Now choose another person from remaining 5 by $^5C_1=5$ and arranged it in line

Similarly, choose another person from remaining 4 by $^4C_1=4$ and arranged it in line

Similarly, choose another person from remaining 3 by $^3C_1=3$ and arranged it in line

Similarly, choose another person from remaining 2 by $^2C_1=2$ and arranged it in line

And choose another person from remaining 1 by $^1C_1=1$ and arranged it in line

So total number of ways is 6! =720

(ii) It is the same as above, by converting line arrangement into the circle but you need to remove some arrangement

Let suppose 6 persons as A, B, C, D ,E, F you need to arrange this 6 persons into a circle.

First, we arranged 6 persons in line(number of ways = 6!)

NOTE: A, B, C, D ,E, F and B, C, D, E, F, A consider as a different line, but when we arranged this 2 combination in circle then it becomes same,

i.e. Let takes us an example we need to arrange A, N, O, D, E.

We arrange it as shown. When we rotate first one, then 1st and 2nd became identical and so on that's why all 5 are identical, and we count it as 1

Now come back to our questions

So total number of arrangement is (6-1)! = 5! = 120

NOTE: When you want to arrange n persons in circle then a total number of ways is n!/n,

i.e. Total number of ways = (n-1)!

Q. 2. There are 5 men and 5 ladies to dine at a round table. In how many ways can they sit so that no ladies are together?

Answer : Let first arranged 5 men in the round table by 4! (By using the formula (n-1)! Mention above)

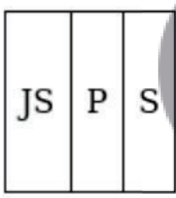
Now there are 5 gaps created between 5 men (check the figure)

So we arrange 5 ladies in this gap by 5!

A total number of ways to arrange 5 men and 5 ladies is 5! × 4! = 2880

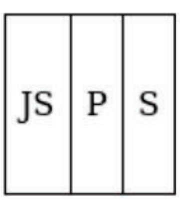
Q. 3. In how many ways can 11 members of a committee sit at a round table so that the secretary and the joint secretary are always the neighbour of the president?

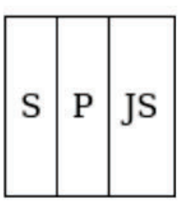
Answer : First assume the president(P), Joint secretary(JS) and secretary(S) to be 1 members(as shown below)



So there are 9 members, a number of ways to arrange this 9 people is 8! (The formula used (n-1)!)

Now we need to look at the internal arrangement. There are 2 arrangement possible





So total number of arrangement are (8!) × 2 = 80,640

Q. 4. In how many ways can 8 persons be seated at a round table so that all shall not have the same neighbour in any two arrangement?

Answer : By using the formula (n-1)! (Mention in Solution-1)

So 8 persons can be arranged by 7!

Now each person have the same neighbour in the clockwise and anticlockwise arrangement

Total number of arrangement are (7!)/2 = 2520

Q. 5. In how many different ways can 20 different pearls be arranged to form a necklace?

Answer : We know that necklace in the form of a circle, So we need to arrange 20 pearls in Circle

20 pearls can be arranged by 19!

Now each pearl have the same neighbour in the clockwise and anticlockwise arrangement

Total number of arrangement are (19!)/2

Q. 6. In how many different ways can a garland of 16 different flowers be made?

Answer : It is also in the form of a circle, So we need to arrange 16flowers in Circle

16 flowers can be arranged by 15!

Now each flower have the same neighbour in the clockwise and anticlockwise arrangement

Total number of arrangement are (15!)/2

Exercise 8H

Q. 1. If (n + 1)! = 12 × [(n – 1)!], find the value of n.

Answer : To Find: Value of n

Given: (n+1)! = 12× [(n-1)!]

Formula Used: n! = (n) × (n-1) × (n-2) × (n-3) ………. 3 × 2 × 1

Now, $(n+1)! = 12\times [(n-1)!]$

$\Rightarrow (n+1) \times (n) \times [(n-1)!] = 12 \times [(n-1)!]$

$\Rightarrow (n+1) \times (n) = 12$

$\Rightarrow n^2+n = 12$

$\Rightarrow n^2+n-12 = 0$

$\Rightarrow (n-3)(n+4) = 0$

$\Rightarrow n = 3$ or, $n = -4$

But, n=-4 is not possible because in case of factorial (!) n cannot be negative.

Hence, n=3 is the correct answer.

Q. 2. If $\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$, find the value of x.

Answer : To Find: Value of n

Given: $\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$

Formula Used: $n! = (n) \times (n-1) \times (n-2) \times (n-3) \ldots\ldots\ldots 3 \times 2 \times 1$

Now, $\frac{1}{4!}+\frac{1}{5!}=\frac{x}{6!}$

$\Rightarrow \frac{1}{24}+\frac{1}{120}=\frac{x}{720}$ (4! = 24, 5! = 120)

$\Rightarrow \frac{5+1}{120}=\frac{x}{720}$

$\Rightarrow \frac{6}{120}=\frac{x}{720}$

$\Rightarrow x = 36$

Q. 3. How many 3-digit numbers are there with no digit repeated?

Answer : Given: We have 10 numbers i.e. 0,1,2,3,4,5,6,7,8,9

To Find: Number of 3-digit numbers formed with no repetition of digits.

Conditions: No digit is repeated

Let us represent the 3-digit number

9 ways	9 ways	8 ways

First place can be filled with 9 numbers, i.e. 1,2,3,4,5,6,7,8,9 (0 cannot be placed as it will make it a 2-digit number) = 9 ways

Second place can be filled with remaining 9 numbers (as one number is used already) = 9 ways

Similarly, third place can be filled with 8 numbers = 8 ways

Total number of 3-digit numbers which can be formed

$= 9 \times 9 \times 8 = 648$

Q. 4. How many 3-digit numbers above 600 can be formed by using the digits 2, 3, 4, 5, 6, if repetition of digits is allowed?

Answer : Given: We have 5 digits i.e. 2,3,4,5,6

To Find: Number of 3-digit numbers

Condition: (i) Number should be greater than 600

(ii) Repetition of digits is allowed

For forming a 3 digit number, we have to fill 3 vacant spaces.

But as the number should be above 600, hence the first place must be occupied with 6 only because no other number is greater than 6.

Let us represent the 3-digit number

6	2,3,4,5,6	2,3,4,5,6

So the first place is filled with 6 = 1 ways

Second place can be filled with 5 numbers = 5 ways

Third place can be filled with 5 numbers = 5 ways

Total number of ways = 1 × 5 × 5 = 25

Total number of 3-digit numbers above 600 which can be formed by using the digits 2, 3, 4, 5, 6 with repetition allowed is 25

Q. 5. How many numbers divisible by 5 and lying between 4000 and 5000 can be formed from the digits 4, 5, 6, 7, 8 if repetition of digits is allowed?

Answer : Given: We have 5 digits, i.e. 4,5,6,7,8

To Find: Number of numbers divisible by 5

Condition: (i) Number should be between 4000 and 5000

(ii) Repetition of digits is allowed

Here as the number is lying between 4000 and 5000, we can conclude that the number is of 4-digits and the number must be starting with 4.

Now, for a number to be divisible by 5 must ends with 5

Let us represent the 4-digit number

4	4,5,6,7,8	4,5,6,7,8	5

Therefore,

The first place is occupied by 4 = 1 way

The fourth (last) place is occupied by 5 = 1 way

The second place can be filled by 5 numbers = 5 ways

The third place can be filled by 5 numbers = 5 ways

Total numbers formed = 1 × 5 × 5 × 1 = 25

There are 25 numbers which are divisible by 5 and lying between 4000 and 5000 and can be formed from the digits 4, 5, 6, 7, 8 with repetition of digits.

Q. 6. In how many ways can the letters of the word 'CHEESE' be arranged?

Answer : Given: We have 6 letters

To Find: Number of words formed with Letter of the word 'CHEESE.'

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest if any, are of a different kind is $= \frac{n!}{p_1!p_2!\ldots\ldots\ldots\ldots p_k!}$

Suppose we have these words – C,H,E_1,E_2,S,E_3

Now if someone makes two words as $CHE_1E_3SE_2$ and $CHE_2E_3SE_1$

These two words are different because E_1, E2 and E_3 are different but we have three similar E's hence, in our case these arrangements will be a repetition of same words.

In the word CHEESE, 3 E's are similar

∴ n = 6, p_1 = 3

$$\Rightarrow \frac{6!}{3!} = \frac{720}{6} = 120$$

In 120 ways the letters of the word 'CHEESE' can be arranged.

Q. 7. In how many ways can the letters of the word 'PERMUTATIONS' be arranged if each word starts with P and ends with S?

Answer : Given: We have 12 letters

To Find: Number of words formed with Letter of the word 'PERMUTATIONS.'

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest if any, are of a different kind is $= \frac{n!}{p_1!p_2!\ldots\ldots\ldots p_k!}$

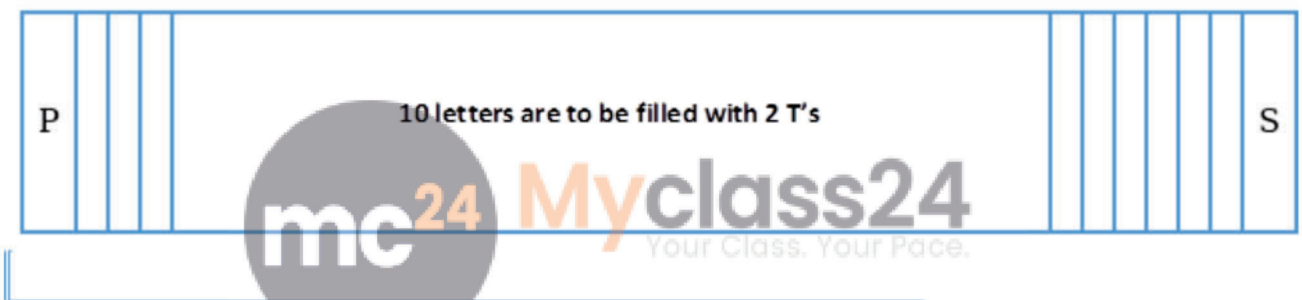
In the word 'PERMUTATIONS' we have 2 T's.

We have to start the word with P and end it with S, hence the first and last position is occupied with P and S respectively.

As two positions are occupied the remaining 10 positions are to be filled with 10 letters in which we have 2 T's.

NOTE:- Unless specified , assume that repetition is not allowed.

Let us represent the arrangement



Hence,

The first place is occupied by P = 1 way

The last place (12^{th}) is occupied by S = 1 way

For the remaining 10 places:

Using the above formula

Where,

n=10

p_1=2

$$\Rightarrow \frac{10!}{2!} = 1814400$$

Total number of ways are $1\times1814400\times1=1814400$ ways.

In 1814400 ways the letters of the word ‘PERMUTATIONS’ can be arranged if each word starts with P and ends with S.

Q. 8. How many different words can be formed by using all the letters of the word ‘ALLAHABAD’?

Answer : Given: We have 9 letters

To Find: Number of words formed with Letter of the word ‘ALLAHABAD.’

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest if any, are of a different kind is $=\frac{n!}{p_1!p_2!\ldots\ldots\ldots p_k!}$

‘ALLAHABAD’ consist of 9 letters out of which we have 4 A’s and 2 L’s.

Using the above formula

Where,

n=9

p_1=4

p_2=2

$\Rightarrow\frac{9!}{4!2!}=7560$

7560 different words can be formed by using all the letters of the word ‘ALLAHABAD.’

Q. 9. How many permutations of the letters of the word ‘APPLE’ are there?

Answer : Given: We have 5 letters

To Find: Number of words formed with Letter of the word ‘APPLE.’

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest if any, are of a different kind is $=\frac{n!}{p_1!p_2!\ldots\ldots\ldots p_k!}$

‘APPLE’ consists of 5 letters out of which we have 2 Ps.

Using the above formula

Where,

n=5

$p_1=2$

$$\Rightarrow \frac{5!}{2!} = 60$$

There are 60 permutations of the letters of the word ‘APPLE’.

Q. 10. How many words can be formed by the letters of the word ‘SUNDAY’?

Answer : Given: We have 6 letters

To Find: Number of words formed with Letter of the word ‘SUNDAY.’

‘SUNDAY’ consist of 6 letters.

NOTE: - Unless specified, assume that repetition is not allowed.

Let us represent the arrangement with an example

U (s,u,n,d,a,y)	N (s,n,d,a,y)	D (s,d,a,y)	A (s,a,y)	S (s,y)	Y

<u>6 ways</u> <u>5 ways</u> <u>4 ways</u> <u>3 ways</u> <u>2 ways</u> <u>1 way</u>

We have 6 places

First place can be filled with 6 letters, i.e. S,U,N,D,A,Y = 6 ways

Second place can be filled with 5 letters (as one letter is already used in the first place) = 5 ways

Similarly,

Third place can be filled with 4 letters = 4 ways

The fourth place can be filled with 3 letters = 3 ways

The fifth place can be filled with 2 letters = 2 ways

The sixth place can be filled with 1 letters = 1 ways

Total number of letters = 6 × 5 × 4 × 3 × 2 × 1 = 720

720 words can be formed by the letters of the word 'SUNDAY.'

Q. 11. In how many ways can 4 letters be posted in 5 letter boxes?

Answer : Given: We have 4 letters and 5 letter boxes

To Find: Number of ways of posting letters.

One letter can be posted in any of 5 letter boxes.

We have to assume that all the letters are different.

So for first letter i.e. L_1, we have 5 ways

Similarly for,

L_2=5 ways

L_3=5 ways

L_4=5 ways

Total number of ways = 5 × 5 × 5 × 5 = 625

In 625 ways 4 letters can be posted in 5 letter boxes.

Q. 12. In how many ways can 4 women draw water from 4 taps if no tap remains unused?

Answer : Given: We have 4 women and 4 taps

To Find: Number of ways of drawing water

Condition: No tap remains unused

Let us represent the arrangement

4 ways	3 ways	2 ways	1 way

The first woman can use any of the four taps = 4 Ways

The second woman can use the remaining three taps = 3 ways

The third woman can use the remaining two taps = 2 ways

The fourth woman can use the remaining one tap = 1 way

Total number of ways = 4 × 3 × 2 × 1 = 24

There is 24 number of ways in which 4 women can draw water from 4 taps such that no tap remains unused.

Q. 13. How many 5-digit numbers can be formed by using the digits 0, 1 and 2?

Answer : Given: We have 3 digits, i.e. 0, 1 and 2

To Find: Number of 5-digit numbers formed

Let us represent the arrangement

2 ways, i.e. 1,2	3 ways	3 ways	3 ways	3 ways

For forming a 5-digit number, we have to fill 5 vacant spaces.

But the first place cannot be filled with 0, hence for filling first place, we have only 1 and 2

First place can be filled with 2 numbers, i.e. 1, 2 = 2 ways

Second place can be filled with 3 numbers = 3 ways

Third place can be filled with 3 numbers = 3 ways

The fourth place can be filled with 3 numbers = 3 ways

The fifth place can be filled with 3 numbers = 3 ways

Total number of ways = 2 × 3 × 3 × 3 × 3 = 162

162 5-digit numbers can be formed by using the digits 0, 1 and 2.

Q. 14. In how many ways can 5 boys and 3 girls be seated in a row so that each girl is between 2 boys?

Answer : Given: We have 5 boys and 3 girls

To Find: Number of ways of seating so that 5 boys and 3 girls are seated in a row and each girl is between 2 boys

The formula used: The number of permutations of n different objects taken r at a time (object does not repeat) is $^nP_r = \frac{n!}{(n-r)!}$

The only arrangement possible is

B__B__B__B__B

Number of ways for boys $= {}^nP_r$

$= {}^5P_5$

$= \frac{5!}{(5-5)!}$

$= \frac{5!}{0!}$

=120

There are 3 girls, and they have 4 vacant positions

Number of ways for girls $= {}^4P_3 = 24$ ways

$$= \frac{4!}{(4-3)!}$$

$$= \frac{4!}{1!}$$

= 24

Total number of ways = 24 × 120 = 2880

In 2880 ways 5 boys and 3 girls can be seated in a row so that each girl is between 2 boys.

Q. 15. A child has plastic toys bearing the digits 4, 4 and 5. How many 3-digit numbers can he make using them?

Answer : Given: We have toys with bearing 4, 4 and 5

To Find: Number of 3-digit numbers he can make

The formula used: The number of permutations of n objects, where p_1 objects are of one kind, p_2 are of the second kind, ..., p_k is of a k^{th} kind and the rest, if any, are of a different kind is $= \frac{n!}{p_1!p_2!\ldots\ldots p_k!}$

The child has to form a 3-digit number.

Here the child has two 4's.

We have to use the above formula

Where,

n=3

p_1=2

$$\Rightarrow \frac{3!}{2!} = 3 \text{ ways}$$

The numbers are 544, 454 and 445.

He can make 3 3-digit numbers.

Q. 16. In how many ways can the letters of the word 'PENCIL' be arranged so that N is always next to E?

Answer : Given: We have 6 letters

To Find: Number of ways to arrange letters P,E,N,C,I,L

Condition: N is always next to E

Here we need EN together in all arrangements.

So, we will consider EN as a single letter.

Now, we have 5 letters, i.e. P,C,I,L and 'EN'.

5 letters can be arranged in ${}^{5}P_{5}$ways

$\Rightarrow {}^{5}P_{5}$

$\Rightarrow \frac{5!}{(5-5)!}$

$\Rightarrow \frac{5!}{0!}$

$\Rightarrow 120$

In 120 ways we can arrange the letters of the word 'PENCIL' so that N is always next to E.