\frac{1.8 \times 10^6 + 25 \times 10^{-9}}{50}$

=  $0.36 \times 10^5 + 5 \times 10^{-8} = 10 \text{ hours} + 0.5 \text{ ns.}$

8. a)  $u = 5c/13$

$$\Delta t = \frac{t}{\sqrt{1 - v^2/c^2}} = \frac{1y}{\sqrt{1 - \frac{25c^2}{169c^2}}} = \frac{y \times 13}{12} = \frac{13}{12}y.$$

The time interval between the consecutive birthday celebration is 13/12 y.

b) The friend on the earth also calculates the same speed.

9. The birth timings recorded by the station clocks is proper time interval because it is the ground frame. That of the train is improper as it records the time at two different places. The proper time interval  $\Delta T$  is less than improper.

i.e.  $\Delta T' = v \Delta T$

Hence for – (a) up train → Delhi baby is elder (b) down train → Howrah baby is elder.

10. The clocks of a moving frame are out of synchronization. The clock at the rear end leads the one at front by  $L_0 v/C^2$  where  $L_0$  is the rest separation between the clocks, and  $v$  is speed of the moving frame. Thus, the baby adjacent to the guard cell is elder.

11.  $v = 0.9999 C$  ;  $\Delta t = \text{One day in earth}$  ;  $\Delta t' = \text{One day in heaven}$

$$v = \frac{1}{\sqrt{1 - v^2/c^2}} = \frac{1}{\sqrt{1 - \frac{(0.9999)^2 C^2}{C^2}}} = \frac{1}{0.014141782} = 70.712$$

$\Delta t' = v \Delta t$  ;

Hence,  $\Delta t' = 70.7 \text{ days in heaven.}$

12.  $t = 100 \text{ years}$  ;  $V = 60/100 C$  ;  $C = 0.6 C$ .

$$\Delta t = \frac{t}{\sqrt{1 - V^2/C^2}} = \frac{100y}{\sqrt{1 - \frac{(0.6)^2 C^2}{C^2}}} = \frac{100y}{0.8} = 125 y.$$

13. We know

$$f' = f\sqrt{1 - V^2/C^2}$$

$f'$  = apparent frequency ;

$f$  = frequency in rest frame

$v = 0.8 C$

$$f' = \sqrt{1 - \frac{0.64C^2}{C^2}} = \sqrt{0.36} = 0.6 \text{ s}^{-1}$$

14.  $V = 100 \text{ km/h}$ ,  $\Delta t = \text{Proper time interval} = 10 \text{ hours}$

$$\Delta t' = \frac{\Delta t}{\sqrt{1 - V^2/C^2}} = \frac{10 \times 3600}{\sqrt{1 - \left(\frac{1000}{36 \times 3 \times 10^8}\right)^2}}$$

$$\Delta t' - \Delta t = 10 \times 3600 \left[ \frac{1}{1 - \left(\frac{1000}{36 \times 3 \times 10^8}\right)^2} - 1 \right]$$

By solving we get,  $\Delta t' - \Delta t = 0.154 \text{ ns}$ .

$\therefore$  Time will lag by 0.154 ns.

15. Let the volume (initial) be  $V$ .

$$V' = V/2$$

$$\text{So, } V/2 = v\sqrt{1 - V^2/C^2}$$

$$\Rightarrow C/2 = \sqrt{C^2 - V^2} \Rightarrow C^2/4 = C^2 - V^2$$

$$\Rightarrow V^2 = C^2 - \frac{C^2}{4} = \frac{3}{4}C^2 \Rightarrow V = \frac{\sqrt{3}}{2}C.$$

16.  $d = 1 \text{ cm}$ ,  $v = 0.995 C$

$$\text{a) time in Laboratory frame} = \frac{d}{v} = \frac{1 \times 10^{-2}}{0.995C}$$

$$= \frac{1 \times 10^{-2}}{0.995 \times 3 \times 10^8} = 33.5 \times 10^{-12} = 33.5 \text{ PS}$$

- b) In the frame of the particle

$$t' = \frac{t}{\sqrt{1 - V^2/C^2}} = \frac{33.5 \times 10^{-12}}{\sqrt{1 - (0.995)^2}} = 335.41 \text{ PS.}$$

17.  $x = 1 \text{ cm} = 1 \times 10^{-2} \text{ m}$ ;  $K = 500 \text{ N/m}$ ,  $m = 200 \text{ g}$

$$\text{Energy stored} = \frac{1}{2} Kx^2 = \frac{1}{2} \times 500 \times 10^{-4} = 0.025 \text{ J}$$

$$\text{Increase in mass} = \frac{0.025}{C^2} = \frac{0.025}{9 \times 10^{16}}$$

$$\text{Fractional Change of max} = \frac{0.025}{9 \times 10^{16}} \times \frac{1}{2 \times 10^{-1}} = 0.01388 \times 10^{-16} = 1.4 \times 10^{-8}.$$

18.  $Q = MS \Delta\theta \Rightarrow 1 \times 4200 (100 - 0) = 420000 \text{ J}$ .

$$E = (\Delta m)C^2$$

$$\Rightarrow \Delta m = \frac{E}{C^2} = \frac{Q}{C^2} = \frac{420000}{(3 \times 10^8)^2}$$

$$= 4.66 \times 10^{-12} = 4.7 \times 10^{-12} \text{ kg.}$$

19. Energy possessed by a monoatomic gas =  $3/2 nRdt$ .

$$\text{Now } dT = 10, n = 1 \text{ mole, } R = 8.3 \text{ J/mol-K.}$$

$$E = 3/2 \times t \times 8.3 \times 10$$

$$\text{Loss in mass} = \frac{1.5 \times 8.3 \times 10}{C^2} = \frac{124.5}{9 \times 10^{15}}$$

$$= 1383 \times 10^{-16} = 1.38 \times 10^{-15} \text{ Kg.}$$

20. Let initial mass be  $m$

$$\frac{1}{2} mv^2 = E$$

$$\Rightarrow E = \frac{1}{2} m \left( \frac{12 \times 5}{18} \right)^2 = \frac{m \times 50}{9}$$

$$\Delta m = E/C^2$$

$$\Rightarrow \Delta m = \frac{m \times 50}{9 \times 9 \times 10^{16}} \Rightarrow \frac{\Delta m}{m} = \frac{50}{81 \times 10^{16}}$$

$$\Rightarrow 0.617 \times 10^{-16} = 6.17 \times 10^{-17}$$

21. Given : Bulb is 100 Watt = 100 J/s

So, 100 J is expended per 1 sec.

Hence total energy expended in 1 year =  $100 \times 3600 \times 24 \times 365 = 3153600000$  J

$$\text{Change in mass recorded} = \frac{\text{Total energy}}{C^2} = \frac{3153600000}{9 \times 10^{16}}$$

$$= 3.504 \times 10^8 \times 10^{-16} \text{ kg} = 3.5 \times 10^{-8} \text{ Kg.}$$

22.  $I = 1400 \text{ w/m}^2$

$$\text{Power} = 1400 \text{ w/m}^2 \times A$$

$$= (1400 \times 4\pi R^2)w = 1400 \times 4\pi \times (1.5 \times 10^{11})^2$$

$$= 1400 \times 4\pi \times (1/5)^2 \times 10^{22}$$

$$\text{a) } \frac{E}{t} = \frac{\Delta m C^2}{t} = \frac{\Delta m}{t} = \frac{E/t}{C^2}$$

$$C^2 = \frac{1400 \times 4\pi \times 2.25 \times 10^{22}}{9 \times 10^{16}} = 1696 \times 10^{66} = 4.396 \times 10^9 = 4.4 \times 10^9.$$

b)  $4.4 \times 10^9$  Kg disintegrates in 1 sec.

$$2 \times 10^{30} \text{ Kg disintegrates in } \frac{2 \times 10^{30}}{4.4 \times 10^9} \text{ sec.}$$

$$= \left( \frac{1 \times 10^{21}}{2.2 \times 365 \times 24 \times 3600} \right) = 1.44 \times 10^{-8} \times 10^{21} \text{ y} = 1.44 \times 10^{13} \text{ y.}$$

23. Mass of Electron = Mass of positron =  $9.1 \times 10^{-31}$  Kg

Both are oppositely charged and they annihilate each other.

Hence,  $\Delta m = m + m = 2 \times 9.1 \times 10^{-31}$  Kg

Energy of the resulting  $\gamma$  particle =  $\Delta m C^2$

$$= 2 \times 9.1 \times 10^{-31} \times 9 \times 10^{16} \text{ J} = \frac{2 \times 9.1 \times 9 \times 10^{-15}}{1.6 \times 10^{-19}} \text{ ev}$$

$$= 102.37 \times 10^4 \text{ ev} = 1.02 \times 10^6 \text{ ev} = 1.02 \text{ Mev.}$$

24.  $m_e = 9.1 \times 10^{-31}$ ,  $v_0 = 0.8 C$

$$\text{a) } m' = \frac{m_e}{\sqrt{1 - V^2/C^2}} = \frac{9.1 \times 10^{-31}}{\sqrt{1 - 0.64C^2/C^2}} = \frac{9.1 \times 10^{-31}}{0.6}$$

$$= 15.16 \times 10^{-31} \text{ Kg} = 15.2 \times 10^{-31} \text{ Kg.}$$

b) K.E. of the electron :  $m'C^2 - m_e C^2 = (m' - m_e) C^2$

$$= (15.2 \times 10^{-31} - 9.1 \times 10^{-31})(3 \times 10^8)^2 = (15.2 \times 9.1) \times 9 \times 10^{-31} \times 10^{18} \text{ J}$$

$$= 54.6 \times 10^{-15} \text{ J} = 5.46 \times 10^{-14} \text{ J} = 5.5 \times 10^{-14} \text{ J.}$$

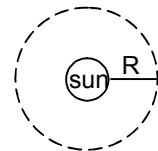
c) Momentum of the given electron = Apparent mass  $\times$  given velocity

$$= 15.2 \times 10^{-31} - 0.8 \times 3 \times 10^8 \text{ m/s} = 36.48 \times 10^{-23} \text{ kg m/s}$$

$$= 3.65 \times 10^{-22} \text{ kg m/s}$$

$$25. \text{ a) } ev - m_0 C^2 = \frac{m_0 C^2}{2\sqrt{1 - \frac{V^2}{C^2}}} \Rightarrow ev - 9.1 \times 10^{-31} \times 9 \times 10^{16}$$

$$= \frac{9.1 \times 9 \times 10^{-31} \times 10^{16}}{2\sqrt{1 - \frac{0.36C^2}{C^2}}} \Rightarrow ev - 9.1 \times 9 \times 10^{-15}$$



$$= \frac{9.1 \times 9 \times 10^{-15}}{2 \times 0.8} \Rightarrow eV - 9.1 \times 9 \times 10^{-15} = \frac{9.1 \times 9 \times 10^{-15}}{1.6}$$

$$\Rightarrow eV = \left( \frac{9.1 \times 9}{1.6} + 9.1 \times 9 \right) \times 10^{-15} = eV \left( \frac{81.9}{1.6} + 81.9 \right) \times 10^{-15}$$

$$eV = 133.0875 \times 10^{-15} \Rightarrow V = 83.179 \times 10^4 = 831 \text{ KV.}$$

$$\text{b) } eV - m_0 C^2 = \frac{m_0 C^2}{2\sqrt{1 - \frac{V^2}{C^2}}} \Rightarrow eV - 9.1 \times 9 \times 10^{-19} \times 9 \times 10^{16} = \frac{9.1 \times 9 \times 10^{-15}}{2\sqrt{1 - \frac{0.81C^2}{C^2}}}$$

$$\Rightarrow eV - 81.9 \times 10^{-15} = \frac{9.1 \times 9 \times 10^{-15}}{2 \times 0.435}$$

$$\Rightarrow eV = 12.237 \times 10^{-15}$$

$$\Rightarrow V = \frac{12.237 \times 10^{-15}}{1.6 \times 10^{-19}} = 76.48 \text{ kV.}$$

$$V = 0.99 C = eV - m_0 C^2 = \frac{m_0 C^2}{2\sqrt{1 - \frac{V^2}{C^2}}}$$

$$\Rightarrow eV = \frac{m_0 C^2}{2\sqrt{1 - \frac{V^2}{C^2}}} + m_0 C^2 = \frac{9.1 \times 10^{-31} \times 9 \times 10^{16}}{2\sqrt{1 - (0.99)^2}} + 9.1 \times 10^{-31} \times 9 \times 10^{16}$$

$$\Rightarrow eV = 372.18 \times 10^{-15} \Rightarrow V = \frac{372.18 \times 20^{-15}}{1.6 \times 10^{-19}} = 272.6 \times 10^4$$

$$\Rightarrow V = 2.726 \times 10^6 = 2.7 \text{ MeV.}$$

$$26. \text{ a) } \frac{m_0 C^2}{\sqrt{1 - \frac{V^2}{C^2}}} - m_0 C^2 = 1.6 \times 10^{-19}$$

$$\Rightarrow m_0 C^2 \left( \frac{1}{\sqrt{1 - V^2/C^2}} - 1 \right) = 1.6 \times 10^{-19}$$

$$\Rightarrow \frac{1}{\sqrt{1 - V^2/C^2}} - 1 = \frac{1.6 \times 10^{-19}}{9.1 \times 10^{-31} \times 9 \times 10^{16}}$$

$$\Rightarrow V = C \times 0.001937231 = 3 \times 0.001967231 \times 0^8 = 5.92 \times 10^5 \text{ m/s.}$$

$$\text{b) } \frac{m_0 C^2}{\sqrt{1 - \frac{V^2}{C^2}}} - m_0 C^2 = 1.6 \times 10^{-19} \times 10 \times 10^3$$

$$\Rightarrow m_0 C^2 \left( \frac{1}{\sqrt{1 - V^2/C^2}} - 1 \right) = 1.6 \times 10^{-15}$$

$$\Rightarrow \frac{1}{\sqrt{1 - V^2/C^2}} - 1 = \frac{1.6 \times 10^{-15}}{9.1 \times 9 \times 10^{15}}$$

$$\Rightarrow V = 0.584475285 \times 10^8 = 5.85 \times 10^7 \text{ m/s.}$$

$$\text{c) } \text{K.E.} = 10 \text{ Mev} = 10 \times 10^6 \text{ eV} = 10^7 \times 1.6 \times 10^{-19} \text{ J} = 1.6 \times 10^{-12} \text{ J}$$

$$\Rightarrow \frac{m_0 C^2}{\sqrt{1 - \frac{V^2}{C^2}}} - m_0 C^2 = 1.6 \times 10^{-12} \text{ J}$$

$$\Rightarrow V^2 = 8.999991359 \times 10^{16} \Rightarrow V = 2.999987038 \times 10^8.$$

27.  $\Delta m = m - m_0 = 2m_0 - m_0 = m_0$

Energy  $E = m_0 c^2 = 9.1 \times 10^{-31} \times 9 \times 10^{16} \text{ J}$

$E$  in e.v. =  $\frac{9.1 \times 9 \times 10^{-15}}{1.6 \times 10^{-19}} = 51.18 \times 10^4 \text{ ev} = 511 \text{ Kev.}$

28. 
$$\frac{\left( \frac{m_0 C^2}{\sqrt{1 - \frac{V^2}{C^2}}} - m_0 C^2 \right) - \frac{1}{2} m v^2}{\frac{1}{2} m_0 v^2} = 0.01$$

$$\Rightarrow \left[ \frac{m_0 C^2 \left( 1 + \frac{V^2}{2C^2} + \frac{1}{2} \times \frac{3}{4} \frac{V^2}{C^2} + \frac{1}{2} \times \frac{3}{4} \times \frac{5}{6} \frac{V^6}{C^6} \right) - m_0 C^2}{\frac{1}{2} m_0 v^2} \right] - \frac{1}{2} m v^2 = 0.1$$

$$\Rightarrow \frac{\frac{1}{2} m_0 v^2 + \frac{3}{8} m_0 \frac{V^4}{C^2} + \frac{15}{96} m_0 \frac{V^4}{C^2} - \frac{1}{2} m_0 v^2}{\frac{1}{2} m_0 v^2} = 0.01$$

$$\Rightarrow \frac{3}{4} \frac{V^4}{C^2} + \frac{15}{96 \times 2} \frac{V^4}{C^4} = 0.01$$

Neglecting the  $v^4$  term as it is very small

$$\Rightarrow \frac{3}{4} \frac{V^2}{C^2} = 0.01 \Rightarrow \frac{V^2}{C^2} = 0.04 / 3$$

$$\Rightarrow V/C = 0.2/\sqrt{3} = V = 0.2/\sqrt{3} C = \frac{0.2}{1.732} \times 3 \times 10^8$$
  

$$= 0.346 \times 10^8 \text{ m/s} = 3.46 \times 10^7 \text{ m/s.}$$

