

## QUADRILATERALS - CHAPTER 10

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### EXERCISE 10A

#### **Answer 1:**

Given: Three angles of a quadrilateral are  $75^\circ$ ,  $90^\circ$  and  $75^\circ$ .

Let the fourth angle be  $y$ .

Using angle sum property of quadrilateral,

$$75^\circ + 90^\circ + 75^\circ + y = 360^\circ$$

$$\Rightarrow 240^\circ + y = 360^\circ$$

$$\Rightarrow y = 360^\circ - 240^\circ$$

$$\Rightarrow y = 120^\circ$$

So, the measure of the fourth angle is  $120^\circ$

#### **Answer 2:**

Let  $\angle A = 2y^\circ$ .

Then  $\angle B = (4y)^\circ$ ;  $\angle C = (5y)^\circ$  and  $\angle D = (7y)^\circ$

Since the sum of the angles of a quadrilateral is  $360^\circ$ , as,

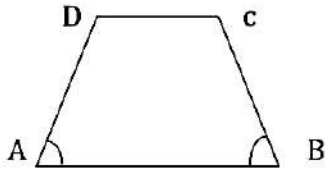
$$2y + 4y + 5y + 7y = 360^\circ$$

$$\Rightarrow 18y = 360^\circ$$

$$\Rightarrow y = 20^\circ$$

$$\therefore \angle A = 40^\circ; \angle B = 80^\circ; \angle C = 100^\circ; \angle D = 140^\circ$$

**Answer 3:**



Given ,  $AB \parallel DC$ . As we know that the interior angles on the same side of transversal line, then  $\angle A = 55^\circ$  and  $\angle B = 70^\circ$

$$\angle A + \angle D = 180^\circ$$

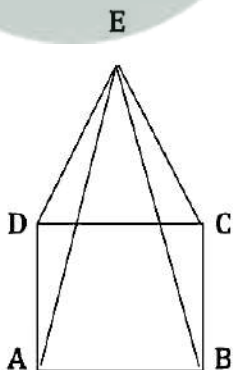
$$\Rightarrow \angle D = 180^\circ - \angle A = 180^\circ - 55^\circ = 125^\circ$$

$$\text{Also, } \angle B + \angle C = 180^\circ$$

$$\Rightarrow \angle C = 180^\circ - \angle B = 180^\circ - 70^\circ = 110^\circ$$

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**Answer 4:**



Given: ABCD is a square in which  $AB = BC = CD = DA$ .  $\triangle EDC$  is an equilateral triangle in which  $ED = EC = DC$  and  $\angle EDC = \angle DEC = \angle DCE = 60^\circ$ .

To prove:  $AE = BE$  and  $\angle DAE = 15^\circ$

Proof: In  $\triangle ADE$  and  $\triangle BCE$ , as,

$AD = BC$  [Sides of a square]

$DE = EC$  [Sides of an equilateral triangle]

$\angle ADE = \angle BCE = 90^\circ + 60^\circ = 150^\circ$

$\therefore \triangle ADE \cong \triangle BCE$

i.e.,  $AE = BE$

Now,  $\angle ADE = 150^\circ$

$DA = DC$  [Sides of a square]

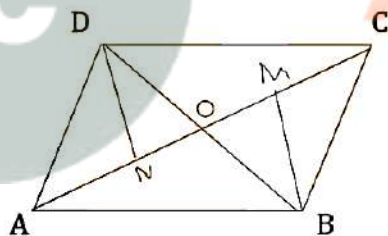
$DC = DE$  [Sides of an equilateral triangle]

So,  $DA = DE$

$\triangle ADE$  and  $\triangle BCE$  are isosceles triangles.

i.e.,  $\angle DAE = \angle DEA = \frac{1}{2}(180^\circ - 150^\circ) = \frac{30}{2} = 15^\circ$

Answer 5:



Given: by fig, both the diagonals intersect at  $O$  and  $BM \perp AC$  then

Let the diagonals intersect each other at  $O$

Now, in  $\triangle OND$  and  $\triangle OMB$ ,

$\angle OND = \angle OMB$  ( $90^\circ$  each)

$\angle DON = \angle BOM$  (Vertically opposite angles)

Also,  $DN = BM$  (Given)

As we know that by parallelogram

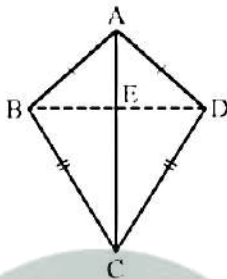
$$\triangle OND \cong \triangle OMB$$

$$\therefore OD = OB$$

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Hence, AC bisects BD.

**Answer 6:**



Given: ABCD is a quadrilateral in which  $AB = AD$  and  $BC = DC$

(i) To prove : AC bisects  $\angle A$  and  $\angle C$

In  $\triangle ABC$  and  $\triangle ADC$ ,

$$AB = AD$$

$$BC = DC$$

AC is common in both the triangles.

i.e.,  $\triangle ABC \cong \triangle ADC$  (SSS congruence rule)

$$\therefore \angle BAC = \angle DAC \text{ and } \angle BCA = \angle DCA \quad (\text{By CPCT})$$

Hence proved, AC bisects both the angles,  $\angle A$  and  $\angle C$ .

(ii) To prove  $BE = DE$

In  $\triangle ABE$  and  $\triangle ADE$ ,

$$AB = AD$$

$$\angle BAE = \angle DAE$$

AE is common.

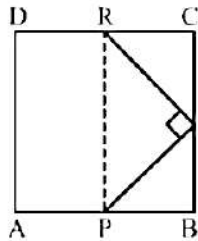
$$\therefore \triangle ABE \cong \triangle ADE \quad (\text{SAS congruence rule})$$

$$\Rightarrow \text{hence proved } BE = DE$$

(iii) To prove :  $\angle ABC = \angle ADC$

$\triangle ABC \cong \triangle ADC$  (Given)  
Hence proved,  $\angle ABC = \angle ADC$

**Answer 7:**



Given: ABCD is a square and  $\angle PQR = 90^\circ$ .

$$PB = QC = DR$$

(i) To prove:  $QB = DR$

$$\therefore BC = CD \quad (\text{Sides of square})$$

$$\text{and } CQ = DR \quad (\text{Given})$$

$$\text{so, by fig } BC = BQ + CQ$$

$$\Rightarrow CQ = BC - BQ$$

$$\therefore DR = BC - BQ \quad \dots(i)$$

$$\text{Also, } CD = RC + DR$$

$$\therefore DR = CD - RC = BC - RC \quad \dots(ii)$$

From (i) and (ii), we get

$$BC - BQ = BC - RC$$

$$\therefore BQ = RC$$

(ii) To prove,  $PQ = QR$

In  $\triangle RCQ$  and  $\triangle QBP$ ,

$$PB = QC \quad (\text{Given})$$

$$BQ = RC \quad (\text{Given})$$

$$\angle RCQ = \angle QBP \quad (90^\circ \text{ each})$$

By parallelogram theorem

$$\triangle RCQ \cong \triangle QBP \quad (\text{SAS congruence rule})$$

$$\therefore QR = PQ \quad \text{hence proved}$$

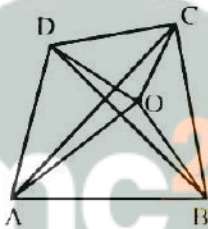
(iii) To prove,  $\angle QPR = 45^\circ$

$\triangle RCQ \cong \triangle QBP$  and  $QR = PQ$

$$\therefore \text{In } \triangle RPQ, \angle QPR = \angle QRP = \frac{1}{2}(180^\circ - 90^\circ) = \frac{90}{2} = 45^\circ$$

Hence proved,  $\angle QPR = 45^\circ$

**Answer 8:**



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Let ABCD be a quadrilateral with diagonals AC and BD and O is a point within the quadrilateral.

Suppose

In  $\triangle AOC$ ,  $OA + OC > AC$ .....(1)

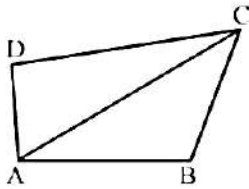
And, in  $\triangle BOD$ ,  $OB + OD > BD$ .....(2)

Adding these ,

$$(OA + OC) + (OB + OD) > (AC + BD)$$

$$\Rightarrow OA + OB + OC + OD > AC + BD$$

**Answer 9:**



Given: ABCD is a quadrilateral and AC is its diagonal.

(i) As sum of any two sides of any triangle is greater than the third side.

$$\text{In } \triangle ABC, AB + BC > AC \quad \dots(1)$$

$$\text{In } \triangle ACD, CD + DA > AC \quad \dots(2)$$

Adding (1) and (2),

$$AB + BC + CD + DA > 2AC \dots\dots\dots\text{hence proved}$$

(ii) In  $\triangle ABC$ ,

$$AB + BC > AC \quad \dots(1)$$

In  $\triangle ACD$ ,

$$AC > |DA - CD| \quad \dots(2)$$

From (1) and (2),

$$AB + BC > |DA - CD|$$

$$\Rightarrow AB + BC + CD > DA \dots\dots\dots\text{hence proved}$$

(iii) In  $\triangle ABC$ , we know that  $AB + BC > AC$

Same as, In  $\triangle ACD$ ,  $CD + DA > AC$

And

In  $\triangle BCD$ ,

$$BC + CD > BD$$

In  $\triangle ABD$ ,

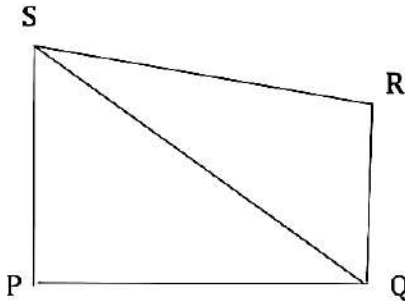
$$DA + AB > BD$$

Adding these ,

$$2(AB + BC + CD + DA) > 2(AC + BD)$$

$$\Rightarrow (AB + BC + CD + DA) > (AC + BD)$$

Answer 10:



Let PQRS be a quadrilateral and  $\angle 1, \angle 2, \angle 3$  and  $\angle 4$  are its four angles .  
Join QR which divides PQRS in two triangles,  $\Delta PQR$  and  $\Delta QRS$ .

In  $\Delta PQR$ ,

$$\angle 1 + \angle 2 + \angle P = 180^\circ \quad \dots(i)$$

In  $\Delta QRS$ ,

$$\angle 3 + \angle 4 + \angle R = 180^\circ \quad \dots(ii)$$

On adding (i) and (ii),

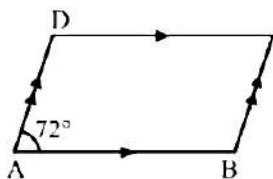
$$(\angle 1 + \angle 3) + \angle P + \angle R + (\angle 4 + \angle 2) = 360^\circ$$
$$\Rightarrow \angle P + \angle R + \angle Q + \angle S = 360^\circ \quad \therefore \angle 1 + \angle 3 = \angle Q ; \angle 4 + \angle 2 = \angle S$$

Hence proved

$$\therefore \angle P + \angle R + \angle Q + \angle S = 360^\circ$$

## EXERCISE 10B

Answer 1:



Given, ABCD is parallelogram and  $\angle A = 72^\circ$ .

Then, as we know that opposite angles are equals.

$$\therefore \angle A = \angle C \text{ and } \angle B = \angle D$$

$$\therefore \angle C = 72^\circ$$

$\angle A$  and  $\angle B$  are the adjacent angles.

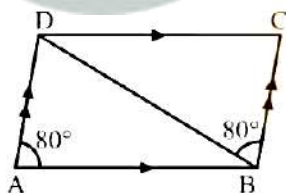
$$\text{as, } \angle A + \angle B = 180^\circ$$

$$\Rightarrow \angle B = 180^\circ - \angle A = 180^\circ - 72^\circ = 108^\circ$$

As above,  $\angle B = \angle D = 108^\circ$

Hence,  $\angle B = \angle D = 108^\circ$  and  $\angle C = 72^\circ$

Answer 2:



Given: ABCD is parallelogram and  $\angle DAB = 80^\circ$  and  $\angle DBC = 60^\circ$

To find: Measure of  $\angle CDB$  and  $\angle ADB$

In parallelogram ABCD,  $AD \parallel BC$

$$\therefore \angle DBC = \angle ADB = 60^\circ \text{ (Alternate interior angles) } \dots(i)$$

As  $\angle DAB$  and  $\angle ADC$  are the adjacent angles,

$$\angle DAB + \angle ADC = 180^\circ$$

$$\therefore \angle ADC = 180^\circ - \angle DAB = 180^\circ - 80^\circ = 100^\circ$$

Also,  $\angle ADC = \angle ADB + \angle CDB$

$$\therefore \angle ADC = 100^\circ$$

Then,

$$\Rightarrow \angle ADB + \angle CDB = 100 \quad \dots(\text{ii})$$

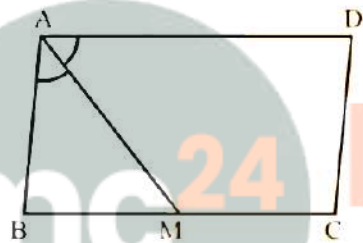
From (i) and (ii),

$$60^\circ + \angle CDB = 100^\circ$$

$$\Rightarrow \angle CDB = 100^\circ - 60^\circ = 40$$

Hence,  $\angle CDB = 40^\circ$  and  $\angle ADB = 60^\circ$

**Answer 3:**



Given: parallelogram ABCD, M is the midpoint of side BC and  $\angle BAM = \angle DAM$ .

To prove:  $AD = 2CD$

Proof:

Since,  $AD \parallel BC$  and AM is the transversal.

So,  $\angle DAM = \angle AMB$  (Alternate interior angles)

But,  $\angle DAM = \angle BAM$  (Given)

Thus,  $\angle AMB = \angle BAM$

$$\Rightarrow AB = BM$$

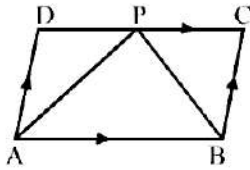
As we know angles opposite to equal sides are equal and opposite sides of parallelogram are equal

Now,  $AB = CD$

$$\Rightarrow 2AB = 2CD$$

So,  $\Rightarrow (AB + AB) = 2CD$   
 $\Rightarrow BM + MC = 2CD$  (AB = BM and MC = BM)  
 $\Rightarrow BC = 2CD$   
 $\therefore AD = 2CD$  (AD=BC) hence proved

**Answer 4:**



ABCD is a parallelogram.

$\therefore \angle A = \angle C$  and  $\angle B = \angle D$  (Opposite angles)

And  $\angle A + \angle B = 180^\circ$  (Adjacent angles are supplementary)

$\therefore \angle B = 180^\circ - \angle A$

$\Rightarrow 180^\circ - 60^\circ = 120^\circ$  ( $\angle A = 60^\circ$ )

$\therefore \angle A = \angle C = 60^\circ$  and  $\angle B = \angle D = 120^\circ$

(i) In  $\Delta APB$ ,  $\angle PAB = \frac{60}{2} = 30^\circ$

and  $\angle PBA = \frac{120}{2} = 60^\circ$

$\therefore \angle APB = 180^\circ - (30^\circ + 60^\circ) = 90^\circ$

(ii) In  $\Delta ADP$ ,  $\angle PAD = 30^\circ$  and  $\angle ADP = 120^\circ$

$\therefore \angle APD = 180^\circ - (30^\circ + 120^\circ) = 30^\circ$

Thus,  $\angle PAD = \angle APD = 30^\circ$

Hence,  $\Delta ADP$  is an isosceles triangle and  $AD = DP$ .

In  $\Delta PBC$ ,  $\angle PBC = 60^\circ$ ,  $\angle BPC = 180^\circ - (90^\circ + 30^\circ) = 60^\circ$  and  $\angle BCP = 60^\circ$

(Opposite angle of  $\angle A$ )

$\therefore \angle PBC = \angle BPC = \angle BCP$

Hence,  $\Delta PBC$  is an equilateral triangle and, therefore,  $PB = PC = BC$ .

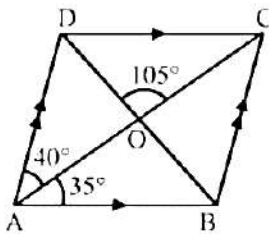
(iii)  $DC = DP + PC$

From (ii), as,

$$\begin{aligned} DC &= AD + BC \\ \Rightarrow DC &= AD + AD \\ \Rightarrow DC &= 2 AD \end{aligned}$$

[AD = BC, opposite sides of a parallelogram]

**Answer 5:**



ABCD is a parallelogram.

$\therefore AB \parallel DC$  and  $BC \parallel AD$

(i) In  $\triangle AOB$ ,  $\angle BAO = 35^\circ$ ,

As we know that, vertically opposite angles are equals

$$\angle AOB = \angle COD = 105^\circ$$

$$\therefore \angle ABO = 180^\circ - (35^\circ + 105^\circ) = 40^\circ$$

(ii) As we know that these angles are  $\angle ODC$  and  $\angle ABO$  are alternate interior angles.

$$\therefore \angle ODC = \angle ABO = 40^\circ$$

(iii) These are Alternate interior angles

$$\angle ACB = \angle CAD = 40^\circ$$

(iv) In  $\triangle ABC$ , we get

$$\angle CBD = \angle ABC - \angle ABD \quad \dots(i)$$

$$\angle ABC = 180^\circ - \angle BAD \quad (\text{Adjacent angles are supplementary})$$

$$\Rightarrow \angle ABC = 180^\circ - 75^\circ = 105^\circ$$

In  $\triangle CBD$ , we have

$$\text{Then, } \angle CBD = \angle ABC - \angle ABD$$

$$\Rightarrow \angle CBD = 105^\circ - \angle ABD \quad (\angle ABD = \angle ABO)$$

$$\Rightarrow \angle CBD = 105^\circ - 40^\circ = 65^\circ$$

### Answer 6:

ABCD is a parallelogram.

i.e.,  $\angle A = \angle C$  and  $\angle B = \angle D$  (Opposite angles)

Also,  $\angle A + \angle B = 180^\circ$  (Adjacent angles are supplementary)

$$\therefore (2x + 25)^\circ + (3x - 5)^\circ = 180^\circ$$

$$\Rightarrow 5x + 20 = 180^\circ$$

$$\Rightarrow 5x = 180 - 20$$

$$\Rightarrow 5x = 160^\circ$$

$$\Rightarrow x = \frac{160}{5} = 32^\circ$$

$$\therefore \angle A = 2 \times 32 + 25 = 89^\circ \text{ and } \angle B = 3 \times 32 - 5 = 91^\circ$$

Hence,  $x = 32^\circ$ ,  $\angle A = \angle C = 89^\circ$  and  $\angle B = \angle D = 91^\circ$

### Answer 7:

Let PQRS be a parallelogram.

$$\therefore \angle P = \angle R \text{ and } \angle Q = \angle S$$

$$\text{Let } \angle P = y^\circ \text{ and } \angle B = \left(\frac{4y}{5}\right)^\circ$$

$$\text{Now, } \angle P + \angle Q = 180^\circ$$

$$\Rightarrow y + \left(\frac{4y}{5}\right)^\circ = 180^\circ \Rightarrow \left(\frac{9y}{5}\right)^\circ = 180^\circ \Rightarrow y = 100^\circ$$

$$\text{Now, } \angle P = 100^\circ \text{ and } \angle B = \left(\frac{4}{5}\right) \times 100^\circ = 80^\circ$$

Hence,  $\angle P = \angle R = 100^\circ$ ;  $\angle B = \angle S = 80^\circ$

**Answer 8:**

Let PQRS be a parallelogram.

$$\therefore \angle P = \angle R \text{ and } \angle Q = \angle S \quad (\text{Opposite angles})$$

Let  $\angle P$  be the smallest angle whose measure is  $y^\circ$ .

$$\therefore \angle Q = (2y - 30)^\circ$$

Now,  $\angle P + \angle Q = 180^\circ$  (Adjacent angles are supplementary)

$$\Rightarrow y + 2y - 30^\circ = 180^\circ$$

$$\Rightarrow 3y = 210^\circ$$

$$\Rightarrow y = \frac{210}{3} = 70$$

$$\Rightarrow y = 70^\circ$$

$$\therefore \angle Q = 2 \times 70^\circ - 30^\circ = 110^\circ$$

Hence,  $\angle P = \angle R = 70^\circ$ ;  $\angle Q = \angle S = 110^\circ$

**Answer 9:**

ABCD is a parallelogram.

The opposite sides of a parallelogram are parallel and equal.

$$\therefore AB = DC = 9.5 \text{ cm}$$

Let  $BC = AD = y$

$$\therefore \text{Perimeter of ABCD} = AB + BC + CD + DA = 30 \text{ cm}$$

$$\Rightarrow 9.5 + y + 9.5 + y = 30$$

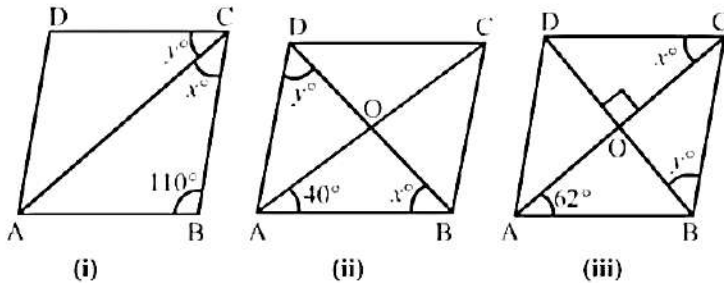
$$\Rightarrow 19 + 2y = 30$$

$$\Rightarrow 2y = 11$$

$$\Rightarrow y = \frac{11}{2} = 5.5 \text{ cm}$$

Hence,  $AB = DC = 9.5 \text{ cm}$  and  $BC = DA = 5.5 \text{ cm}$

**Answer 10:**



ABCD is a rhombus and a rhombus is also a parallelogram. A rhombus has four equal sides.

(i) In  $\triangle ABC$ ,

$$\angle BAC = \angle BCA = \frac{1}{2}(180 - 110)^\circ = 35^\circ$$

i.e.,  $x = 35^\circ$

Now by Adjacent angles are supplementary we get,

$$\angle B + \angle C = 180^\circ$$

$$\text{As, } \angle C = x + y = 70^\circ$$

$$\Rightarrow y = 70^\circ - x$$

$$\Rightarrow y = 70^\circ - 35^\circ = 35^\circ$$

Hence,  $x = 35^\circ$ ;  $y = 35^\circ$

(ii) The diagonals of a rhombus are perpendicular bisectors of each other.

So, in  $\triangle AOB$ ,  $\angle OAB = 40^\circ$ ,  $\angle AOB = 90^\circ$  and

$$\angle ABO + \angle BOA + \angle OAB = 180$$

$$\angle ABO = 180^\circ - (40^\circ + 90^\circ) = 50^\circ$$

$$\therefore x = 50^\circ$$

In  $\triangle ABD$ ,  $AB = AD$

So,  $\angle ABD = \angle ADB = 50^\circ$

Hence,  $x = 50^\circ$ ;  $y = 50^\circ$

(iii)  $\angle BAC = \angle DCA$  (Alternate interior angles)

i.e.,  $x = 62^\circ$

In  $\triangle BOC$ ,  $\angle BCO = 62^\circ$

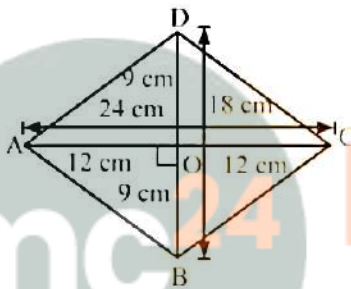
Also,  $\angle BOC = 90^\circ$

$\angle BCO + \angle BOC + \angle OBC = 180$

$$\therefore \angle OBC = 180^\circ - (90^\circ + 62^\circ) = 28^\circ$$

Hence,  $x = 62^\circ$ ;  $y = 28^\circ$

**Answer 11:**



Let PQRS be a rhombus.

$\therefore PQ = QR = RS = SP$

Here, PR and QS are the diagonals of PQRS, where  $PR = 24$  cm and  $QS = 18$  cm.

Let the diagonals intersect each other at M.

$\therefore \triangle PMQ$  is a right angle triangle in which  $MP = \frac{AC}{2} = \frac{24}{2} = 12$  cm and  $MQ =$

$$\frac{QS}{2} = \frac{18}{2} = 9 \text{ cm.}$$

Now,  $PQ^2 = MP^2 + MQ^2$  [Pythagoras theorem]

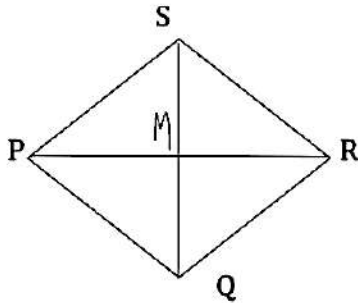
$$\Rightarrow PQ^2 = (12)^2 + (9)^2$$

$$\Rightarrow PQ^2 = 144 + 81 = 225$$

$$\Rightarrow PQ = 15 \text{ cm}$$

Hence, the side of the rhombus is 15 cm.

Answer 12:



Let PQRS be a rhombus.

$\therefore PQ = QR = RS = SP = 10 \text{ cm}$

Let PR and QS are the diagonals of PQRS. Let  $PR = y$  and  $QS = 16 \text{ cm}$  and M be the intersection point of the diagonals.

$\therefore \triangle PMQ$  is a right angle triangle, in which

$$MP = \frac{PR}{2} = \frac{y}{2} \text{ and } MQ = \frac{QS}{2} = \frac{16}{2} = 8 \text{ cm.}$$

Now,  $PQ^2 = MP^2 + MQ^2$  [Pythagoras theorem]

$$\Rightarrow 10^2 = \left(\frac{y}{2}\right)^2 + 8^2 \Rightarrow 100 - 64 = \frac{y^2}{4} \Rightarrow 36 \times 4 = y^2$$

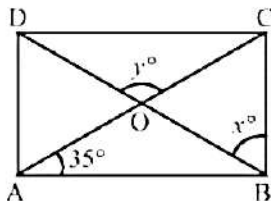
$$\Rightarrow y^2 = 144$$

$$\therefore y = 12 \text{ cm}$$

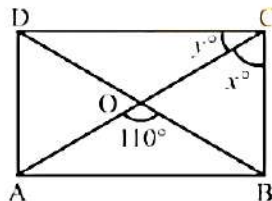
Hence, the other diagonal of the rhombus is 12 cm.

$\therefore$  Area of the rhombus =  $12 \times (12 \times 16) = 96 \text{ cm}^2$

Answer 13:



(i)



(ii)

(i) ABCD is a rectangle.

The diagonals of a rectangle are congruent and bisect each other. Therefore, in  $\Delta AOB$ , as,

$$OA = OB$$

$$\therefore \angle OAB = \angle OBA = 35^\circ$$

$$\therefore x = 90^\circ - 35^\circ = 55^\circ$$

In  $\Delta AOB$

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$\text{And } \angle AOB = 180^\circ - (35^\circ + 35^\circ) = 110^\circ$$

$$\therefore y = \angle AOB = 110^\circ \quad [\text{Vertically opposite angles}]$$

$$\text{Hence, } x = 55^\circ \text{ and } y = 110^\circ$$

(ii) In  $\Delta AOB$ , as,

$$\text{Given, } \angle AOB = 100^\circ$$

$$OA = OB$$

$$\text{As, } \angle OAB = \angle OBA$$

$$\text{Then, } \angle AOB + \angle OBA + \angle OAB = 180$$

$$\Rightarrow 2\angle AOB = 180 - \angle AOB \dots\dots\dots(\angle OAB = \angle OBA)$$

$$\Rightarrow 2\angle AOB = 180 - 110 = 70^\circ$$

$$\Rightarrow \angle AOB = \frac{1}{2} \times 70 = 35^\circ$$

$$\text{so, } \therefore y = \angle BAC = 35^\circ \quad [\text{Interior alternate angles}]$$

Here at  $\angle C$  is at right angle  $\Delta$  by fig,

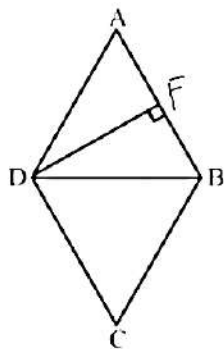
$$\Rightarrow 90^\circ = x + y$$

$$\Rightarrow x = 90^\circ - y$$

$$\Rightarrow x = 90^\circ - 35^\circ = 55^\circ$$

$$\text{Thus, } x = 55^\circ \text{ and } y = 35^\circ$$

Answer 14:



Given: ABCD is a rhombus, DF is altitude which bisects AB i.e.  $AF = FB$

In  $\triangle AFD$  and  $\triangle BFD$ ,

$DF = DF$  (Common side)

$\angle DFA = \angle DFB = 90^\circ$  (Given)

$AF = FB$  (Given)

$\therefore \triangle AFD \cong \triangle BFD$  (By SAS congruence Criteria)

$\Rightarrow AD = BD$  (CPCT)

Also,  $AD = AB$  (Sides of rhombus are equal)

$\Rightarrow AD = AB = BD$

Thus,  $\triangle ABD$  is an equilateral triangle.

Therefore,  $\angle A = 60^\circ$

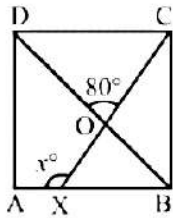
$\Rightarrow \angle C = \angle A = 60^\circ$  (Opposite angles of rhombus are equal)

And,  $\angle ABC + \angle BCD = 180^\circ$  (Adjacent angles of rhombus are supplementary.)

$\Rightarrow \angle ABC + 60^\circ = 180^\circ \Rightarrow \angle ABC = 180^\circ - 60^\circ \Rightarrow \angle ABC = 120^\circ \Rightarrow \angle ADC = \angle ABC = 120^\circ$

Hence, the angles of rhombus are  $60^\circ, 120^\circ, 60^\circ$  and  $120^\circ$

**Answer 15:**



The angles of a square are bisected by the diagonals.

$$\angle OBX = \frac{1}{2} \times \angle CBA = \frac{1}{2} \times 90 = 45^\circ$$

$$\therefore \angle OBX = 45^\circ$$

$$\text{Given, } \angle COD = 80^\circ$$

$$\text{And } \angle BOX = \angle COD = 80^\circ \quad [\text{Vertically opposite angles}]$$

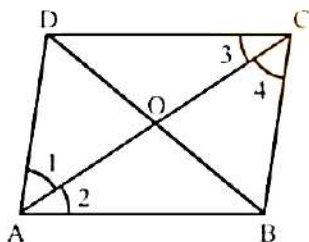
$\therefore$  In  $\triangle BOX$ , as we know that exterior angle is sum of both interior angles.

$$\angle AXO = \angle OBX + \angle BOX$$

$$\Rightarrow \angle AXO = 45^\circ + 80^\circ = 125^\circ$$

$$\therefore x = 125^\circ$$

**Answer 16:**



Given: A rhombus ABCD.

To prove: Diagonal AC bisects  $\angle A$  as well as  $\angle C$  and diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

Proof:

In  $\triangle ABC$ ,

$AB = BC$  (Sides of rhombus are equal.)

$\angle ACB = \angle CAB$  (Angles opposite to equal sides are equal.) ... (1)

$AD \parallel BC$  (Opposite sides of rhombus are parallel.)

AC is transversal.

$\angle DAC = \angle ACB$  (Alternate interior angles) ... (2)

From (1) and (2),

$\angle DAC = \angle CAB$

Thus, AC bisects  $\angle A$ .

As,  $AB \parallel DC$  and AC is transversal.

$\angle CAB = \angle DCA$  (Alternate interior angles) ... (3)

From (1) and (3),

$\angle ACB = \angle DCA$

Thus, AC bisects  $\angle C$ .

Thus, AC bisects  $\angle C$  and  $\angle A$

In  $\triangle DAB$ ,

$AD = AB$  (Sides of rhombus are equal.)

$\angle ADB = \angle ABD$  (Angles opposite to equal sides are equal.) ... (4)

Also,

$DC \parallel AB$  (Opposite sides of rhombus are parallel.)

BD is transversal.

$\angle CDB = \angle DBA$  (Alternate interior angles) ... (5)

From (4) and (5),

$\angle ADB = \angle CDB$

Therefore, DB bisects  $\angle D$ .

As,  $AD \parallel BC$  and BD is transversal.

$\angle CBD = \angle ADB$  (Alternate interior angles) ... (6)

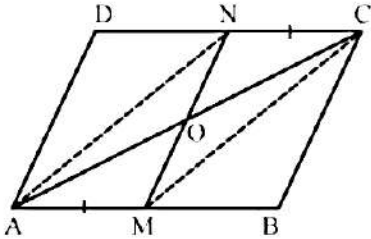
From (4) and (6)

$\angle CBD = \angle ABD$

Therefore, BD bisects  $\angle B$ .

Thus, BD bisects  $\angle D$  and  $\angle B$ .

**Answer 17:**



Given: In a parallelogram ABCD,  $AM = CN$ .

To prove: AC and MN bisect each other.

Construction: Join AN and MC.

Proof:

As, ABCD is a parallelogram.

$\Rightarrow AB \parallel DC \Rightarrow AM \parallel NC$

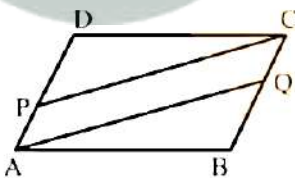
And,  $AM = CN$  (Given)

Therefore, AMCN is a parallelogram.

As, the diagonals of a parallelogram bisect each other.

Thus, AC and MN also bisect each other.

**Answer 18:**



As, per by given fig,

$\angle B = \angle D$  [Opposite angles of parallelogram ABCD]

$AD = BC$  and  $AB = DC$  [Opposite sides of parallelogram ABCD]

Also,  $AD \parallel BC$  and  $AB \parallel DC$

Given,  $AP = \frac{1}{3}AD$  and  $CQ = \frac{1}{3}BC$

So, we get

$$\therefore AP = CQ \quad [AD = BC]$$

In  $\triangle DPC$  and  $\triangle BQA$ ,

$$AB = CD, \angle B = \angle D \text{ and } DP = QB \quad [DP = \frac{2}{3}AD \text{ and } QB = \frac{2}{3}BC]$$

i.e.,  $\triangle DPC \cong \triangle BQA$

$$\therefore PC = QA$$

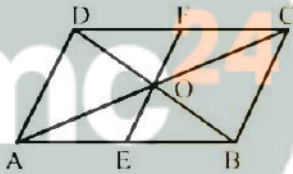
Thus, in quadrilateral AQCP,

$$AP = CQ \quad \dots(i)$$

$$PC = QA \quad \dots(ii)$$

$\therefore$  AQCP is a parallelogram.

Answer 19:



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**Given,** ABCD is a parallelogram whose diagonals intersect each other at O. A line segment EOF is drawn to meet AB at E and DC at F.

So in  $\triangle ODF$  and  $\triangle OBE$ ,

$$OD = OB \quad (\text{Diagonals bisect each other})$$

$$\angle DOF = \angle BOE \quad (\text{Vertically opposite angles})$$

$$\angle FDO = \angle OBE \quad (\text{Alternate interior angles})$$

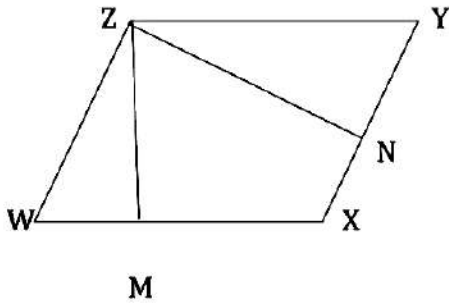
By parallelogram theorem

$$\triangle ODF \cong \triangle OBE$$

$$\therefore OF = OE$$

Hence, proved.

Answer 20:



Given:  $\square$  parallelogram WXYZ,  $ZM \perp WX$ ,  $WN \perp XY$  and  $\angle MZN = 60^\circ$

In quadrilateral ZMXN, by angle sum property,

$$\angle MZN + \angle ZMX + \angle X + \angle XNZ = 360^\circ$$

$$\Rightarrow 60^\circ + 90^\circ + \angle X + 90^\circ = 360^\circ$$

$$\Rightarrow \angle X = 360^\circ - 240^\circ \Rightarrow \angle X = 120^\circ \Rightarrow \angle X = 120^\circ$$

Also,

$$\angle X = \angle Z = 120^\circ \quad (\text{Opposite angles of a parallelogram are equal.})$$

$$\angle W + \angle X = 180^\circ \quad (\text{Adjacent angles of a parallelogram are supplementary.})$$

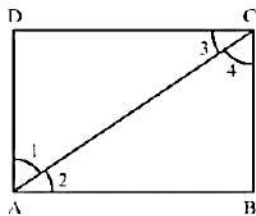
$$\Rightarrow \angle W + 120^\circ = 180^\circ \Rightarrow \angle W = 180^\circ - 120^\circ \Rightarrow \angle W = 60^\circ$$

Also,

$$\angle W = \angle Y = 60^\circ \quad (\text{Opposite angles of a parallelogram are equal.})$$

Thus, the angles of a parallelogram are  $60^\circ$ ,  $120^\circ$ ,  $60^\circ$  and  $120^\circ$ .

Answer 21:



Given: In

rectangle ABCD, AC bisects  $\angle A$ , i.e.  $\angle DAC = \angle CAB$  and AC bisects  $\angle C$ , i.e.  $\angle DCA = \angle ACB$ .

To prove:

- (i) ABCD is a square,
- (ii) diagonal BD bisects  $\angle B$  as well as  $\angle D$ .

Proof:

(i) Since,  $AD \parallel BC$  (Opposite sides of a rectangle are parallel.)

So,  $\angle DAC = \angle ACB$  (Alternate interior angles)

But,  $\angle DAC = \angle CAB$  (Given)

So,  $\angle CAB = \angle ACB$

In  $\triangle ABC$ ,

Since,  $\angle CAB = \angle ACB$

So,  $BC = AB$  (Sides opposite to equal angles are equal.)

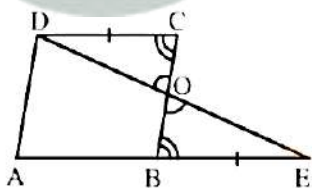
But these are adjacent sides of the rectangle ABCD.

Hence, ABCD is a square.

(ii) Since, the diagonals of a square bisect its angles.

So, diagonals BD bisect  $\angle B$  as well as  $\angle D$ .

**Answer 22:**



**Given,** ABCD is parallelogram in which AB is produced to E.

$BE = AB$  (given)

So in  $\triangle ODC$  and  $\triangle OEB$ , as,

$DC = BE$  (DC = AB)

$$\angle OCD = \angle OBE \quad (\text{Alternate interior angles})$$

$$\angle COD = \angle BOE \quad (\text{Vertically opposite angles})$$

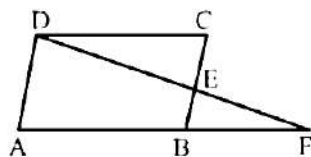
by parallelogram theorem we get,

$$\therefore \triangle ODC \cong \triangle OEB$$

$$\Rightarrow OC = OB$$

Hence, ED bisects BC.

**Answer 23:**



**Given:** ABCD is a parallelogram.

$$BE = CE$$

DE and AB when produced meet at F.

**To prove:**  $AF = 2AB$

**Proof:** In parallelogram ABCD, as,

$$AB \parallel DC$$

$$\angle DCE = \angle EBF \quad (\text{Alternate interior angles})$$

In  $\triangle DCE$  and  $\triangle BFE$ ,

$$\angle DCE = \angle EBF \quad (\text{Proved above})$$

$$\angle DEC = \angle BEF \quad (\text{Vertically opposite angles})$$

And,  $BE = CE$  (Given)

By parallelogram theorem

$$\therefore \triangle DCE \cong \triangle BFE$$

hence  $\therefore DC = BF$

But  $DC = AB$ , as ABCD is a parallelogram.

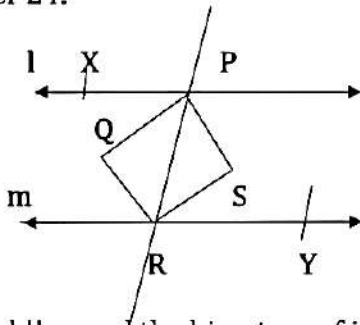
$$\therefore DC = AB = BF \quad \dots(i)$$

can also be written as,  $AF = AB + BF \quad \dots(ii)$

$$AF = AB + AB = 2AB \quad \dots\text{from (i)}$$

Hence, proved.  $AF = 2AB$ .

Answer 24:



Given:  $l \parallel m$  and the bisectors of interior angles intersect at X and Y.  
To prove: PQRS is a rectangle.

Proof:

Since,  $l \parallel m$  (Given)

So,  $\angle XPR = \angle PRY$  (Alternate interior angles)

$$\Rightarrow \frac{1}{2} \angle XPR = \frac{1}{2} \angle PRY$$

$\Rightarrow \angle QPR = \angle PRS$  but, these are a pair of alternate interior angles for PQ and RS.

$\Rightarrow PQ \parallel SR$

Similarly,  $PR \parallel QS$

So, PQRS is a parallelogram.

Also,

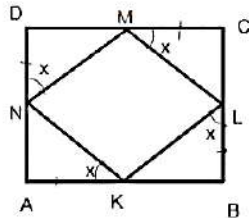
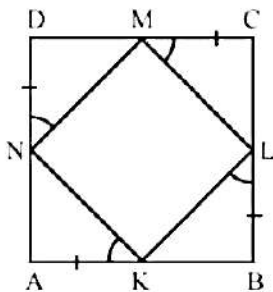
$$\angle XPR + \angle RPZ = 180^\circ \quad (\text{Linear pair})$$

$$\Rightarrow \frac{1}{2} \angle XPR + \frac{1}{2} \angle PRY = 90^\circ \Rightarrow \angle QPR + \angle RPS = 90^\circ \Rightarrow \angle QPS = 90^\circ$$

But, this is an angle of the parallelogram PQRS

Hence, PQRS is a rectangle.

Answer 25:



Given: In square ABCD,  $AK = BL = CM = DN$ .

To prove: KLMN is a square.

Proof:

In square ABCD,

$AB = BC = CD = DA$  (All sides of a square are equal.)

And,  $AK = BL = CM = DN$  (Given)

So,  $AB - AK = BC - BL = CD - CM = DA - DN$

$\Rightarrow KB = CL = DM = AN$  ... (1)

In  $\triangle NAK$  and  $\triangle KBL$ ,

$\angle NAK = \angle KBL = 90^\circ$  (Each angle of a square is a right angle.)

$AK = BL$  (Given)

$AN = KB$  [From (1)]

So, by parallelogram theorem,

$\triangle NAK \cong \triangle KBL$

$\Rightarrow NK = KL$  (CPCT) ... (2)

Similarly,

$\triangle MDN \cong \triangle NAK$   $\triangle DNM \cong \triangle CML$   $\triangle MCL \cong \triangle LBN$

$\Rightarrow MN = NK$  and  $\angle DNM = \angle KNA$  (CPCT) ... (3)

$MN = JM$  and  $\angle DNM = \angle CML$  (CPCT) ... (4)

$ML = LK$  and  $\angle CML = \angle BLK$  (CPCT) ... (5)

From (2), (3), (4) and (5),

$NK = KL = MN = ML$  ... (6)

And,  $\angle DNM = \angle AKN = \angle KLB = \angle LMC$

Now,

In  $\triangle NAK$ ,

$\angle NAK = 90^\circ$

Let  $\angle AKN = y^\circ$

So,  $\angle DNK = 90^\circ + y^\circ$

$\Rightarrow \angle DNM + \angle MNK = 90^\circ + y^\circ \Rightarrow y^\circ + \angle MNK = 90^\circ + y^\circ \Rightarrow \angle MNK = 90^\circ$

Similarly,

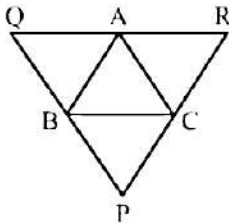
$\angle NKL = \angle KLM = \angle LMN = 90^\circ$  ... (7)

Using (6) and (7),

All sides of quadrilateral KLMN are equal and all angles are  $90^\circ$

So, KLMN is a square.

**Answer 26:**



$\Delta ABC$ , if lines are drawn through A, B, C parallel respectively to the sides BC, CA and AB. So, we get,  $BC \parallel QA$  and  $CA \parallel QB$

i.e., BCQA is a parallelogram.

$$\therefore BC = QA \quad \dots(i)$$

Similarly,  $BC \parallel AR$  and  $AB \parallel CR$ .

i.e., BCRA is a parallelogram.

$$\therefore BC = AR \quad \dots(ii)$$

As  $QR = QA + AR$

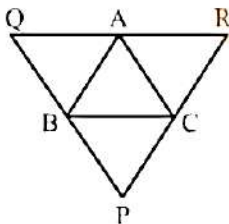
From (i) and (ii),

$$QR = BC + BC$$

$$\Rightarrow QR = 2BC$$

$$\therefore BC = \frac{1}{2}QR$$

**Answer 27:**



In  $\Delta ABC$  A, B, C lines drawn, parallel respectively to BC, CA and AB intersecting at P, Q and R. Acc to question,

$$\text{Perimeter of } \Delta ABC = AB + BC + CA \quad \dots(i)$$

$$\text{Perimeter of } \Delta PQR = PQ + QR + PR \quad \dots(ii)$$

By given figure,

$BC \parallel QA$  and  $CA \parallel QB$

i.e., BCQA is a parallelogram.

$$\therefore BC = QA \quad \dots(\text{iii})$$

Similarly,  $BC \parallel AR$  and  $AB \parallel CR$

i.e., BCRA is a parallelogram.

$$\therefore BC = AR \quad \dots(\text{iv})$$

But,  $QR = QA + AR$

From (iii) and (iv),

$$\Rightarrow QR = BC + BC$$

$$\Rightarrow QR = 2BC$$

$$\therefore BC = \frac{1}{2}QR$$

$$\text{Similarly, } CA = \frac{1}{2}PQ \text{ and } AB = \frac{1}{2}PR$$

From (i) and (ii),

$$\text{Perimeter of } \Delta ABC = \frac{1}{2}QR + \frac{1}{2}PQ + \frac{1}{2}PR$$

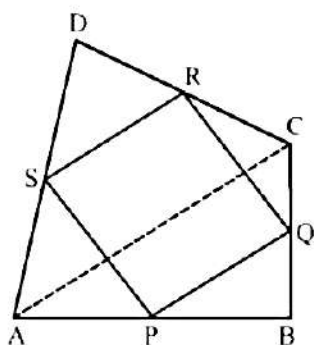
$$= \frac{1}{2}(PR + QR + PQ)$$

$$\text{i.e., Perimeter of } \Delta ABC = \frac{1}{2}(\text{Perimeter of } \Delta PQR)$$

$$\therefore \text{Perimeter of } \Delta PQR = 2 \times \text{Perimeter of } \Delta ABC$$

## EXERCISE – 10C

Answer 1:



Given: In

quadrilateral ABCD, P, Q, R and S are respectively the midpoints of the sides AB, BC, CD and DA.

To prove:

(i)  $PQ \parallel AC$  and  $PQ = \frac{1}{2}AC$

(ii)  $PQ \parallel SR$

(iii) PQRS is a parallelogram.

Proof:

(i) In  $\triangle ABC$ ,

Since, P and Q are the mid points of sides AB and BC, respectively. (Given)

$\Rightarrow AC \parallel PQ$  and  $PQ = \frac{1}{2}AC$  (Using mid-point theorem.)

(ii) In  $\triangle ADC$ ,

Since, S and R are the mid-points of AD and DC, respectively. (Given)

$\Rightarrow SR \parallel AC$  and  $SR = \frac{1}{2}AC$  (Using mid-point theorem.) ... (1)

From (i) and (1), we get

$PQ \parallel SR$

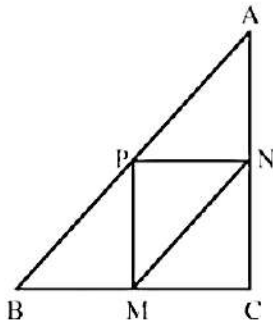
(iii) From (i) and (ii), we get

$PQ = SR = \frac{1}{2}AC$

So, PQ and SR are parallel and equal.

Hence, PQRS is a parallelogram.

**Answer 2:**



Given: In an isosceles right  $\Delta XYZ$ , ZEPG is a square.

To prove: F bisects the hypotenuse XY i.e.,  $XF = FY$ .

Proof:

In square ZEPG,

$\therefore ZE = EP = PG = GZ$  (All sides are equal.)

Also,  $\Delta XYZ$  is an isosceles with  $XZ = YZ$ .

$\Rightarrow XG + GZ = ZE + EY$

$\Rightarrow XG = EY$  ( $GZ = ZE$ ) ... (i)

Now,

In  $\Delta XGF$  and  $\Delta FEY$ ,

$XG = EY$  [From (i)]

$\angle XGF = \angle FEY = 90^\circ$

$FG = FE$  (Sides of square ZEPG)

$\therefore$  By SAS congruence criteria,

$\Delta XGF \cong \Delta FEY$

Hence,  $XF = FY$  (By CPCT)

**Answer 3:**

In parallelogram PQRS,

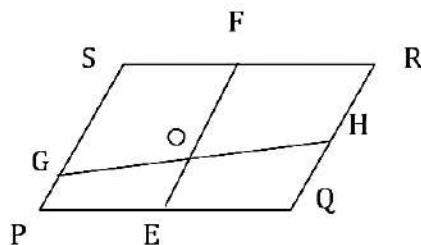
$PS \parallel QR$  and  $PQ \parallel RS$

$PS = QR$  and  $PQ = SR$

$PQ = PE + QE$  and  $RS = SF + FR$

$\therefore PE = QE = SF = FR$

Now,  $SF = PE$  and  $SF \parallel PE$ .



i.e., PEFS is a parallelogram.

$\therefore PS \parallel EF$

Similarly, QEFR is also a parallelogram.

$\therefore EF \parallel QR$

$\therefore PS \parallel EF \parallel QR$

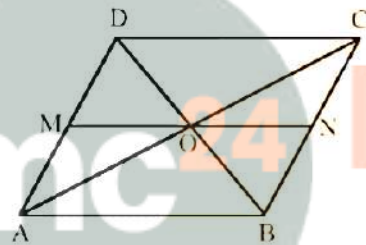
Thus, PS, EF and QR are three parallel lines cut by the transversal line SR at S, F and R, such that  $SF = FR$ .

These lines PS, EF and QR are also cut by the transversal PQ at P, E and Q, such that  $PE = QE$ .

Similarly, they also cut by GH.

$\therefore GO = OH$  (By intercept theorem)

**Answer 4:**



Given: A parallelogram ABCD

To prove: MN is bisected at O

Proof:

In  $\triangle OAM$  and  $\triangle OCN$ , we get by fig,

$OA = OC$  (Diagonals of parallelogram bisect each other)

$\angle AOM = \angle CON$  (Vertically opposite angles)

$\angle MAO = \angle OCN$  (Alternate interior angles)

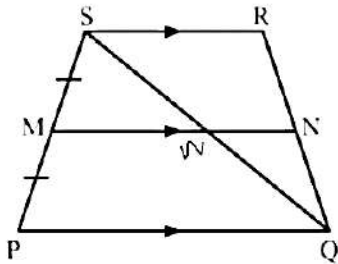
$\therefore$  By ASA congruence criteria, and parallelogram theorem

$\triangle OAM \cong \triangle OCN$

$\Rightarrow OM = ON$

Hence proved, MN is bisected at O.

**Answer 5:**



**Given:** In trapezium PQRS,  $PQ \parallel SR$ , M is the midpoint of PS and  $MN \parallel PQ$ .

**To prove:** N is the midpoint of QR.

**Construction:** Join QS.

**Proof:**

In  $\Delta SPQ$ , we get

M is the mid-point of SP and  $MW \parallel PQ$ .

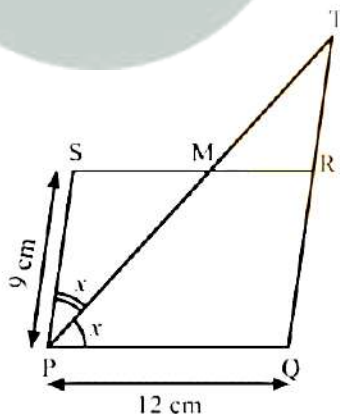
Therefore, W is the mid-point of SQ. (By Mid-point theorem)

Also, in  $\Delta SRQ$ ,

As, W is mid-point of SQ and  $WN \parallel SR$

Therefore, N is the mid-point of QR. (By Mid-point theorem)

**Answer 6:**



**Given:** In parallelogram PQRS,  $PQ = 12$  cm and  $PS = 9$  cm. The bisector of  $\angle SPQ$  meets SR at M.

Let  $\angle SPQ = 2y$ .

$\Rightarrow \angle SRQ = 2y$  and  $\angle TPQ = y$ .

Also,  $PQ \parallel SR$

$\Rightarrow \angle TMR = \angle TPQ = y$ .

In  $\angle TMR$ ,  $\angle SRQ$  is an exterior angle.

$\Rightarrow \angle SRQ = \angle TMR + \angle MTR$

$\Rightarrow 2y = y + \angle MTR$

$\Rightarrow \angle MTR = y$

$\Rightarrow \angle TPQ$  is an isosceles triangle.

$\Rightarrow TQ = PQ = 12 \text{ cm}$

Now,

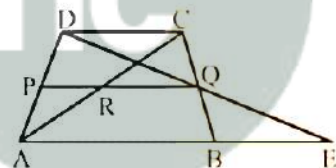
$RT = TQ - QR$

$$= TQ - PS$$

$$= 12 - 9$$

$$= 3 \text{ cm}$$

Answer 7:



Given:  $AB \parallel DC$ ,  $AP = PD$  and  $BQ = CQ$

(i) In  $\triangle QCD$  and  $\triangle QBE$ ,

$\angle DQC = \angle BQE$  (Vertically opposite angles)

$\angle DCQ = \angle EBQ$  (Alternate angles, as  $AE \parallel DC$ )

$BQ = CQ$  (P is the midpoints)

By parallelogram theorem

$\therefore \triangle QCD \cong \triangle QBE$

hence  $\therefore, DQ = QE$

(ii) Now, in  $\triangle ADE$ , P and Q are the midpoints of AD and DE, respectively.

$\therefore PQ \parallel AE$  from above

From fig we get,

$\Rightarrow PQ \parallel AB \parallel DC$

R is intersect point on AC and PQ then,

$\Rightarrow AB \parallel PR \parallel DC$

(iii) PQ, AB and DC are the three lines cut by transversal AD at P such that

$AP = PD$ .

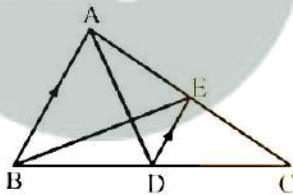
These lines PQ, AB, DC are also cut by transversal BC at Q such that

$BQ = QC$ .

Also, lines PQ, AB and DC are also cut by AC at R.

$\therefore AR = RC$

**Answer 8:**



AD is a median of  $\triangle ABC$ .

D is the mid point BC

$\therefore BD = DC$

It is clear that the line drawn through the midpoint of one side of triangle and parallel to another side bisects the third side.

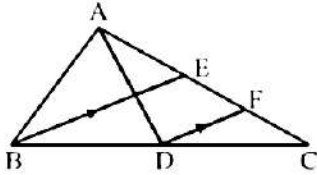
Then DE bisects AC.

$\therefore AE = EC$

$\therefore E$  is midpoint of AC.

$\Rightarrow BE$  is median in  $\triangle ABC$ .

**Answer 9:**



In  $\triangle ABC$ , by fig, we get  
 $AC = AE + EC$  ... (i)  
E is point of AC, then

$$AE = EC$$

Can also be written as

$$\therefore AC = 2EC \quad \dots \text{(ii)}$$

In  $\triangle BEC$ ,  $DF \parallel BE$ .

F is mid point of EC

$$\therefore EF = CF$$

As,  $EC = EF + CF$

$$\Rightarrow EC = 2 \times CF \quad \dots \text{(iii)}$$

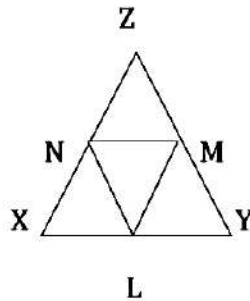
From (ii) and (iii),

$$AC = 2 \times (2 \times CF)$$

$$AC = 4 \times CF$$

$$\therefore CF = \frac{1}{4}AC$$

**Answer 10**



$\triangle XYZ$  is given. L, M and N are the midpoints of sides XY, YZ and ZX, respectively.

As, L and M are the mid points of sides XY, and YZ of  $\triangle XYZ$ .

$\therefore LM \parallel XZ$  (By midpoint theorem)

Similarly,  $LN \parallel YZ$  and  $MN \parallel XY$ .

Therefore, XLMN, YLNM and LNZN are all parallelograms.

Now, LM is the diagonal of the parallelogram YLNM.

$\therefore \triangle YLM \cong \triangle NML$

Similarly, LN is the diagonal of the parallelogram XLMN.

$\therefore \triangle LXN \cong \triangle NML$

And, MN is the diagonal of the parallelogram LNZN.

$\therefore \triangle MNZ \cong \triangle NML$

So, all the four triangles are congruent.

**Answer 11:**

D, E and F are the midpoints of sides BC, CA and AB, respectively.

As F and E are the mid points of sides AB and AC of  $\triangle ABC$ .

$\therefore FE \parallel BC$  (By mid point theorem)

Similarly,  $DE \parallel FB$  and  $FD \parallel AC$ .

Therefore, AFDE, BDEF and DCEF are all parallelograms.

In parallelogram AFDE, as,

$\angle A = \angle EDF$  (Opposite angles are equal)

In parallelogram BDEF, as,

$\angle B = \angle DEF$  (Opposite angles are equal)

In parallelogram DCEF, as ,

$\angle C = \angle DFE$  (Opposite angles are equal)

**Answer 12:**

Let LMNO be the rectangle and E, F, G and H be the midpoints of LM, MN, NO and OL, respectively.

Join LN, a diagonal of the rectangle.

In  $\Delta LMN$ , as ,

$\therefore EF \parallel LN$  and  $EF = \frac{1}{2} LN$  [By midpoint theorem]

Again, in  $\Delta OLN$ , the points G and H are the mid points of LO and ON, respectively.

$\therefore GH \parallel LN$  and  $GH = \frac{1}{2} LN$  [By midpoint theorem]

Now,  $EF \parallel LN$  and  $GH \parallel LN$

$\Rightarrow EF \parallel GH$

Also,  $EF = GH$  [Each equal to  $\frac{1}{2} LN$ ] ... (i)

So, EFGH is a parallelogram.

Now, in  $\Delta HLE$  and  $\Delta FME$ , as ,

$LH = MF$

$\angle L = \angle M = 90^\circ$

$LE = ME$

i.e.,  $\Delta HLE \cong \Delta FME$

$\therefore EH = EF$  ... (ii)

Similarly,  $\Delta HOG \cong \Delta FNG$

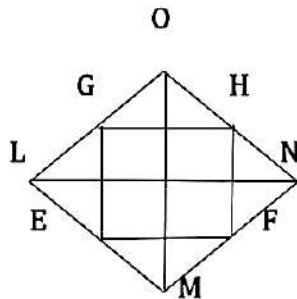
$\therefore HG = GF$  ... (iii)

From (i), (ii) and (iii), as ,

$EF = EH = HG = GF$

Hence, EFGH is a rhombus.

Answer 13:



Let LMNO be the rectangle and E, F, G and H be the midpoints of LM, MN, NO and OL.

Join the diagonals, LN and MO.

In  $\triangle LMN$ ,

$\therefore EF \parallel LN$  and  $EF = \frac{1}{2} LN$  [By midpoint theorem]

Now, in  $\triangle OLN$ , the points G and H are mid points of LO and ON.

$\therefore GH \parallel LN$  and  $GH = \frac{1}{2} LN$  [By midpoint theorem]

As,  $EF \parallel LN$  and  $GH \parallel LN$

$\Rightarrow EF \parallel GH$

Also,  $EF = GH$  ... (i)

$\therefore$  EFGH is a parallelogram.

$\therefore \angle YKX = 90^\circ$

Now,  $XG \parallel KM$

$\Rightarrow GY \parallel FK$

Also,  $HG \parallel LN$

$\Rightarrow XG \parallel KY$

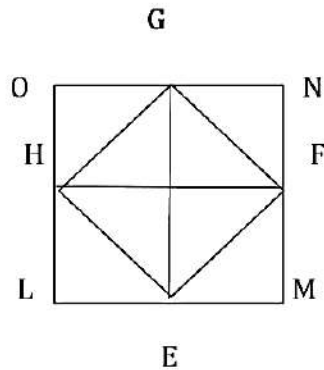
$\therefore$  KYGX is a parallelogram.

$\therefore \angle XGY = \angle YKX = 90^\circ$

Thus, EFGH is a parallelogram with  $\angle G = 90^\circ$ .

$\therefore$  EFGH is a rectangle.

Answer 14:



Let LMNO be the rectangle and E, F, G and H be the midpoints of LM, MN, NO and OL, respectively.

Join the diagonals LN and MO. Let OM cut HG at X and LN cut FG at Y. Let K be the intersection point of LN and OM.

In  $\triangle LMN$ , as,

$$\therefore EF \parallel LN \text{ and } EF = \frac{1}{2} LN \quad [\text{By midpoint theorem}]$$

Again, in  $\triangle OLN$ , the points G and H are the mid points of LO and ON respectively.

$$\therefore GH \parallel LN \text{ and } GH = \frac{1}{2} LN \quad [\text{By midpoint theorem}]$$

$$\begin{aligned} \text{Now, } EF \parallel LN \text{ and } GH \parallel LN \\ \Rightarrow EF \parallel GH \end{aligned}$$

$$\text{Also, } EF = GH \quad [\text{Each equal to } \frac{1}{2} LN] \quad \dots(i)$$

So, EF GH is a parallelogram.

Now, in  $\triangle HLE$  and  $\triangle FME$ , as,

$$LH = MF$$

$$\angle L = \angle M = 90^\circ$$

$$LE = ME$$

i.e.,  $\triangle HLE \cong \triangle FME$

$$\therefore EH = EF \quad \dots(ii)$$

Similarly,  $\triangle SDR \cong \triangle RCQ$

$$\therefore HG = FG \quad \dots(iii)$$

From (i), (ii) and (iii), as,

$$EF = EH = HG = FG \dots(iv)$$

We know that the diagonals of a square bisect each other at right angles.

$$\therefore \angle XOY = 90^\circ$$

Now,  $GQ \parallel ON$

$\Rightarrow GX \parallel YO$

Also,  $HG \parallel LN$

$\Rightarrow YG \parallel KX$

$\therefore KXRY$  is a parallelogram.

So,  $\angle YRX = \angle XKY = 90^\circ$  (Opposite angles are equal)

Thus,  $EFGH$  is a parallelogram with  $\angle G = 90^\circ$  and  $EF = EF = HG = HG$ .

$\therefore EFGH$  is a square.

### Answer 15:

Let  $LMNO$  be the rectangle and  $E, F, G$  and  $H$  be the midpoints of  $LM, MN, NO$  and  $OL$ , respectively.

Join  $EF, FG, GH, HE$  and  $NO$ .  $NO$  is a diagonal of  $LMNO$ .

In  $\triangle LMN$ , as,

$\therefore EF \parallel LN$  and  $EF = \frac{1}{2} LN$  (i) (By midpoint theorem)

Similarly in  $\triangle MNO$ , as,

$\therefore GH \parallel LN$  and  $GH = \frac{1}{2} MO$  (ii) (By midpoint theorem)

From equations (i) and (ii), we get:

$HE \parallel MO \parallel FG \therefore HE \parallel FG$  and  $HE = FG$  [Each equal to  $\frac{1}{2} MO$ ]

In quadrilateral  $HEFG$ , one pair of the opposite sides is equal and parallel to each other.

$\therefore HEFG$  is a parallelogram.

We know that the diagonals of a parallelogram bisect each other.

$\therefore EG$  and  $FH$  bisect each other.

### Answer 16

Given: In quadrilateral  $ABCD$ ,  $BD = AC$  and  $E, F, G$  and  $H$  are the mid-points of  $AD, CD, BC$  and  $AB$ , respectively.

To prove:  $EFGH$  is a rhombus.

Proof:

In  $\triangle ADC$ ,

Since, E and F are the mid-points of sides AD and CD, respectively.

$$\text{So, } EF \parallel AC \text{ and } EF = \frac{1}{2} AC \quad \dots(1)$$

Similarly, in  $\triangle ABC$ ,

Since, G and H are the mid-points of sides BC and AB, respectively.

$$\text{So, } GH \parallel AC \text{ and } GH = \frac{1}{2} AC \quad \dots(2)$$

From (1) and (2), we get

$$EF = GH \text{ and } EF \parallel GH$$

But this a pair of opposite sides of the quadrilateral EFGH.

So, EFGH is a parallelogram.

Now, in  $\triangle ABD$ ,

Since, F and G are the mid-points of sides AD and AB, respectively.

$$\text{So, } FG \parallel BD \text{ and } FG = \frac{1}{2} BD \quad \dots(3)$$

$$\text{But } BD = AC \quad (\text{Given})$$

$$\Rightarrow \frac{1}{2} BD = \frac{1}{2} AC$$

$$\Rightarrow FG = GH \quad [\text{From (2) and (3)}]$$

But these are a pair of adjacent sides of the parallelogram EFGH.

Hence, EFGH is a rhombus.

### Answer 17:

Given: In quadrilateral ABCD,  $AC \perp BD$ . E, F, G and H are the mid-points of AB, BC, CD and AD, respectively.

To prove: EFGH is a rectangle.

Proof:

In  $\triangle ABC$ , E and F are mid-points of AB and BC, respectively.

$$\therefore EF \parallel AC \text{ and } EF = \frac{1}{2} AC \quad (\text{Mid-point theorem}) \quad \dots(1)$$

Similarly, in  $\triangle ACD$ ,

So, G and H are mid-points of sides CD and AD, respectively.

$$\therefore GH \parallel AC \text{ and } GH = \frac{1}{2} AC \quad (\text{Mid-point theorem}) \quad \dots(2)$$

From (1) and (2), we get

$$EF \parallel GH \text{ and } EF = GH$$

But this is a pair of opposite sides of the quadrilateral EFGH,

So, EFGH is parallelogram.

Now, in  $\triangle BCD$ , F and G are mid-points of BC and CD, respectively.

$$\therefore FG \parallel BD \text{ and } FG = \frac{1}{2} BD \quad (\text{Mid-point theorem}) \quad \dots(3)$$

From (2) and (3), we get

$GH \parallel AC$  and  $FG \parallel BD$   
But,  $AC \perp BD$  (Given)  
 $\therefore GH \perp FG$   
Hence, EFGH is a rectangle.

**Answer 18:**

Given: In quadrilateral ABCD,  $AC = BD$  and  $AC \perp BD$ . E, F, G and H are the mid-points of AB, BC, CD and AD, respectively.

To prove: EFGH is a square.

Construction: Join AC and BD.

Proof: In  $\triangle ABC$ ,

$\because$  E and F are mid-points of AB and BC, respectively.

$\therefore EF \parallel AC$  and  $EF = \frac{1}{2}AC$  (Mid-point theorem) ... (1)

Similarly, in  $\triangle ACD$ ,

$\because$  G and H are mid-points of sides CD and AD, respectively.

$\therefore GH \parallel AC$  and  $GH = \frac{1}{2}AC$  (Mid-point theorem) ... (2)

From (1) and (2), we get

$EF \parallel GH$  and  $EF = GH$

But this a pair of opposite sides of the quadrilateral EFGH.

So, EFGH is parallelogram.

Now, in  $\triangle BCD$ ,

$\because$  F and G are mid-points of sides BC and CD, respectively.

$\therefore FG \parallel BD$  and  $FG = \frac{1}{2}BD$  (Mid-point theorem) ... (3)

From (2) and (3), we get

$GH \parallel AC$  and  $FG \parallel BD$

But,  $AC \perp BD$  (Given)

$\therefore GH \perp FG$

But this a pair of adjacent sides of the parallelogram EFGH.

So, EFGH is a rectangle.

Again,  $AC = BD$  (Given)

$\Rightarrow \frac{1}{2}AC = \frac{1}{2}BD$

$\Rightarrow GH = FG$  [From (2) and (3)]

But this a pair of adjacent sides of the rectangle EFGH.

Hence, EFGH is a square.

## MULTIPLE CHOICE QUESTIONS

**Answer 1:**

(b)  $73^\circ$

Let the measure of the fourth angle be  $y^\circ$ .

Since the sum of the angles of a quadrilateral is  $360^\circ$ , as ,

$$80^\circ + 95^\circ + 112^\circ + y = 360^\circ$$

$$\Rightarrow 287^\circ + y = 360^\circ$$

$$\Rightarrow y = 73^\circ$$

Hence, the measure of the fourth angle is  $73^\circ$ .

**Answer 2:**

(b)  $60^\circ$

Let  $\angle A = 3y$ ,  $\angle B = 4y$ ,  $\angle C = 5y$  and  $\angle D = 6y$ .

Since the sum of the angles of a quadrilateral is  $360^\circ$ , as ,

$$3y + 4y + 5y + 6y = 360^\circ$$

$$\Rightarrow 18y = 360^\circ$$

$$\Rightarrow y = 20^\circ$$

$$\therefore \angle A = 60^\circ, \angle B = 80^\circ, \angle C = 100^\circ \text{ and } \angle D = 120^\circ$$

**Answer 3:**

(c)  $45^\circ$

Given,  $\angle BAD = 75^\circ$  and  $\angle CBD = 60^\circ$

$$\Rightarrow \angle B = 180^\circ - \angle A = 180^\circ - 75^\circ = 105^\circ$$

Thus,  $\angle B = \angle ABD + \angle CBD$

$$\Rightarrow 105^\circ = \angle ABD + 60^\circ$$

$$\Rightarrow \angle ABD = 105^\circ - 60^\circ = 45^\circ$$

$$\Rightarrow \angle ABD = \angle BDC = 45^\circ$$

**Answer 4:**

Given,  $\angle ACB = 50^\circ$  and  $\angle A = 90^\circ$  as it is rhombus  $\Delta$

In  $\Delta BOC$ ,

$$90^\circ + 50^\circ + \angle OBC = 180^\circ$$

$$\Rightarrow \angle OBC = 180^\circ - (90 + 50) = 180 - 140^\circ$$

$$\Rightarrow \angle OBC = 40^\circ$$

As  $\angle OBC = \angle ADB$

Thus,  $\angle ADB = 40^\circ$

Hence, (a) is the correct answer.

**Answer 5:**

(d) Rectangle.  
rectangle has diagonals of equal length.

**Answer 6:**

(d) rhombus  
rhombus diagonals bisect each other at right angles.

**Answer 7:**

(a) 10 cm

Let PQRS be the rhombus.

$$\therefore PQ = QR = RS = SP$$

Here, PR and QS are the diagonals of PQRS, where PR = 16 cm and QS = 12 cm.

Let the diagonals intersect each other at M.

We know that the diagonals of a rhombus are perpendicular bisectors of each

other.

$\therefore \triangle PMQ$  is a right angle triangle, in which  $MP = \frac{1}{2} PR = \frac{16}{2} = 8$  cm and  $MQ =$

$$\frac{1}{2} QS = \frac{12}{2} = 6 \text{ cm.}$$

Now,  $PQ^2 = MP^2 + MQ^2$  [Pythagoras theorem]

$$\Rightarrow PQ^2 = (8)^2 + (6)^2$$

$$\Rightarrow PQ^2 = 64 + 36 = 100$$

$$\Rightarrow PQ = 10 \text{ cm}$$

Hence, the side of the rhombus is 10 cm.

**Answer 8:**

(b) 12 cm

Let PQRS be the rhombus.

$$\therefore PQ = QR = RS = SP = 10 \text{ cm}$$

Let PR and QS be the diagonals of the rhombus.

Let PR be  $y$  and QS be 16 cm and M be the intersection point of the diagonals.

We know that the diagonals of a rhombus are perpendicular bisectors of each other.

$\therefore \triangle AOB$  is a right angle triangle in  $MP = \frac{1}{2} PR = \frac{y}{2}$  and  $MQ = \frac{1}{2} QS = \frac{16}{2} = 8$

cm.

Now,  $PQ^2 = MP^2 + MQ^2$  [Pythagoras theorem]

$$\Rightarrow 10^2 = \left(\frac{y}{2}\right)^2 + 8^2 \Rightarrow \left(\frac{y}{2}\right)^2 = 36 = 6^2 \Rightarrow y = 2 \times 6 = 12 \text{ cm}$$

**Answer 9:**

Given: In rectangle PQRS,  $\angle MPD = 35^\circ$ .

Since,  $\angle QPS = 90^\circ$

$$\Rightarrow \angle MPQ = 90^\circ - 35^\circ = 55^\circ$$

In  $\triangle MPQ$ ,

Since,  $MP = MQ$  (Diagonals of a rectangle are equal and bisect each other)

$$\Rightarrow \angle MPQ = \angle MQP = 55^\circ \text{ (Angles opposite to equal sides are equal)}$$

Now, in  $\triangle MSP$ ,

$$55^\circ + 55^\circ + \angle SMP = 180^\circ \text{ (Angle sum property of a triangle)}$$

$$\Rightarrow \angle SMP = 180^\circ - 110^\circ$$

$$\Rightarrow \angle SMP = 70^\circ$$

Thus, the acute angle between the diagonals is  $70^\circ$ .

Hence, the correct option is (b).

**Answer 10:**

(c) Rectangle

ABCD is parallelogram with two adjacent side

$$\angle A = \angle B \dots\dots\dots(\text{given})$$

$$\text{Then } \angle A + \angle B = 180^\circ$$

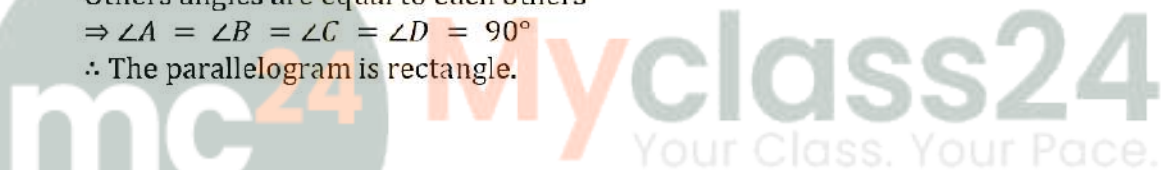
$$\Rightarrow 2\angle A = 180^\circ$$

$$\Rightarrow \angle A = 90^\circ$$

Others angles are equal to each others

$$\Rightarrow \angle A = \angle B = \angle C = \angle D = 90^\circ$$

$\therefore$  The parallelogram is rectangle.



**Answer 11:**

(b)  $50^\circ$

in quadrilateral ABCD, AO and BO are the bisectors of  $\angle C = 70^\circ$  and  $\angle D = 30^\circ$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\angle A + \angle B = 360^\circ - (70 + 30)^\circ = 260^\circ$$

$$\therefore \frac{1}{2}(\angle A + \angle B) = \frac{1}{2}(260)^\circ = 130^\circ$$

In  $\Delta AOB$ ,

$$\angle AOB = 180^\circ - \left[\frac{1}{2}(\angle A + \angle B)\right]$$

$$\Rightarrow \angle AOB = 180^\circ - 130^\circ = 50^\circ$$

**Answer 12:**

(d)  $90^\circ$

Sum of any two adjacent angles of a rectangle is  $180^\circ$

$\therefore$ , sum of angle bisectors of two adjacent angles =  $\frac{1}{2} \times 180^\circ = 90^\circ$

$\therefore$  Intersection angle of bisectors of two adjacent angles =  $180^\circ - 90^\circ = 90^\circ$

**Answer 13:**

(c) Rectangle

parallelograms angle bisectors enclose a rectangle

**Answer 14:**

Given: In quadrilateral ABCD, AS, BQ, CQ and DS are angle bisectors of angles A, B, C and D.

$$\angle QPS = \angle APB \quad \dots(1)$$

In  $\triangle APB$ ,

$$\angle APB + \angle PAB + \angle ABP = 180^\circ$$

$$\Rightarrow \angle APB = 180^\circ - \angle PAB - \angle ABP$$

$$\Rightarrow \angle APB = 180 - \frac{1}{2}\angle A - \frac{1}{2}\angle B$$

$$\Rightarrow \angle APB = 180^\circ - \frac{1}{2}(\angle A + \angle B) \quad \dots(2)$$

From (1) and (2),

$$\angle QPS = 180^\circ - \frac{1}{2}(\angle A + \angle B) \quad \dots(3)$$

$$\text{Also, } \angle QRS = 180^\circ - \frac{1}{2}(\angle C + \angle D) \quad \dots(4)$$

From (3) and (4), we get

$$\angle QPS + \angle QRS = 360^\circ - \frac{1}{2}(\angle A + \angle B + \angle C + \angle D)$$

$$= 360^\circ - \frac{1}{2}(360^\circ)$$

$$= 360^\circ - 180^\circ$$

$$= 180^\circ$$

Thus, PQRS is a quadrilateral whose opposite angles are supplementary.  
Hence, (d) is the correct option.

**Answer 15:**

(d) parallelogram

parallelogram is formed after joining the mid points of the adjacent sides of a quadrilateral.

**Answer 16:**

(b) Square

Square is formed after joining the mid points of the adjacent sides of a square of the sides.

**Answer 17:**

(d) parallelogram.

parallelogram is formed after joining the mid points of the adjacent sides of a parallelogram

**Answer 18:**

(a) rhombus

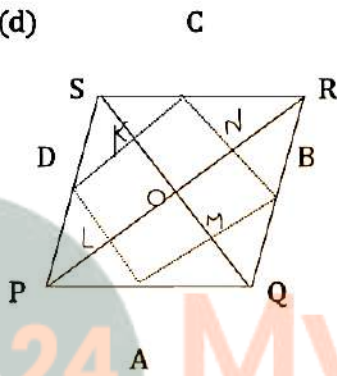
Rhombus is formed after joining the mid points of the adjacent sides of a rectangle

**Answer 19:**

(c) Rectangle

Rectangle quadrilateral formed after joining the mid points of the adjacent sides of a rhombus.

**Answer 20: (d)**



ABCD is always parallelogram

By midpoint theorem,

$DA \parallel QS$  and  $AB \parallel PR$

$\Rightarrow LA \parallel OM$  and  $OM \parallel LA \Rightarrow LMOA$  is a parallelogram.

$\Rightarrow \angle LAM = \angle LOM = 90^\circ$  [  $PR \perp SQ$  (given) ]

Now, ABCD is parallelogram with one angle  $\angle A = 90^\circ$

$\therefore$  ABCD is rectangle if  $PR \perp SQ$

**Answer 21:**

Given:

The quadrilateral PQRS is a rhombus.

Thus, the sides PQ, QR, RS and SP are equal.

In  $\triangle LMO$ ,

$$RS = \frac{1}{2}MN \quad \dots(1)$$

Also, in  $\triangle LON$ ,

$$QR = \frac{1}{2}LN \quad \dots(2)$$

And,  $QR = RS$

$$\Rightarrow \frac{1}{2}MO = \frac{1}{2}LN \quad [\text{From (1) and (2)}]$$

$$\therefore MO = LN$$

Thus, the diagonals of LMNO are equal.

Hence, (c) is the correct option.

**Answer 22: (d)**

Square the quadrilateral formed after joining the mid points of the quadrilateral with diagonals perpendicular and equal to each other

Hence, (d) is the correct option.

**Answer 23:**

(c)  $72^\circ$

Let PQRS is a parallelogram.

$$\therefore \angle P = \angle R \text{ and } \angle Q = \angle S \quad (\text{Opposite angles})$$

$$\text{Let } \angle P = y \text{ and } \angle Q = \frac{2}{3}y$$

$$\therefore \angle P + \angle Q = 180^\circ$$

$$\Rightarrow y + \frac{2}{3}y = 180^\circ$$

$$\Rightarrow \frac{5}{3}y = 180^\circ$$

$$\Rightarrow y = 108^\circ$$

$$\therefore \angle Q = \frac{2}{3} \times (108^\circ) = 72^\circ$$

Hence,  $\angle P = \angle R = 108^\circ$  and  $\angle Q = \angle S = 72^\circ$

**Answer 24:**

(c)  $112^\circ$

Let PQRS is a parallelogram.

$$\therefore \angle P = \angle R \text{ and } \angle Q = \angle S$$

Let  $\angle P = y$

$$\therefore \angle Q = (2y - 24)^\circ$$

$$\text{Now, } \angle P + \angle Q = 180^\circ$$

$$\Rightarrow y + 2y - 24^\circ = 180^\circ$$

$$\Rightarrow 3y = 204^\circ$$

$$\Rightarrow y = 68^\circ$$

$$\therefore \angle Q = 2 \times 68^\circ - 24^\circ = 112^\circ$$

Hence,  $\angle P = \angle R = 68^\circ$  and  $\angle Q = \angle S = 112^\circ$

**Answer 25:**

(c) Trapezium

Let the angles be  $(3y)$ ,  $(7y)$ ,  $(6y)$  and  $(4y)$ .

$$\text{Now } 3y + 7y + 6y + 4y = 360^\circ$$

$$\therefore y = 18^\circ$$

Thus, angles will be

$$3 \times 18^\circ = 54^\circ$$

$$7 \times 18^\circ = 126^\circ,$$

$$6 \times 18^\circ = 108^\circ,$$

$$4 \times 18^\circ = 72^\circ$$

$$\text{As, } 54^\circ + 126^\circ = 180^\circ \text{ and } 72^\circ + 108^\circ = 180^\circ$$

$\therefore$  ABCD is a trapezium.

**Answer 26:**

(c) The opposite angles in a parallelogram are bisected by the diagonals.

**Answer 27:**

(c) Rectangle

It is obvious that the bisectors will enclose a rectangle.

If  $AMB$  and  $CND$  are two parallel lines, then the bisectors of  $\angle AMN$ ,  $\angle BMN$ ,  $\angle NMP$  and  $\angle NMD$  enclose a rectangle.

**Answer 28:**

(c)  $60^\circ$

$\angle ABD = \angle CDB = 45^\circ$  .....alternative interior angles

$\angle BAD = \angle BCD = 75^\circ$

In  $\triangle BCD$ ,  $\angle C = 75^\circ$

$\Rightarrow \angle CBD + \angle BCD + \angle BDC = 180^\circ$

$\therefore \angle CBD = 180^\circ - (75^\circ + 45^\circ) = 60^\circ$

**Answer 29:**

(c)  $A < B$

Let us assume that  $x$  be height of the parallelogram.

Now clearly,  $x < b$

$\therefore A = a \times x < a \times b = B$

$\therefore A < B.$

**Answer 30:**

(b)  $AF = 2 AB$

In parallelogram  $ABCD$ ,

$AB \parallel DC$

$\angle DCE = \angle EBF$

In  $\triangle DCE$  and  $\triangle BFE$ ,

$\angle DCE = \angle EBF$  (Proved above)

$$\angle DEC = \angle BEF$$

$$BE = CE \quad (\text{Given})$$

By parallelogram theorem

$$\therefore \triangle DCE \cong \triangle BFE$$

$$\therefore DC = BF$$

Now  $DC = AB$ , since ABCD is a parallelogram.

$$\therefore DC = AB = BF \quad \dots(\text{i})$$

$$\text{Now, } AF = AB + BF \quad \dots(\text{ii})$$

From (i),

$$\therefore AF = AB + AB = 2AB$$

### Answer 31:

Given: In  $\triangle ABC$ , R, S, D and E are the mid-points of BP, CP, AB and AC

In  $\triangle ABP$ ,

$$\therefore BR = \frac{1}{2}AP \text{ and } BR \parallel AP \quad \dots(\text{i})$$

In  $\triangle ACP$ ,

$$\therefore ES = \frac{1}{2}AP \text{ and } ES \parallel AP \quad \dots(\text{ii})$$

From (i) and (ii)

$$BR = ES \text{ and } BR \parallel ES$$

As BR and ES are opposite sides of the quadrilateral, thus it is a parallelogram.

Thus, (b) is the correct answer.

### Answer 32:

$$(b) \frac{1}{2}(a+b)$$

Suppose PQRS is a trapezium.

Draw YZ parallel to PQ.

Join QS to cut YZ at X.

Now, in  $\triangle SPQ$ , Y is the midpoint of PS and  $YX \parallel PQ$ .

$$\therefore M \text{ is the mid point of QS and } YX = \frac{1}{2}(a)$$

Similarly, M is the mid point of QS and  $XZ \parallel DC$ .

$$\text{i.e., } Z \text{ is the midpoint of QR and } XZ = \frac{1}{2}(b)$$

$$\therefore YZ = YX + XZ = \frac{1}{2}(a+b)$$

**Answer 33:**

$$(d) \frac{1}{2}(AB - CD)$$

Join CF and produce it to cut AB at M.

Then  $\triangle CDF \cong \triangle MBF$  [DF = BF,  $\angle DCF = \angle BMF$  and  $\angle CDF = \angle MBF$ ]

$$\therefore CD = MB$$

Thus, in  $\triangle CAM$ , the points E and F are the mid points of AC and CM, respectively.

$$\therefore EF = \frac{1}{2}(AM) = \frac{1}{2}(AB - MB) = \frac{1}{2}(AB - CD)$$

**Answer 34:**

$$(c) 90^\circ$$

$$\angle B = \angle D$$

$$\Rightarrow \frac{1}{2}\angle B = \frac{1}{2}\angle D$$

$$\Rightarrow \angle ADB = \angle ABD$$

$\therefore \triangle ABD$  is an isosceles triangle and M is midpoint of BD.

$$\therefore AM \perp BD \text{ thus, } \angle AMB = 90^\circ$$

**Answer 35:**

$$(c) AC^2 + BD^2 = 4AB^2$$

As diagonals of a rhombus bisect each other at right angles.

$$\Rightarrow OA = \frac{1}{2}AC$$

$$OB = \frac{1}{2}BD \text{ and } \angle AOB = 90^\circ$$

By Pythagoras theorem,  $\triangle AOB$

$$\text{Now, } (AB)^2 = (OA)^2 + (OB)^2$$

$$\Rightarrow \frac{1}{4}(AC)^2 + \frac{1}{4}(BD)^2$$

$$\therefore 4AB^2 = (AC^2 + BD^2)$$

**Answer 36:**

(c)  $BC^2 + AD^2 + 2AB.CD$

Draw perpendicular from D and C on AB which meets AB at M and N, respectively.

$\therefore$  DMNC is a parallelogram and  $MN = CD$ .

In  $\triangle ABC$ ,  $\angle B$  is acute.

$$\therefore AC^2 = BC^2 + AB^2 - 2AB.AM$$

In  $\triangle ABD$ ,  $\angle A$  is acute.

$$\therefore BD^2 = AD^2 + AB^2 - 2AB.AN$$

$$\therefore AC^2 + BD^2$$

$$= (BC^2 + AD^2) + (AB^2 + AB^2) - 2AB(AM + BN)$$

$$= (BC^2 + AD^2) + 2AB(AB - AM - BN) \quad [AB = AM + MN + NB \text{ and } AB - AM = BN]$$

$$= (BC^2 + AD^2) + 2AB(BM - BN)$$

$$= (BC^2 + AD^2) + 2AB.MN$$

$$\therefore AC^2 + BD^2 = (BC^2 + AD^2) + 2AB.CD$$

**Answer 37:**

(d) 1:1

Area of a parallelogram = base  $\times$  height

The height will be same for any pair of parallelograms with same base and same parallel lines.

**Answer 38:**

(b)  $\frac{1}{3}AC$

Let X be the mid point of FC. Join DX.

In  $\triangle BCF$ , D is the mid point of BC and X is the mid point of FC.

$$\therefore DX \parallel BF$$

$$\Rightarrow DX \parallel EF$$

In  $\triangle ADX$ , E is the mid point of AD and  $EF \parallel DX$ .

i.e., F is the mid point of AX.

Now,  $AF = FG = GC$

$$\therefore AF = \frac{1}{3}AC$$

**Answer 39: (A)**

Given,  $\angle AOB = 70^\circ$

$\angle OAD = \angle OCB = 30^\circ$  (Alternate interior angles)  
As we know that Linear pair of angles is  $180^\circ$

$$\angle AOB + \angle BOC = 180^\circ$$
$$\therefore \angle BOC = 180^\circ - 70^\circ = 110^\circ$$

In  $\triangle BOC$ ,

$$\angle OBC + \angle BOC + \angle OCB = 180^\circ$$

$$\angle OBC = 180^\circ - \angle BOC - \angle OCB$$

$$\angle OBC = 180^\circ - (110^\circ + 30^\circ) = 40^\circ$$
$$\therefore \angle DBC = 40^\circ$$

**Answer 40:**

(c) I and II

The statement III false, any triangle that will be formed on joining midpoints of sides of an isosceles triangle will be an isosceles triangle.

**Answer 41:**

(b) II and III

The statement I is not true as diagonal of rectangle does not bisect  $\angle A$  and  $\angle C$ .

### SHORT ANSWER QUESTIONS

**Answer 42:**

Given,  $SR = 2\text{cm}$  and  $PR = 5\text{cm}$ .

As, the opposite angles of quadrilateral are equal, so PQRS is a parallelogram.

$\Rightarrow SR = PQ$

$\therefore SR = PQ = 2\text{ cm}$

**Answer 43:**

The parallelogram diagonals bisect each other , thus the statement is not true.

**Answer 44:**

Given :  $\angle P + \angle S = 180^\circ$ .

i.e. the sum of the adjacent angles is equal to  $180^\circ$ .

$PQ \parallel RS$  and also  $\angle R + \angle S = 180^\circ$

Hence PQRS is a parallelogram.

**Answer 45:**

Acute angles is less than  $90^\circ$ . It is clear if all angles are less than  $90^\circ$ , then sum all angles will be less than  $360^\circ$ , thus a quadrilateral cannot be formed.

**Answer 46:**

It mean all angles is  $90^\circ$ . As rectangle and square have all angles as right angles , thus the statement holds true.

**Answer 47:**

It means obtuse angles is greater than  $90^\circ$ . It is clear if all angles are greater than  $90^\circ$ , then sum all angles will be greater than  $360^\circ$ , thus a quadrilateral cannot be formed.

**Answer 48:**

As the sum of all the angles given is  $70^\circ + 115^\circ + 60^\circ + 120^\circ = 365^\circ$   
Thus, a quadrilateral with these angles cannot be formed.

Sum of all the angles should be exact  $360^\circ$ .

**Answer 49:**

As, the sum of all angles is equal to  $360^\circ$  in a quadrilateral.

Let each angle of the quadrilateral be  $y$ .

$$y + y + y + y = 360^\circ$$

$$\Rightarrow 4y = 360^\circ$$

$$\Rightarrow y = 90^\circ$$

$\Rightarrow$  All the angles of the quadrilateral are  $90^\circ$ .

Thus, the given quadrilateral is a rectangle.

**Answer 50:**

Given,  $AB=7.2\text{cm}$ ,  $BC= 9.8\text{cm}$ ,  $AC = 3.6\text{cm}$

In  $\triangle ABC$ ,

As, D and E are the mid-points of sides AB and BC .

$$DE = \frac{1}{2}(AC) = \frac{1}{2}(3.6)$$

$$\Rightarrow DE = 1.8 \text{ cm}$$

Thus, DE is equal to 1.8 cm.

**Answer 51:**

As the diagonals of the quadrilateral bisect each other, thus PQRS is a parallelogram. And given,  $\angle Q = 56^\circ$

Angles at liner equations,

Thus,  $\angle Q + \angle R = 180^\circ$

$\Rightarrow 56^\circ + \angle R = 180^\circ$

$\Rightarrow \angle R = 180^\circ - 56^\circ$

$\Rightarrow \angle R = 124^\circ$

**Answer 52:**

Given: Parallelograms BDEF and AFDE.

F is mid point of AB, A

As,  $BF = DE$

...(i)

And,  $AF = DE$

...(ii)

From (i) and (ii)

$AF = FB$

**Answer 53:**

As it is clear that when the diagonals of a quadrilateral bisects each other, then it is a parallelogram and when the diagonals of a quadrilateral are equal, then it is not necessarily a parallelogram.

$\therefore$  I gives the answer and II does not give the answer.

Thus, (a) is the correct answer.

**Answer 54:**

It is clear that neither I alone nor II alone is sufficient to answer.

On the other hand, on considering both I and II together it will give the

answer.

∴, (c) is the correct answer.

**Answer 55:**

As it is clear that when the diagonals of a parallelogram are equal, and intersect each other at right angle then the parallelogram is a square.

Thus, (c) is the correct answer.

**Answer 56:**

It is clear that when I or II holds true, the quadrilateral is a parallelogram.

Thus, (b) is the correct answer.

**Answer 57:**

(a) Both Assertion and Reason are true and Reason is a correct explanation of Assertion.

$$\text{Fourth angle} = 360^\circ - (130^\circ + 70^\circ + 60^\circ) = 100^\circ$$

It is obvious that the assertion (A) and reason(R) is absolutely true.

On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

As, the reason (R) hold the assertion (A).

Thus, (a) is the correct answer .

**Answer 58:**

(a) Both Assertion and Reason are true and Reason is a correct explanation of Assertion.

It is obvious that the assertion (A) and reason(R) is absolutely true.

On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

As, the reason (R) hold the assertion (A).

Thus, (a) is the correct answer .

**Answer 59:**

(b) Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.

It is obvious that the assertion (A) is absolutely true.

On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

As, the reason (R) does not hold the assertion (A).

Thus, (b) is the correct answer .

**Answer 60:**

(d) Assertion is false and Reason is true.

It is obvious that the assertion (A) is absolutely false.

On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

Thus, (d) is the correct answer.

**Answer 61:**

(b) Both Assertion and Reason are true but Reason is not a correct explanation of Assertion.

It is obvious that the assertion (A) is absolutely true.

On the same hand the reason (R) can be proved easily. Thus, (R) is true as well.

As, the reason (R) does not hold the assertion (A).

Thus, (b) is the correct answer .

**Answer 62:**

(a) will go with (q),

(b) will go with (r),

(c) will go with (s),

(d) will go with (p)

**Answer 63:**

(a) - (r), (b) - (s), (c) - (p), (d) - (q)

$$(a) PQ = \frac{1}{2}(AB + CD) = \frac{1}{2}(17) = 8.5 \text{ cm}$$

$$(b) OR = \frac{1}{2}(PR) = \frac{1}{2}(13) = 6.5 \text{ cm}$$