

EXERCISE 2.2

1. Write each of the following in exponential form:

(i) $(3/2)^{-1} \times (3/2)^{-1} \times (3/2)^{-1} \times (3/2)^{-1}$

(ii) $(2/5)^{-2} \times (2/5)^{-2} \times (2/5)^{-2}$

Solution:

(i) $(3/2)^{-1} \times (3/2)^{-1} \times (3/2)^{-1} \times (3/2)^{-1}$

$(3/2)^{-4}$ (we know that $a^{-n} = 1/a^n$, $a^n = a \times a \dots n$ times)

(ii) $(2/5)^{-2} \times (2/5)^{-2} \times (2/5)^{-2}$

$(2/5)^{-6}$ (we know that $a^{-n} = 1/a^n$, $a^n = a \times a \dots n$ times)

2. Evaluate:

(i) 5^{-2}

(ii) $(-3)^{-2}$

(iii) $(1/3)^{-4}$

(iv) $(-1/2)^{-1}$

Solution:

(i) 5^{-2}

$1/5^2 = 1/25$ (we know that $a^{-n} = 1/a^n$)

(ii) $(-3)^{-2}$

$(1/-3)^2 = 1/9$ (we know that $a^{-n} = 1/a^n$)

(iii) $(1/3)^{-4}$

$3^4 = 81$ (we know that $1/a^{-n} = a^n$)

(iv) $(-1/2)^{-1}$

$-2^1 = -2$ (we know that $1/a^{-n} = a^n$)

3. Express each of the following as a rational number in the form p/q:

(i) 6^{-1}

(ii) $(-7)^{-1}$

(iii) $(1/4)^{-1}$

(iv) $(-4)^{-1} \times (-3/2)^{-1}$

(v) $(3/5)^{-1} \times (5/2)^{-1}$

Solution:

(i) 6^{-1}

$$1/6^1 = 1/6 \text{ (we know that } a^{-n} = 1/a^n)$$

(ii) $(-7)^{-1}$

$$1/-7^1 = -1/7 \text{ (we know that } a^{-n} = 1/a^n)$$

(iii) $(1/4)^{-1}$

$$4^1 = 4 \text{ (we know that } 1/a^{-n} = a^n)$$

(iv) $(-4)^{-1} \times (-3/2)^{-1}$

$$1/-4^1 \times (2/-3)^1 \text{ (we know that } a^{-n} = 1/a^n, 1/a^{-n} = a^n)$$

$$1/-2 \times -1/3$$

$$1/6$$

(v) $(3/5)^{-1} \times (5/2)^{-1}$

$$(5/3)^1 \times (2/5)^1$$

$$5/3 \times 2/5$$

$$2/3$$

4. Simplify:

(i) $(4^{-1} \times 3^{-1})^2$

(ii) $(5^{-1} \div 6^{-1})^3$

(iii) $(2^{-1} + 3^{-1})^{-1}$

(iv) $(3^{-1} \times 4^{-1})^{-1} \times 5^{-1}$

(v) $(4^{-1} - 5^{-1}) \div 3^{-1}$

Solution:

(i) $(4^{-1} \times 3^{-1})^2$

$$(1/4 \times 1/3)^2 \text{ (we know that } a^{-n} = 1/a^n)$$

$$(1/12)^2$$

$$1/144$$

(ii) $(5^{-1} \div 6^{-1})^3$

$$(1/5 \div 1/6)^3 \text{ (we know that } a^{-n} = 1/a^n)$$

$$(1/5 \times 6)^3 \text{ (we know that } 1/a \div 1/b = 1/a \times b/1)$$

$$(6/5)^3$$

$$216/125$$

(iii) $(2^{-1} + 3^{-1})^{-1}$

$$(1/2 + 1/3)^{-1} \text{ (we know that } a^{-n} = 1/a^n)$$

LCM of 2 and 3 is 6

$$\begin{aligned} & ((3+2)/6)^{-1} \\ & (5/6)^{-1} \text{ (we know that } 1/a^{-n} = a^n) \\ & 6/5 \end{aligned}$$

$$\begin{aligned} \text{(iv)} & (3^{-1} \times 4^{-1})^{-1} \times 5^{-1} \\ & (1/3 \times 1/4)^{-1} \times 1/5 \text{ (we know that } a^{-n} = 1/a^n) \\ & (1/12)^{-1} \times 1/5 \text{ (we know that } 1/a^{-n} = a^n) \\ & 12 \times 1/5 \\ & 12/5 \end{aligned}$$

$$\begin{aligned} \text{(v)} & (4^{-1} - 5^{-1}) \div 3^{-1} \\ & (1/4 - 1/5) \div 1/3 \text{ (we know that } a^{-n} = 1/a^n) \\ & \text{LCM of 4 and 5 is 20} \\ & (5-4)/20 \times 3/1 \text{ (we know that } 1/a \div 1/b = 1/a \times b/1) \\ & 1/20 \times 3 \\ & 3/20 \end{aligned}$$

5. Express each of the following rational numbers with a negative exponent:

(i) $(1/4)^3$

(ii) 3^5

(iii) $(3/5)^4$

(iv) $((3/2)^4)^{-3}$

(v) $((7/3)^4)^{-3}$

Solution:

(i) $(1/4)^3$

$(4)^{-3}$ (we know that $1/a^n = a^{-n}$)

(ii) 3^5

$(1/3)^{-5}$ (we know that $1/a^n = a^{-n}$)

(iii) $(3/5)^4$

$(5/3)^{-4}$ (we know that $(a/b)^{-n} = (b/a)^n$)

(iv) $((3/2)^4)^{-3}$

$(3/2)^{-12}$ (we know that $(a^n)^m = a^{nm}$)

(v) $((7/3)^4)^{-3}$

$(7/3)^{-12}$ (we know that $(a^n)^m = a^{nm}$)

6. Express each of the following rational numbers with a positive exponent:

(i) $(3/4)^{-2}$

(ii) $(5/4)^{-3}$

(iii) $4^3 \times 4^{-9}$

(iv) $((4/3)^{-3})^{-4}$

(v) $((3/2)^4)^{-2}$

Solution:

(i) $(3/4)^{-2}$

$(4/3)^2$ (we know that $(a/b)^{-n} = (b/a)^n$)

(ii) $(5/4)^{-3}$

$(4/5)^3$ (we know that $(a/b)^{-n} = (b/a)^n$)

(iii) $4^3 \times 4^{-9}$

$(4)^{3-9}$ (we know that $a^n \times a^m = a^{n+m}$)

4^{-6}

$(1/4)^6$ (we know that $1/a^n = a^{-n}$)

(iv) $((4/3)^{-3})^{-4}$

$(4/3)^{12}$ (we know that $(a^n)^m = a^{nm}$)

(v) $((3/2)^4)^{-2}$

$(3/2)^{-8}$ (we know that $(a^n)^m = a^{nm}$)

$(2/3)^8$ (we know that $1/a^n = a^{-n}$)

7. Simplify:

(i) $((1/3)^{-3} - (1/2)^{-3}) \div (1/4)^{-3}$

(ii) $(3^2 - 2^2) \times (2/3)^{-3}$

(iii) $((1/2)^{-1} \times (-4)^{-1})^{-1}$

(iv) $(((-1/4)^2)^{-2})^{-1}$

(v) $((2/3)^2)^3 \times (1/3)^{-4} \times 3^{-1} \times 6^{-1}$

Solution:

(i) $((1/3)^{-3} - (1/2)^{-3}) \div (1/4)^{-3}$

$(3^3 - 2^3) \div 4^3$ (we know that $1/a^n = a^{-n}$)

$(27-8) \div 64$

$19 \div 64$

$19 \times 1/64$ (we know that $1/a \div 1/b = 1/a \times b/1$)

$19/64$

$$\begin{aligned} & \text{(ii)} (3^2 - 2^2) \times (2/3)^{-3} \\ & (9 - 4) \times (3/2)^3 \text{ (we know that } 1/a^n = a^{-n}\text{)} \\ & 5 \times (27/8) \\ & 135/8 \end{aligned}$$

$$\begin{aligned} & \text{(iii)} ((1/2)^{-1} \times (-4)^{-1})^{-1} \\ & (2^1 \times (1/-4))^{-1} \text{ (we know that } 1/a^n = a^{-n}\text{)} \\ & (1/-2)^{-1} \text{ (we know that } 1/a^n = a^{-n}\text{)} \\ & -2^1 \\ & -2 \end{aligned}$$

$$\begin{aligned} & \text{(iv)} (((-1/4)^2)^{-2})^{-1} \\ & ((-1/16)^{-2})^{-1} \text{ (we know that } 1/a^n = a^{-n}\text{)} \\ & ((-16)^2)^{-1} \text{ (we know that } 1/a^n = a^{-n}\text{)} \\ & (256)^{-1} \text{ (we know that } 1/a^n = a^{-n}\text{)} \\ & 1/256 \end{aligned}$$

$$\begin{aligned} & \text{(v)} ((2/3)^2)^3 \times (1/3)^{-4} \times 3^{-1} \times 6^{-1} \\ & (4/9)^3 \times 3^4 \times 1/3 \times 1/6 \text{ (we know that } 1/a^n = a^{-n}\text{)} \\ & (64/729) \times 81 \times 1/3 \times 1/6 \\ & (64/729) \times 27 \times 1/6 \\ & 32/729 \times 27 \times 1/3 \\ & 32/729 \times 9 \\ & 32/81 \end{aligned}$$

8. By what number should 5^{-1} be multiplied so that the product may be equal to $(-7)^{-1}$?

Solution:

Let us consider a number x

$$\text{So, } 5^{-1} \times x = (-7)^{-1}$$

$$1/5 \times x = 1/-7 \text{ (we know that } 1/a^n = a^{-n}\text{)}$$

$$x = (-1/7) / (1/5)$$

$$= (-1/7) \times (5/1) \text{ (we know that } 1/a \div 1/b = 1/a \times b/1\text{)}$$

$$= -5/7$$

9. By what number should $(1/2)^{-1}$ be multiplied so that the product may be equal to $(-4/7)^{-1}$?

Solution:

Let us consider a number x

$$\begin{aligned} \text{So, } (1/2)^{-1} \times x &= (-4/7)^{-1} \\ 1/(1/2) \times x &= 1/(-4/7) \text{ (we know that } 1/a^n = a^{-n}\text{)} \\ x &= (-7/4) / (2/1) \\ &= (-7/4) \times (1/2) \text{ (we know that } 1/a \div 1/b = 1/a \times b/1\text{)} \\ &= -7/8 \end{aligned}$$

10. By what number should $(-15)^{-1}$ be divided so that the quotient may be equal to $(-5)^{-1}$?

Solution:

Let us consider a number x

$$\text{So, } (-15)^{-1} \div x = (-5)^{-1} \text{ (we know that } 1/a \div 1/b = 1/a \times b/1\text{)}$$

$$1/-15 \times 1/x = 1/-5 \text{ (we know that } 1/a^n = a^{-n}\text{)}$$

$$1/x = (1 \times -15)/-5$$

$$1/x = 3$$

$$x = 1/3$$

11. By what number should $(5/3)^{-2}$ be multiplied so that the product may be $(7/3)^{-1}$?

Solution:

Let us consider a number x

$$\text{So, } (5/3)^{-2} \times x = (7/3)^{-1}$$

$$1/(5/3)^2 \times x = 1/(7/3) \text{ (we know that } 1/a^n = a^{-n}\text{)}$$

$$x = (3/7) / (3/5)^2$$

$$= (3/7) / (9/25)$$

$$= (3/7) \times (25/9) \text{ (we know that } 1/a \div 1/b = 1/a \times b/1\text{)}$$

$$= (1/7) \times (25/3)$$

$$= 25/21$$

12. Find x , if

(i) $(1/4)^{-4} \times (1/4)^{-8} = (1/4)^{-4x}$

Solution:

$$(1/4)^{-4} \times (1/4)^{-8} = (1/4)^{-4x}$$

$$(1/4)^{-4-8} = (1/4)^{-4x} \text{ (we know that } a^n \times a^m = a^{n+m}\text{)}$$

$$(1/4)^{-12} = (1/4)^{-4x}$$

When the bases are same we can directly equate the coefficients

$$-12 = -4x$$

$$x = -12/-4$$

$$= 3$$

(ii) $(-1/2)^{-19} \div (-1/2)^8 = (-1/2)^{-2x+1}$

Solution:

$$(-1/2)^{-19} \div (-1/2)^8 = (-1/2)^{-2x+1}$$

$$(1/2)^{-19-8} = (1/2)^{-2x+1} \text{ (we know that } a^n \div a^m = a^{n-m}\text{)}$$

$$(1/2)^{-27} = (1/2)^{-2x+1}$$

When the bases are same we can directly equate the coefficients

$$-27 = -2x+1$$

$$-2x = -27-1$$

$$x = -28/-2$$

$$= 14$$

(iii) $(3/2)^{-3} \times (3/2)^5 = (3/2)^{2x+1}$

Solution:

$$(3/2)^{-3} \times (3/2)^5 = (3/2)^{2x+1}$$

$$(3/2)^{-3+5} = (3/2)^{2x+1} \text{ (we know that } a^n \times a^m = a^{n+m}\text{)}$$

$$(3/2)^2 = (3/2)^{2x+1}$$

When the bases are same we can directly equate the coefficients

$$2 = 2x+1$$

$$2x = 2-1$$

$$x = 1/2$$

(iv) $(2/5)^{-3} \times (2/5)^{15} = (2/5)^{2+3x}$

Solution:

$$(2/5)^{-3} \times (2/5)^{15} = (2/5)^{2+3x}$$

$$(2/5)^{-3+15} = (2/5)^{2+3x} \text{ (we know that } a^n \times a^m = a^{n+m}\text{)}$$

$$(2/5)^{12} = (2/5)^{2+3x}$$

When the bases are same we can directly equate the coefficients

$$12 = 2+3x$$

$$3x = 12-2$$

$$x = 10/3$$

(v) $(5/4)^{-x} \div (5/4)^{-4} = (5/4)^5$

Solution:

$$(5/4)^{-x} \div (5/4)^{-4} = (5/4)^5$$

$$(5/4)^{-x+4} = (5/4)^5 \text{ (we know that } a^n \div a^m = a^{n-m}\text{)}$$

When the bases are same we can directly equate the coefficients

$$-x+4 = 5$$

$$-x = 5-4$$

$$-x = 1$$

$$x = -1$$

$$(vi) (8/3)^{2x+1} \times (8/3)^5 = (8/3)^{x+2}$$

Solution:

$$(8/3)^{2x+1} \times (8/3)^5 = (8/3)^{x+2}$$

$$(8/3)^{2x+1+5} = (8/3)^{x+2} \text{ (we know that } a^n \times a^m = a^{n+m}\text{)}$$

$$(8/3)^{2x+6} = (8/3)^{x+2}$$

When the bases are same we can directly equate the coefficients

$$2x+6 = x+2$$

$$2x-x = -6+2$$

$$x = -4$$

13. (i) If $x = (3/2)^2 \times (2/3)^4$, find the value of x^{-2} .

Solution:

$$x = (3/2)^2 \times (2/3)^4$$

$$= (3/2)^2 \times (3/2)^4 \text{ (we know that } 1/a^n = a^{-n}\text{)}$$

$$= (3/2)^{2+4} \text{ (we know that } a^n \times a^m = a^{n+m}\text{)}$$

$$= (3/2)^6$$

$$x^{-2} = ((3/2)^6)^{-2}$$

$$= (3/2)^{-12}$$

$$= (2/3)^{12}$$

(ii) If $x = (4/5)^{-2} \div (1/4)^2$, find the value of x^{-1} .

Solution:

$$x = (4/5)^{-2} \div (1/4)^2$$

$$= (5/4)^2 \div (1/4)^2 \text{ (we know that } 1/a^n = a^{-n}\text{)}$$

$$= (5/4)^2 \times (4/1)^2 \text{ (we know that } 1/a \div 1/b = 1/a \times b/1\text{)}$$

$$= 25/16 \times 16$$

$$= 25$$

$$x^{-1} = 1/25$$

14. Find the value of x for which $5^{2x} \div 5^{-3} = 5^5$

Solution:

$$5^{2x} \div 5^{-3} = 5^5$$

$$5^{2x+3} = 5^5 \text{ (we know that } a^n \div a^m = a^{n-m}\text{)}$$

When the bases are same we can directly equate the coefficients

$$2x+3 = 5$$

$$2x = 5-3$$

$$2x = 2$$

$$x = 1$$