Exercise 8(D)

1. If 3/2 log a + 2/3 log b - 1 = 0, find the value of $a^9.b^4$

Solution:

Given equation,

3/2 log a + 2/3 log b - 1 = 0

log $a^{3/2}$ + log $b^{2/3}$ - 1 = 0

log $a^{3/2} \times b^{2/3}$ - 1 = 0

log $a^{3/2}.b^{2/3}$ = 1

Removing logarithm, we have

$a^{3/2}.b^{2/3} = 10$

On manipulating,

$(a^{3/2}.b^{2/3})^6 = 10^6$

Hence,

$a^9.b^4 = 10^6$

2. If x = 1 + log 2 - log 5, y = 2log3 and z = log a - log 5; find the value of a if x + y = 2z.

Solution:

Given, x = 1 + log 2 - log 5, y = 2log3 and z = log a - log 5

Now, considering the given equation x + y = 2z

(1 + log 2 – log 5) + (2log 3) = 2(log a - log 5)

1 + log 2 – log 5 + 2log 3 = 2log a – 2log 5

1 + log 2 – log 5 + 2log 3 + 2log 5 = 2log a

log 10 + log 2 + log 3^2 + log 5 = log a^2

log 10 + log 2 + log 9 + log 5 = log a^2

log (10 x 2 x 9 x 5) = log a^2

log 900 = log a^2

On removing logarithm on both sides, we have

$900 = a^2$

Taking square root, we get

a = ±30

Since, a cannot be a negative value,

Hence, a = 30

3. If x = log 0.6; y = log 1.25 and z = log 3 - 2log 2, find the values of:

(i) x + y - z (ii) $5^{x + y - z}$

Solution:

Given,

x = log 0.6, y = log 1.25 and z = log 3 - 2log 2

So, z = log 3 - log 2^2

= log 3 – log 4

= log ¾

= log 0.75 … (1)

(i) Considering,
$x + y - z = \log 0.6 + \log 1.25 - \log 0.75$ … [From (1)]
$= \log (0.6 \times 1.25)/0.75$
$= \log 0.75/0.75$
$= \log 1$
$= 0$ … (2)

(ii) Now, considering
$5^{x+y-z} = 5^0$ … [From (2)]
$= 1$

4. If $a^2 = \log x$, $b^3 = \log y$ and $3a^2 - 2b^3 = 6 \log z$, express y in terms of x and z.
Solution:

We have, $a^2 = \log x$ and $b^3 = \log y$
Now, considering the equation
$3a^2 - 2b^3 = 6\log z$
$3\log x - 2\log y = 6\log z$
$\log x^3 - \log y^2 = \log z^6$
$\log x^3/y^2 = \log z^6$
On removing logarithm on both sides, we get
$x^3/y^2 = z^6$
So,
$y^2 = x^3/z^6$
Taking square root on both sides, we get
$y = \sqrt{(x^3/z^6)}$
Hence, $y = x^{3/2}/z^3$

5. If $\log (a - b)/2 = ½ (\log a + \log b)$, show that: $a^2 + b^2 = 6ab$.
Solution:

We have, $\log (a - b)/2 = ½ (\log a + \log b)$
$\log (a - b)/2 = ½ \log a + ½ \log b$
$= \log a^{1/2} + \log b^{1/2}$
$= \log \sqrt{a} + \log \sqrt{b}$
$= \log \sqrt{(ab)}$
Now, removing logarithm on both sides, we get
$(a - b)/2 = \sqrt{(ab)}$
Squaring on both sides, we get
$[(a - b)/2]^2 = [\sqrt{(ab)}]^2$
$(a - b)^2/4 = ab$
$(a - b)^2 = 4ab$
$a^2 + b^2 - 2ab = 4ab$
$a^2 + b^2 = 4ab + 2ab$
$a^2 + b^2 = 6ab$
- Hence proved

6. If $a^2 + b^2 = 23ab$, show that: log (a + b)/5 = ½ (log a + log b).
Solution:

Given, $a^2 + b^2 = 23ab$
Adding 2ab on both sides,
$a^2 + b^2 + 2ab = 23ab + 2ab$
$(a + b)^2 = 25ab$
$(a + b)^2/25 = ab$
$[(a + b)/5]^2 = ab$
Taking logarithm on both sides, we have
$\log [(a + b)/5]^2 = \log ab$
2log (a + b)/5 = log ab
2log (a + b)/5 = log a + log b
Thus,
log (a + b)/5 = ½ (log a + log b)

7. If m = log 20 and n = log 25, find the value of x, so that: 2log (x - 4) = 2m - n.
Solution:

Given, m = log 20 and n = log 25
Now, considering the given expression
2log (x - 4) = 2m - n
2log (x - 4) = 2log 20 – log 25
$\log (x - 4)^2 = \log 20^2 - \log 25$
$\log (x - 4)^2 = \log 400 - \log 25$
$\log (x - 4)^2 = \log 400/25$
Removing logarithm on both sides,
$(x - 4)^2 = 400/25$
$x^2 - 8x + 16 = 16$
$x^2 - 8x = 0$
x(x - 8) = 0
So,
x = 0 or x = 8
If x = 0, then log (x - 4) doesn't exist
Hence, x = 8

8. Solve for x and y; if x > 0 and y > 0; log xy = log x/y + 2log 2 = 2.
Solution:

We have,
log xy = log x/y + 2log 2 = 2
Considering the equation,
log xy = 2
log xy = 2log 10
$\log xy = \log 10^2$

log xy = log 100
On removing logarithm,
xy = 100 … (1)

Now, consider the equation
log x/y + 2log 2 = 2
log x/y + log 2^2 = 2
log x/y + log 4 = 2
log 4x/y = 2
Removing logarithm, we get
4x/y = 10^2
4x/y = 100
x/y = 25
(xy)/y^2 = 25
100/y^2 = 25 … [From (1)]
y^2 = 100/25
y^2 = 4
y = 2 [Since, y > 0]

From log xy = 2
Substituting the value of y, we get
log 2x = 2
On removing logarithm,
2x = 10^2
2x = 100
x = 100/2
x = 50
Thus, the values x and y are 50 and 2 respectively

9. Find x, if:
(i) $\log_x 625 = -4$
(ii) $\log_x (5x - 6) = 2$
Solution:

(i) We have, $\log_x 625 = -4$
On removing logarithm,
$x^{-4} = 625$
$(1/x)^4 = 5^4$
Taking the fourth root on both sides,
1/x = 5
Hence, x = 1/5

(ii) We have, $\log_x (5x - 6) = 2$
On removing logarithm,
$x^2 = 5x – 6$
$x^2 – 5x + 6 = 0$

$x^2 - 3x - 2x + 6 = 0$
$x(x - 3) - 2(x - 2) = 0$
$(x - 2)(x - 3) = 0$
Hence,
$x = 2$ or 3

10. If p = log 20 and q = log 25, find the value of x, if 2log (x + 1) = 2p – q.
Solution:

Given, p = log 20 and q = log 25
Considering the equation,
$2\log (x + 1) = 2p - q$
$\log (x + 1)^2 = 2p - q$
$\log (x + 1)^2 = 2\log 20 - \log 25$
$\log (x + 1)^2 = \log 20^2 - \log 25$
$\log (x + 1)^2 = \log 400 - \log 25$
$\log (x + 1)^2 = \log 400/25$
Removing logarithm on both sides, we have
$(x + 1)^2 = 400/25 = 16$
$(x + 1)^2 = (4)^2$
Taking square root on both sides, we have
$x + 1 = 4$
$x = 4 - 1$
Hence, $x = 3$

11. If $\log_2 (x + y) = \log_3 (x - y) = \log 25/\log 0.2$, find the value of x and y.
Solution:

Considering the relation, $\log_2 (x + y) = \log 25/\log 0.2$
$\log_2 (x + y) = \log_{0.2} 25$
$= \log_{2/10} 5^2$
$= 2\log_{1/5} 5$
$= 2\log_{5^{-1}} 5$
$= -2\log_5 5$
$= -2 \times 1$
$= -2$
So, we have
$\log_2 (x + y) = -2$
Removing logarithm, we get
$x + y = 2^{-2}$
$x + y = 1/2^2$
$x + y = ¼$ … (i)

Now, considering the relation $\log_3 (x - y) = \log 25/\log 0.2$
$\log_3 (x - y) = = \log_{0.2} 25$
$= \log_{2/10} 5^2$

$= 2\log_{1/5} 5$
$= 2\log_{5^{-1}} 5$
$= -2\log_5 5$
$= -2 \times 1$
$= -2$

So, we have
$\log_3 (x - y) = -2$
Removing logarithm, we get
$x - y = 3^{-2}$
$x - y = 1/3^2$
$x - y = 1/9$ … (ii)

On adding (i) and (ii), we get
$x + y = ¼$
$x - y = 1/9$
– – – – – – – – –
$2x = ¼ + 1/9$
$2x = (9 + 4)/36$
$2x = 13/36$
$x = 13/(36 \times 2)$
$= 13/72$

Now, substituting the value of x in (i), we get
$13/72 + y = ¼$
$y = ¼ - 13/72$
$= (18 - 13)/72$
$= 5/72$

Hence, the values of x and are is 13/72 and 5/72 respectively

12. Given: log x/log y = 3/2 and log xy = 5; find the values of x and y.
Solution:

Given, $\log x/\log y = 3/2$ … (i) and $\log xy = 5$ … (ii)
So,
$\log xy = \log x + \log y = 5$
And, we have $\log y = (2\log x)/3$ … [From (i)]
Now,
$\log x + (2\log x)/3 = 5$
$3\log x + 2\log x = 5 \times 3$
$5\log x = 15$
$\log x = 15/5$
$\log x = 3$
Removing logarithm, we get
$x = 10^3 = 1000$

Substituting value of x in (ii), we get
log xy = 5
Removing logarithm, we get
$xy = 10^5$
$(10^3).\ y = 10^5$
$y = 10^5/10^3$
$y = 10^2$
y = 100

13. Given $\log_{10} x = 2a$ and $\log_{10} y = b/2$
(i) Write 10^a in terms of x
(ii) Write 10^{2b+1} in terms of y
(iii) If $\log_{10} p = 3a – 2b$, express p in terms of x and y.
Solution:

Given, $\log_{10} x = 2a$ and $\log_{10} y = b/2$
(i) Taking $\log_{10} x = 2a$
Removing logarithm on both sides,
$x = 10^{2a}$
Taking square root on both sides, we get
$x^{1/2} = 10^{2a/2}$
Hence, $10^a = x^{1/2}$

(ii) Taking $\log_{10} y = b/2$
Removing logarithm on both sides,
$y = 10^{b/2}$
On manipulating,
$y^4 = 10^{b/2 \times 4}$
$y^4 = 10^{2b}$
$10y^4 = 10^{2b} \times 10$
Hence, $10^{2b+1} = 10y^4$

(iii) We have, $10^a = x^{1/2}$
and $y = 10^{b/2}$
Considering the equation, $\log_{10} p = 3a – 2b$
$\log_{10} p = 3a – 2b$
Removing logarithm, we get
$p = 10^{3a – 2b}$
$p = 10^{3a}/10^{2b}$
$p = (10^a)^3/(10^{b/2})^4$
$p = (x^{1/2})^3/(y)^4$
Hence, $p = x^{3/2}/y^4$

14. Solve:
$\log_5 (x + 1) - 1 = 1 + \log_5 (x - 1)$.
Solution:

Considering the given equation,
$\log_5 (x + 1) - 1 = 1 + \log_5 (x - 1)$
$\log_5 (x + 1) - \log_5 (x - 1) = 1 + 1$
$\log_5 (x + 1)/(x - 1) = 2$
Removing logarithm, we have
$(x + 1)/(x - 1) = 5^2$
$(x + 1)/(x - 1) = 25$
$(x + 1) = 25(x - 1)$
$x + 1 = 25x – 25$
$25x – x = 25 + 1$
$24x = 26$
$x = 26/24$
Hence, $x = 13/12$

15. Solve for x, if:
$\log_x 49 – \log_x 7 + \log_x 1/343 + 2 = 0$
Solution:

We have,
$\log_x 49 – \log_x 7 + \log_x 1/343 + 2 = 0$
$\log_x 49/(7 \times 343) + 2 = 0$
$\log_x 1/49 = -2$
$\log_x 1/7^2 = -2$
$\log_x 7^{-2} = -2$
$-2\log_x 7 = -2$
So,
$\log_x 7 = 1$
Removing logarithm, we get
$x = 7$

16. If $a^2 = \log x$, $b^3 = \log y$ and $a^2/2 – b^3/3 = \log c$, find c in terms of x and y.
Solution:

Given,
$a^2 = \log x$, $b^3 = \log y$
Considering the given equation,
$a^2/2 – b^3/3 = \log c$
$(\log x)/2 – (\log y)/3 = \log c$
$½ \log x – 1/3 \log y = \log c$
$\log x^{1/2} – \log y^{1/3} = \log c$
$\log x^{1/2}/y^{1/3} = \log c$
On removing logarithm, we get
$x^{1/2}/y^{1/3} = c$
Hence, $c = x^{1/2}/y^{1/3}$ is the required relation

17. Given: $x = \log_{10} 12$, $y = \log_4 2 \times \log_{10} 9$ and $z = \log_{10} 0.4$, find
(i) x – y – z
(ii) $13^{x - y - z}$
Solution:

(i) Considering, x – y – z
$= \log_{10} 12 – (\log_4 2 \times \log_{10} 9) - \log_{10} 0.4$
$= \log_{10} 12 – (\log_4 2 \times \log_{10} 9) - \log_{10} 0.4$
$= \log_{10} (4 \times 3) – (\log_{10} 2/ \log_{10} 4 \times \log_{10} 9) - \log_{10} 0.4$
$= \log_{10} 4 + \log_{10} 3 – (\log_{10} 2 \times \log_{10} 3^2)/ \log_{10} 2^2 – \log_{10} 4/10$
$= \log_{10} 4 + \log_{10} 3 – (\log_{10} 2 \times 2\log_{10} 3)/ 2\log_{10} 2 – (\log_{10} 4 – \log_{10} 10)$
$= \log_{10} 4 + \log_{10} 3 – \log_{10} 3 – \log_{10} 4 + \log_{10} 10$
$= \log_{10} 4 + \log_{10} 3 – \log_{10} 3 – \log_{10} 4 + 1$
= 1

(ii) Now,
$13^{x - y - z} = 13^1 = 13$

18. Solve for x, $\log_x 15\sqrt{5} = 2 – \log_x 3\sqrt{5}$
Solution:

Considering the given equation,
$\log_x 15\sqrt{5} = 2 – \log_x 3\sqrt{5}$
$\log_x 15\sqrt{5} + \log_x 3\sqrt{5} = 2$
$\log_x (15\sqrt{5} \times 3\sqrt{5}) = 2$
$\log_x (45 \times 5) = 2$
$\log_x 225 = 2$
Removing logarithm, we get
$x^2 = 225$
Taking square root on both sides,
x = 15

19. Evaluate:
(i) $\log_b a \times \log_c b \times \log_a c$
(ii) $\log_3 8 \div \log_9 16$
(iii) $\log_5 8/(\log_{25} 16 \times \log_{100} 10)$
Solution:

Using $\log_b a = 1/\log_a b$ and $\log_x a/\log_x b = \log_b a$, we have

(i) $\log_b a \times \log_c b \times \log_a c$

$$= \frac{\log_{10} a}{\log_{10} b} \times \frac{\log_{10} b}{\log_{10} c} \times \frac{\log_{10} c}{\log_{10} a}$$

= 1

(ii) $\log_3 8 \div \log_9 16$

$= \log_3 8 / \log_9 16$

$$= \frac{\log_{10}8}{\log_{10}3} \times \frac{\log_{10}9}{\log_{10}16}$$

$$= \frac{3\log_{10}2}{\log_{10}3} \times \frac{2\log_{10}3}{4\log_{10}2}$$

= 3 x ½

= 3/2

(iii) $\log_5 8/(\log_{25} 16 \times \log_{100} 10)$

$$= \frac{\frac{\log_{10}8}{\log_{10}5}}{\frac{\log_{10}16}{\log_{10}25} \times \frac{\log_{10}10}{\log_{10}100}}$$

$$= \frac{\frac{\log_{10}2^3}{\log_{10}5}}{\frac{\log_{10}2^4}{\log_{10}5^2} \times \frac{\log_{10}10}{\log_{10}10^2}}$$

$$= \frac{\log_{10}2^3}{\log_{10}5} \times \frac{\log_{10}5^2}{\log_{10}2^4} \times \frac{\log_{10}10^2}{\log_{10}10}$$

$$= \frac{3\log_{10}2}{\log_{10}5} \times \frac{2\log_{10}5}{4\log_{10}2} \times \frac{2\log_{10}10}{\log_{10}10}$$

= 3 x ½ x 2

= 3

20. Show that:

$\log_a m \div \log_{ab} m = 1 + \log_a b$

Solution:

Considering the L.H.S.,

$\log_a m \div \log_{ab} m = \log_a m/\log_{ab} m$

$= \log_m ab/\log_m a$ [As $\log_b a = 1/\log_a b$]

$= \log_a ab$ [As $\log_x a/\log_x b = \log_b a$]

$= \log_a a + \log_a b$

$= 1 + \log_a b$

21. If $\log_{\sqrt{27}} x = 2\ 2/3$, find x.

Solution:

We have,

$\log_{\sqrt{27}} x = 2\ 2/3$

$\log_{\sqrt{27}} x = 8/3$

Removing logarithm, we get

$x = \sqrt{27^{8/3}}$

$= 27^{1/2 \times 8/3}$

$= 27^{4/3}$

$= 3^{3 \times 4/3}$

$= 3^4$

Hence, x = 81

22. Evaluate:

$1/(\log_a bc + 1) + 1/(\log_b ca + 1) + 1/(\log_c ab + 1)$

Solution:

We have,

$$\frac{1}{\log_a bc + 1} + \frac{1}{\log_b ca + 1} + \frac{1}{\log_c ab + 1}$$

$$= \frac{1}{\log_a bc + \log_a a} + \frac{1}{\log_b ca + \log_b b} + \frac{1}{\log_c ab + \log_c c}$$

$$= \frac{1}{\log_a abc} + \frac{1}{\log_b aba} + \frac{1}{\log_c abc} \qquad [\because \log_a b + \log_b c = \log_a bc]$$

$$= \frac{1}{\frac{\log abc}{\log a}} + \frac{1}{\frac{\log abc}{\log b}} + \frac{1}{\frac{\log abc}{\log c}}$$

$$= \frac{\log a + \log b + \log c}{\log abc}$$

$$= \frac{\log abc}{\log abc} \qquad [\because \log_a b + \log_b c = \log_a bc]$$

$$= 1$$