Now Differentiating

$$\Rightarrow \frac{d\left(\tan^{-1}x - \tan^{-1}\left(\frac{b}{a}\right)\right)}{dx}$$

$$\Rightarrow \frac{d(tan^{-1}x)}{dx} - \frac{d\left(tan^{-1}\left(\frac{b}{a}\right)\right)}{dx}$$

$$\Rightarrow \frac{1}{1+x^2} + 0$$

Ans) $\frac{1}{1+x^2}$

47. Question

Differentiate each of the following w.r.t x:

If $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$, show that $\frac{dy}{dx} = \frac{4}{(1+x^2)}$.

Answer

Given: Value of $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

To Prove: $\frac{dy}{dx} = \frac{4}{(1+x^2)}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx} = \frac{1}{\sqrt{1-x^2}}$

We have, $\sin^{-1}\left(\frac{2x}{1+x^2}\right) + \sec^{-1}\left(\frac{1+x^2}{1-x^2}\right)$

Putting x = tanθ

θ = $\tan^{-1}x$

Dividing numerator and denominator with a

$$\Rightarrow \sin^{-1}\left(\frac{2\tan\theta}{1+(\tan\theta)^2}\right) + \sec^{-1}\left(\frac{1+(\tan\theta)^2}{1-(\tan\theta)^2}\right)$$

$$\Rightarrow \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \sec^{-1}\left(\frac{1+\tan^2\theta}{1-\tan^2\theta}\right)$$

$\Rightarrow \sin^{-1}(\sin 2\theta)+\sec^{-1}\left(\frac{1}{\cos 2\theta}\right)$

$\Rightarrow \sin^{-1}(\sin 2\theta)+\sec^{-1}(\sec 2\theta)$

$\Rightarrow 2\theta+2\theta$

$\Rightarrow 4\theta$

$\Rightarrow 4\tan^{-1}x$

Now Differentiating

$\Rightarrow \frac{d(4\tan^{-1}x)}{dx}$

$\Rightarrow 4\frac{1}{1+x^2}$

Ans) $\frac{4}{1+x^2}$

48. Question

Differentiate each of the following w.r.t x:

If $y=\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$, show that $\frac{dy}{dx}=0.$

Answer

Given: Value of $y=\ \sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$

To Prove: $\frac{dy}{dx}=\ 0$

Formula used: (i) $\cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\sin^{-1}x)}{dx}=\frac{1}{\sqrt{1-x^2}}$

We have, $\sec^{-1}\left(\frac{x+1}{x-1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\Rightarrow \cos^{-1}\left(\frac{x-1}{x+1}\right)+\sin^{-1}\left(\frac{x-1}{x+1}\right)$

$\Rightarrow \frac{\pi}{2}$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{2}\right)}{dx}$$

$\Rightarrow 0$

Ans) $\frac{4}{1+x^2}$

49. Question

Differentiate each of the following w.r.t x:

If $y = \sin\left\{2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right\}$, show that $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$.

Answer

Given: Value of $y = \sin\left\{2\tan^{-1}\left(\sqrt{\frac{1-x}{1+x}}\right)\right\}$

To Prove: $\frac{dy}{dx} = \frac{-x}{\sqrt{1-x^2}}$

Formula used: (i) $\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

Let x = cosθ

θ = $\cos^{-1}x$

Putting x = cosθ in equation

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}\right)\right\}$$

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\sqrt{\frac{2\sin^2\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}}\right)\right\}$$

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\sqrt{\tan^2\frac{\theta}{2}}\right)\right\}$$

$$\Rightarrow \sin\left\{2\tan^{-1}\left(\tan\frac{\theta}{2}\right)\right\}$$

$\Rightarrow \sin\left\{2\frac{\theta}{2}\right\}$

$\Rightarrow \sin\theta$

$\Rightarrow \sin(\cos^{-1}x)$

Now Differentiating

$\Rightarrow \frac{d(\sin(\cos^{-1}x))}{dx}$

$\Rightarrow \frac{d(\sin(\cos^{-1}x))}{d\cos^{-1}x}\frac{d\cos^{-1}x}{dx}$

$\Rightarrow -\cos(\cos^{-1}x)\frac{1}{\sqrt{1-x^2}}$

$\Rightarrow -\frac{x}{\sqrt{1-x^2}}$

Ans) $\frac{4}{1+x^2}$

50. Question

Differentiate each of the following w.r.t x:

If $y=\tan^{-1}\left\{\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right\}$. Prove that $\frac{dy}{dx}=\frac{1}{2\sqrt{1-x^2}}$.

Answer

Given: Value of $y=\tan^{-1}\frac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}$

To Prove: $\frac{dy}{dx}=\frac{1}{2\sqrt{1-x^2}}$

The formula used: (i) $\cos\theta=\sin\left(\frac{\pi}{2}-\theta\right)$

(ii) $\frac{d(\cos^{-1}x)}{dx}=-\frac{1}{\sqrt{1-x^2}}$

Let x = cos2θ

$2\theta=\cos^{-1}x$

$\theta=\frac{1}{2}\cos^{-1}x$

Putting x = cos2θ

$$y= \tan^{-1}\frac{\sqrt{1+\cos2\theta}-\sqrt{1-\cos2\theta}}{\sqrt{1+\cos2\theta}+\sqrt{1-\cos2\theta}}$$

$$y= \tan^{-1}\frac{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}$$

$$y= \tan^{-1}\frac{\sqrt{2}\cos\theta-\sqrt{2}\sin\theta}{\sqrt{2}\cos\theta+\sqrt{2}\sin\theta}$$

$$y= \tan^{-1}\frac{\sqrt{2}(\cos\theta-\sin\theta)}{\sqrt{2}(\cos\theta+\sin\theta)}$$

Dividing by cosθ in the numerator and denominator

$$y= \tan^{-1}\frac{\frac{\cos\theta-\sin\theta}{\cos\theta}}{\frac{\cos\theta+\sin\theta}{\cos\theta}}$$

$$y= \tan^{-1}\frac{1-\tan\theta}{1+\tan\theta}$$

$$y= \tan^{-1}\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}$$

$$y= \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right)$$

$$y= \frac{\pi}{4}-\theta$$

$$y= \frac{\pi}{4}-\frac{1}{2}\cos^{-1}x$$

Now Differentiating

$$\Rightarrow \frac{d\left(\frac{\pi}{4}-\frac{1}{2}\cos^{-1}x\right)}{dx}$$

$$\Rightarrow \frac{d\left(\frac{\pi}{4}\right)}{dx}-\frac{1}{2}\frac{d\cos^{-1}x}{dx}$$

$$\Rightarrow \frac{1}{2}\frac{1}{\sqrt{1-x^2}}$$

$\Rightarrow \frac{1}{2\sqrt{1-x^2}}$

Ans) $\frac{1}{2\sqrt{1-x^2}}$

51. Question

Differentiate each of the following w.r.t x:

Differentiate $\sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$ w. r. t. x

Answer

Given: Value of $y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$

To find: $\frac{dy}{dx}$

The formula used: (i) $\cos\theta = \sin\left(\frac{\pi}{2} - \theta\right)$

(ii) $\frac{d(\tan^{-1}x)}{dx} = \frac{1}{1+x^2}$

$$y = \sin^{-1}\left(\frac{2^{x+1}}{1+4^x}\right)$$

$$y = \sin^{-1}\left(\frac{2^x.2}{1+\left(2^2\right)^x}\right)$$

$$y = \sin^{-1}\left(\frac{2^x.2}{1+(2^x)^2}\right)$$

Let $2^x = \tan\theta$

$\theta = \tan^{-1}(2^x)$

Putting $2^x = \tan\theta$

$$y = \sin^{-1}\left(\frac{\tan\theta.2}{1+(\tan\theta)^2}\right)$$

$$y = \sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right)$$

$y = \sin^{-1}(\sin 2\theta)$

$y = 2\theta$

$y = 2\tan^{-1}(2^x)$

Now Differentiating

$\Rightarrow \frac{d(2\tan^{-1}(2^x))}{dx}$

$\Rightarrow 2\frac{d(\tan^{-1}(2^x))}{d2^x}\frac{d2^x}{dx}$

$\Rightarrow 2\frac{1}{1+(2^x)^2}.\ 2^x \log 2$

$\Rightarrow \frac{2^{1+x}\log 2}{1+4^x}.$

Ans) $\frac{2^{1+x}\log 2}{1+4^x}$

Exercise 10E

1. Question

Find , when:

$x^2 + y^2 = 4$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$d(y^2)/_{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dx} = \frac{d(4)}{dx}$$

$$2x + 2y \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-2x}{2y}$$

$$\frac{dy}{dx} = \frac{-x}{y}$$

2. Question

Find , when:

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(^{y^2}/_{b^2})}{dx} = \frac{d(^{y^2}/_{b^2})}{dy} \times \frac{dy}{dx} = \frac{2y}{b^2} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(^{x^2}/_{a^2})}{dx} + \frac{d(^{y^2}/_{b^2})}{dx} = \frac{d(1)}{dx}$$

$$\frac{2x}{a^2} + \frac{2y}{b^2} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\frac{2x}{a^2}}{\frac{2y}{b^2}}$$

$$\frac{dy}{dx} = \frac{-b^2 x}{a^2 y}$$

3. Question

Find , when:

$$\sqrt{x} + \sqrt{y} = \sqrt{a}$$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(\sqrt{y})}{dx} = \frac{d(\sqrt{y})}{dy} \times \frac{dy}{dx} = \frac{1}{2\sqrt{y}} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(\sqrt{x})}{dx} + \frac{d(\sqrt{y})}{dx} = \frac{d(\sqrt{a})}{dx}$$

$$\frac{1}{2\sqrt{x}} + \frac{1}{2\sqrt{y}} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\frac{1}{2\sqrt{x}}}{\frac{1}{2\sqrt{y}}}$$

$$\frac{dy}{dx} = \frac{-2\sqrt{y}}{2\sqrt{x}}$$

$$\frac{dy}{dx} = \frac{-\sqrt{y}}{\sqrt{x}}$$

4. Question

Find , when:

$$x^{2/3} + y^{2/3} = a^{2/3}$$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(y^{2/3})}{dx} = \frac{d(y^{2/3})}{dy} \times \frac{dy}{dx} = \frac{2}{3y^{1/3}} \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^{2/3})}{dx} + \frac{d(y^{2/3})}{dx} = \frac{d(a^{2/3})}{dx}$$

$$\frac{2}{3\,x^{1/3}} + \frac{2}{3y^{1/3}} \times \frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-\frac{2}{3x^{1/3}}}{\frac{2}{3y^{1/3}}}$$

$$\frac{dy}{dx} = \frac{-y^{1/3}}{x^{1/3}}$$

5. Question

Find , when:

$xy = c^2$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(xy)}{dx} = \frac{d(c^2)}{dx}$$

$$x \times \frac{dy}{dx} + y = 0$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

$$\frac{dy}{dx} = \frac{-xy}{x^2}$$

$$\frac{dy}{dx} = \frac{-c^2}{x^2}$$

6. Question

Find , when:

$x^2 + y^2 - 3xy = 1$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(x^2)}{dx} + \frac{d(y^2)}{dx} - 3\frac{d(xy)}{dx} = \frac{d(1)}{dx}$$

$$2x + 2y \times \frac{dy}{dx} - 3(x \times \frac{dy}{dx} + y) = 0$$

$$(2y - 3x)\frac{dy}{dx} + 2x - 3y = 0$$

$$\frac{dy}{dx} = \frac{-(2x - 3y)}{2y - 3x}$$

$$\frac{dy}{dx} = \frac{2x - 3y}{3x - 2y}$$

7. Question

Find , when:

$xy^2 - x^2y - 5 = 0$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx}=\frac{xd(y)}{dx}+\frac{yd(x)}{dx}=x\times\frac{dy}{dx}+y$$

Therefore ,

$$\frac{d(xy^2)}{dx}-\frac{d(x^2y)}{dx}=\frac{d(5)}{dx}$$

$$x\times\frac{d(y^2)}{dx}+y^2-[x^2\times\frac{d(y)}{dx}+y\times 2x]=0$$

$$x\times(2y\times\frac{dy}{dx})+y^2-[x^2\times\frac{d(y)}{dx}+y\times 2x]=0$$

$$2xy\frac{dy}{dx}-x^2\frac{dy}{dx}+y^2-2xy=0$$

$$\frac{dy}{dx}=\frac{2xy-y^2}{2xy-x^2}\frac{dy}{dx}=\frac{y^2-2xy}{x^2-2xy}$$

8. Question

Find , when:

$(x^2 + y^2)^2 = xy$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx}=n\times x^{(n-1)}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx}=\frac{d(y^2)}{dy}\times\frac{dy}{dx}=2y\times\frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx}=\frac{xd(y)}{dx}+\frac{yd(x)}{dx}=x\times\frac{dy}{dx}+y$$

Therefore ,

$$\frac{d((x^2+y^2)^2)}{dx}=\frac{d(xy)}{dx}$$

$$2(x^2+y^2)\times\frac{d(x^2+y^2)}{dx}=[x\times\frac{d(y)}{dx}+y]$$

$$2(x^2+y^2)\times[2x+2y\times\frac{dy}{dx}]=[x\times\frac{d(y)}{dx}+y]$$

$$4x(x^2+y^2)+4y(x^2+y^2)\frac{dy}{dx}=x\frac{dy}{dx}+y$$

$$\frac{dy}{dx}[4y(x^2+y^2)-x]=y-4x(x^2+y^2)$$

$$\frac{dy}{dx}=\frac{y-4x(x^2+y^2)}{[4y(x^2+y^2)-x]}$$

$$\frac{dy}{dx}=\frac{y-4x^3-4xy^2}{4y^3+4x^2y-x}$$

9. Question

Find , when:

$x^2 + y^2 = \log(xy)$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx}=n\times x^{(n-1)}, \frac{d(\log x)}{dx}=\frac{1}{x}$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx}=\frac{d(y^2)}{dy}\times\frac{dy}{dx}=2y\times\frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx}=\frac{xd(y)}{dx}+\frac{yd(x)}{dx}=x\times\frac{dy}{dx}+y$$

Therefore ,

$$\frac{d(x^2)}{dx}+\frac{d(y^2)}{dx}=\frac{d(\log xy)}{dx}$$

$$2x+2y\frac{d(y)}{dx}=[\frac{1}{xy}\frac{d(xy)}{dx}]$$

$$2x+2y\frac{d(y)}{dx}=\frac{1}{xy}(x\frac{dy}{dx}+y)$$

$$2x+2y\frac{dy}{dx}=\frac{1}{y}\frac{dy}{dx}+\frac{1}{x}$$

$$\frac{dy}{dx}\left[2y-\frac{1}{y}\right]=\frac{1}{x}-2x$$

$$\frac{dy}{dx}\left(\frac{2y^2-1}{y}\right)=\frac{1-2x^2}{x}$$

$$\frac{dy}{dx}=\frac{y(1-2x^2)}{x(2y^2-1)}$$

10. Question

Find , when:

$x^n + y^n = a^n$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx}=n\times x^{(n-1)}$

According to the chain rule of differentiation

$$\frac{d(y^n)}{dx}=\frac{d(y^n)}{dy}\times\frac{dy}{dx}=ny^{n-1}\times\frac{dy}{dx}$$

Therefore ,

$$\frac{d(x^n)}{dx}+\frac{d(y^n)}{dx}=\frac{d(a^n)}{dx}$$

$$nx^{n-1}+ny^{n-1}\times\frac{dy}{dx}=0$$

$$\frac{dy}{dx}=\frac{-nx^{n-1}}{ny^{n-1}}$$

$$\frac{dy}{dx}=\frac{-x^{n-1}}{y^{n-1}}$$

11. Question

Find , when:

x sin 2y = y cos 2x

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin x)}{dx}=\cos x\,,\frac{d(\cos x)}{dx}=-\sin x$

According to the chain rule of differentiation

$$\frac{d(\sin 2y)}{dx} = \frac{d(\sin 2y)}{dy} \times \frac{dy}{dx} = 2\cos 2y \times \frac{dy}{dx}$$

According to the product rule of differentiation

$$\frac{d(x\sin 2y)}{dx} = \frac{xd(\sin 2y)}{dx} + \frac{\sin 2y\, d(x)}{dx} = x \times \frac{d(\sin 2y)}{dx} + \sin 2y$$

Therefore ,

$$\frac{d(x\sin 2y)}{dx} = \frac{d(y\cos 2x)}{dx}$$

$$x \times \frac{d(\sin 2y)}{dx} + \sin 2y = \cos 2x \times \frac{d(y)}{dx} + y(-2\sin 2x)$$

$$x \times 2\cos 2y \times \frac{dy}{dx} + \sin 2y = \cos 2x \times \frac{d(y)}{dx} + y(-2\sin 2x)$$

$$\frac{dy}{dx}[2x\, cos\; 2y - \cos 2x] = -2y\sin 2x - \sin 2y$$

$$\frac{dy}{dx} = \frac{-(2y\sin 2x + \sin 2y)}{2x\; cos\; 2y - \cos 2x}$$

$$\frac{dy}{dx} = \frac{(2y\sin 2x + \sin 2y)}{\cos 2x - 2x\; cos\; 2y}$$

12. Question

Find , when:

$\sin^2 x + 2\cos y + xy$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(sin\, x)}{dx} = cos\; x\; , \frac{d(cos\, x)}{dx} = -sin\; x$

According to chain rule of differentiation

$$\frac{d(\cos y)}{dx} = \frac{d(\cos y)}{dy} \times \frac{dy}{dx} = -\sin y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(sin^2 x)}{dx} + \frac{d(2\cos y)}{dx} + \frac{d(xy)}{dx} = 0$$

$$2\sin x\times\frac{d(\sin x)}{dx}+2\left(-\sin y\times\frac{dy}{dx}\right)+x\times\frac{dy}{dx}+y=0$$

$$2\sin x\times\cos x+y=2\left(\sin y\times\frac{dy}{dx}\right)-x\times\frac{dy}{dx}$$

$$\frac{dy}{dx}[2\sin y-x]=\sin 2x+y$$

$$\frac{dy}{dx}=\frac{\sin 2x+y}{2x\cos 2y-\cos 2x}$$

$$\frac{dy}{dx}=\frac{\sin 2x+y}{2\sin y-x}$$

13. Question

Find , when:

y sec x + tan x + x^2y = 0

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sec x)}{dx}=\sec x\tan x$, $\frac{d(\tan x)}{dx}=sec^2x$

According to product rule of differentiation

$$\frac{d(x^2y)}{dx}=\frac{x^2\mathrm{dy}}{dx}+\frac{yd(x^2)}{dx}=x^2\frac{dy}{dx}+2xy$$

Therefore ,

$$\frac{d(y\sec x)}{dx}+\frac{d(\tan x)}{dx}+\frac{d(x^2y)}{dx}=0$$

$$\sec x\times\frac{d(y)}{dx}+y\sec x\tan x+\sec^2\mathrm{x}+x^2\frac{dy}{dx}+2xy=0$$

$$\frac{dy}{dx}[x^2+\sec x]=-(y\sec x\tan x+\sec^2\mathrm{x}+2\mathrm{xy})$$

$$\frac{dy}{dx}=\frac{-(y\sec x\tan x+\sec^2\mathrm{x}+2\mathrm{xy})}{x^2+\sec x}$$

14. Question

Find , when:

cot (xy) + xy = y

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\cot x)}{dx} = -cosec^2 x$

According to chain rule of differentiation

$$\frac{d(\cot xy)}{dx} = -\mathrm{cosec}^2\, \mathrm{xy} \times \frac{d(xy)}{dx}$$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xdy}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(\cot xy)}{dx} + \frac{d(xy)}{dx} = \frac{dy}{dx}$$

$$-\mathrm{cosec}^2\, \mathrm{xy} \times \frac{d(xy)}{dx} + \frac{d(xy)}{dx} = \frac{dy}{dx}$$

$$\frac{d(xy)}{dx}[-\mathrm{cosec}^2\, \mathrm{xy} + 1] = \frac{dy}{dx}$$

$[x\frac{dy}{dx} + y][-\cot^2 \mathrm{xy}] = \frac{dy}{dx}$ (Since, $1 - \mathrm{cosec}^2\, \mathrm{xy} = -\cot^2 \mathrm{xy}$)

$$x\frac{dy}{dx}(-\cot^2 \mathrm{xy}) - y\cot^2 \mathrm{xy} = \frac{dy}{dx}$$

$$\frac{dy}{dx}[-\mathrm{x}\cot^2 \mathrm{xy} - 1] = y\cot^2 \mathrm{xy}$$

$$\frac{dy}{dx} = \frac{-\,y\cot^2 \mathrm{xy}}{\mathrm{x}\cot^2 \mathrm{xy} + 1}$$

15. Question

Find , when:

y tan x − y^2 cos x + 2x = 0

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan x)}{dx} = sec^2 x$, $\frac{d(\cos x)}{dx} = -\sin x$

According to chain rule of differentiation

$$\frac{d(y^2)}{dx}=\frac{d(y^2)}{dy}\times\frac{dy}{dx}=2y\times\frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(y\tan x)}{dx}=\text{ysec}^2\text{ x}+\tan x\times\frac{dy}{dx}$$

Therefore ,

$$\frac{d(y\tan x)}{dx}-\frac{d(y^2\cos x)}{dx}+\frac{d(2x)}{dx}=0$$

$$\text{ysec}^2\text{ x}+\tan x\times\frac{dy}{dx}-\cos x\frac{d(y^2)}{dx}-y^2(-\sin x)+2=0$$

$$\text{ysec}^2\text{ x}+\tan x\times\frac{dy}{dx}-\cos x\left(2y\frac{dy}{dx}\right)+y^2(\sin x)+2=0$$

$$\text{ysec}^2\text{ x}+\frac{dy}{dx}[\tan x-2y\cos x]+y^2(\sin x)+2=0$$

$$\text{ysec}^2\text{ x}+y^2(\sin x)+2=\frac{dy}{dx}[2y\cos x-\tan x]$$

$$\frac{dy}{dx}=\frac{\text{ysec}^2\text{ x}+y^2\sin x+2}{2y\cos x-\tan x}$$

16. Question

Find , when:

$e^x \log y = \sin^{-1} x + \sin^{-1} y$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin^{-1}x)}{dx}=\frac{1}{\sqrt{1-x^2}}$, $\frac{d(\log x)}{dx}=\frac{1}{x}$

According to chain rule of differentiation

$$\frac{d(\sin^{-1}y)}{dx}=\frac{d(\sin^{-1}y)}{dy}\times\frac{dy}{dx}=\frac{1}{\sqrt{1-y^2}}\times\frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(e^x\log y)}{dx}=e^x\log y+e^x\times\frac{d(\log y)}{dx}=e^x\log y+e^x\times\frac{1}{y}\times\frac{dy}{dx}$$

Therefore ,

$$\frac{d(e^x \log y)}{dx} = \frac{d(\sin^{-1} x)}{dx} + \frac{d(\sin^{-1} y)}{dx}$$

$$e^x \log y + e^x \frac{1}{y}\frac{dy}{dx} = \frac{1}{\sqrt{1-\mathrm{x}^2}} + \frac{1}{\sqrt{1-\mathrm{y}^2}} \times \frac{dy}{dx}$$

$$\frac{dy}{dx}[e^x \frac{1}{y} - \frac{1}{\sqrt{1-\mathrm{y}^2}}] = \frac{1}{\sqrt{1-\mathrm{x}^2}} - e^x \log y$$

$$\frac{dy}{dx}[\frac{e^x\sqrt{1-\mathrm{y}^2} - y}{y\sqrt{1-\mathrm{y}^2}}] = \frac{1-(e^x \log y\, \sqrt{1-\mathrm{x}^2})}{\sqrt{1-\mathrm{x}^2}}$$

$$\frac{dy}{dx} = \frac{y\sqrt{1-\mathrm{y}^2}}{e^x\sqrt{1-\mathrm{y}^2} - y} \times \frac{1-(e^x \log y\, \sqrt{1-\mathrm{x}^2})}{\sqrt{1-\mathrm{x}^2}}$$

$$\frac{dy}{dx} = y \times \sqrt{\frac{1-\mathrm{y}^2}{1-\mathrm{x}^2}} \times \frac{1-(e^x \log y\, \sqrt{1-\mathrm{x}^2})}{(e^x\sqrt{1-\mathrm{y}^2}) - y}$$

17. Question

Find , when:

xy log (x + y) = 1

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xd(y)}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(xy \times \log x + y)}{dx} = \frac{d(1)}{dx}$$

$$\log x + y \times \frac{d(xy)}{dx} + xy \times \frac{d(\log x + y)}{dx} = \frac{d(1)}{dx}$$

$$\log x + y\left[x\frac{dy}{dx} + y\right] + xy[\frac{1}{x+y} \times \left(1 + \frac{dy}{dx}\right)] = 0$$

$$\frac{dy}{dx}[x \times \log x + y] + y \times \log(x+y) + \frac{xy}{x+y}\left(1 + \frac{dy}{dx}\right) = 0$$

$$\frac{dy}{dx}\left(x\log(x+y)+\frac{xy}{x+y}\right)=-\left(y\log(x+y)+\frac{xy}{x+y}\right)$$

$$\frac{dy}{dx}[(x^2+xy)\log(x+y)+xy]=-[(y^2+xy)\log(x+y)+xy]$$

$\frac{dy}{dx}=\frac{-y^2\log(x+y)-xy\log(x+y)-xy}{x[(x+y)\log(x+y)+y]}\times\frac{x}{x}$ (Multiply and divide by x)

$$\frac{dy}{dx}=\frac{-y\;xy\log(x+y)-x\;xy\log(x+y)-x^2y}{x^2[(x+y)\log(x+y)+y]}$$

$$\frac{dy}{dx}=\frac{-y\;(1)-x\;(1)-x^2y}{x^2[(x+y)\log(x+y)+y]}$$

$$\frac{dy}{dx}=\frac{-\left(x+y+x^2y\right)}{x^2\left\{y+(x+y)\log(x+y)\right\}}$$

18. Question

Find , when:

tan (x + y) + tan (x — y) = 1

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan x)}{dx}=sec^2x$

Therefore ,

$$\frac{d(\tan(x+y))}{dx}+\frac{d(\tan(x-y))}{dx}=\frac{d(1)}{dx}$$

$$\sec^2(x+y)[1+\frac{dy}{dx}]+\sec^2(x-y)[1-\frac{dy}{dx}]=0$$

$$\sec^2(x+y)+\sec^2(x+y)\frac{dy}{dx}+\sec^2(x-y)-\sec^2(x-y)\frac{dy}{dx}=0$$

$$\sec^2(x+y)+\sec^2(x-y)=\frac{dy}{dx}[\sec^2(x-y)-\sec^2(x+y)]$$

$$\frac{dy}{dx}=\frac{\sec^2(x+y)+\sec^2(x-y)}{\sec^2(x+y)-\sec^2(x-y)}$$

19. Question

Find , when:

$$\log\sqrt{x^2+y^2}=\tan^{-1}\frac{y}{x}$$

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan^{-1}x)}{dx}=\frac{1}{1+x^2}$, $\frac{d(\log x)}{dx}=\frac{1}{x}$

According to quotient rule of differentiation

$$\frac{d(^y/_x)}{dx}=\frac{\frac{xd(y)}{dx}-\frac{yd(x)}{dx}}{x^2}=\frac{x\frac{dy}{dx}-y}{x^2}$$

Therefore ,

$$\frac{d(\log\sqrt{x^2+y^2})}{dx}=\frac{d(\tan^{-1}\frac{y}{x})}{dx}$$

$$\frac{1}{\sqrt{x^2+y^2}}\times\frac{d(\sqrt{x^2+y^2})}{dx}=\frac{1}{1+(\frac{y}{x})^2}\times\frac{d(\frac{y}{x})}{dx}$$

$$\frac{1}{\sqrt{x^2+y^2}}\times\frac{1}{2\sqrt{x^2+y^2}}\times[2x+2y\frac{d(y)}{dx}]=\frac{1}{1+(\frac{y}{x})^2}\times\frac{x\frac{dy}{dx}-y}{x^2}$$

$$\frac{1}{x^2+y^2}\times[x+y\frac{dy}{dx}]=\frac{x^2}{x^2+y^2}\times\frac{x\frac{dy}{dx}-y}{x^2}$$

$$x+y\frac{dy}{dx}=x\frac{dy}{dx}-y$$

$$\frac{dy}{dx}[x-y]=x+y$$

$$\frac{dy}{dx}=\frac{x+y}{x-y}$$

20. Question

Find , when:

If y = x sin y , prove that $\left(x.\frac{dy}{dx}\right)=\frac{y}{(1-x\cos y)}$.

There is correction in question …. Prove that should be $\frac{dy}{dx} = \frac{\sin y}{1 - x\cos y}$ instead of

$\left(x.\frac{dy}{dx}\right) = \frac{y}{(1 - x\cos y)}$ to get the required answer.

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\sin x)}{dx} = \cos x$

According to chain rule of differentiation

$$\frac{d(\sin y)}{dx} = \frac{d(\sin y)}{dy} \times \frac{dy}{dx} = \cos y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(y)}{dx} = \frac{d(x\sin y)}{dx}$$

$$\frac{dy}{dx} = x\frac{d(\sin y)}{dx} + \sin y$$

$$\frac{dy}{dx} = x\cos y\frac{dy}{dx} + \sin y$$

$$\frac{dy}{dx}[1 - x\cos y] = \sin y$$

$$\frac{dy}{dx} = \frac{\sin y}{1 - x\cos y}$$

21. Question

Find , when:

If xy = tan (xy), show that $\frac{dy}{dx} = \frac{-y}{x}$.

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\tan x)}{dx} = sec^2 x$

According to product rule of differentiation

$$\frac{d(xy)}{dx} = \frac{xdy}{dx} + \frac{yd(x)}{dx} = x \times \frac{dy}{dx} + y$$

Therefore ,

$$\frac{d(xy)}{dx} = \frac{d(\tan xy)}{dx}$$

$$x\frac{dy}{dx} + y = sec^2(xy) \times \frac{d(xy)}{dx}$$

$$x\frac{dy}{dx} + y = sec^2(xy) \times [x\frac{dy}{dx} + y]$$

$$\frac{dy}{dx}[x - xsec^2(xy)] = ysec^2(xy) - y$$

$$x\frac{dy}{dx}(1 - \sec^2 xy) = y(sec^2(xy) - 1)$$

$$\frac{dy}{dx} = \frac{-y(1 - sec^2(xy))}{x(1 - \sec^2 xy)}$$

$$\frac{dy}{dx} = \frac{-y}{x}$$

22. Question

Find , when:

If y log x = (x — y), prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(x^n)}{dx} = n \times x^{(n-1)}$, $\frac{d(\log x)}{dx} = \frac{1}{x}$

According to product rule of differentiation

$$\frac{d(y\log x)}{dx} = \frac{\log x\, d(y)}{dx} + \frac{y\, d(\log x)}{dx} = \log x \times \frac{dy}{dx} + \frac{y}{x}$$

Therefore ,

$$\frac{d(y \times \log x)}{dx} = \frac{d(x - y)}{dx}$$

$$\log x \times \frac{d(y)}{dx} + \frac{y}{x} = 1 - \frac{d(y)}{dx}$$

$$\frac{dy}{dx}[\log x \ + 1] = 1 - \frac{y}{x}$$

$$\frac{dy}{dx}[(1 + \log x)^2] = 1 - \frac{y}{x}(1 + \log x)$$

(Multiply by 1+log x on both sides)

$$\frac{dy}{dx}[(1 + \log x)^2] = 1 + \log x - \frac{y}{x} - \frac{y}{x}\log x$$

$\frac{dy}{dx}[(1 + \log x)^2] = 1 + \log x - \frac{y}{x} - \frac{(x-y)}{x}$ (y log x = x - y)

$$\frac{dy}{dx}[(1 + \log x)^2] = 1 + \log x - \frac{y}{x} - 1 + \frac{y}{x}$$

$$\frac{dy}{dx} = \frac{\log x}{(1 + \log x)^2}$$

23. Question

Find , when:

If cos y = x cos (y + a), prove that $\frac{dy}{dx} = \frac{\cos^2(y + a)}{\sin a}$.

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\cos x)}{dx} = -\sin x$

According to chain rule of differentiation

$$\frac{d(\cos y)}{dx} = \frac{d(\cos y)}{dy} \times \frac{dy}{dx} = -\sin y \times \frac{dy}{dx}$$

According to product rule of differentiation

$$\frac{d(x \cos (y + a))}{dx} = x\frac{d(\cos y + a)}{dx} + \cos(y + a)$$

Therefore ,

$$\frac{d(\cos y)}{dx} = \frac{d(x \cos(y + a))}{dx}$$

$$-\sin y \frac{dy}{dx} = x\frac{d(\cos(y + a))}{dx} + \cos(y + a)$$

$$-\sin y\frac{dy}{dx} = x(-\sin(y+a)\frac{dy}{dx}) + \cos(y+a)$$

$$\frac{dy}{dx}[-\sin y + x sin\,(y+a)] = \cos(y+a)$$

$$\frac{dy}{dx} = \frac{\cos(y+a)}{x\sin(y+a) - \sin y}$$

$$\frac{dy}{dx} = \frac{cos^2(y+a)}{x\cos(y+a)\sin(y+a) - \cos(y+a)\sin y}$$

(Multiply and divide by cos (y+a))

$\frac{dy}{dx} = \frac{cos^2(y+a)}{\sin(y+a)\cos y - \cos(y+a)\sin y}$ (Since cos y = x cos (y + a))

$\frac{dy}{dx} = \frac{cos^2(y+a)}{\sin(y+a-y)}$ (Formula sin(a-b)=sin a cos b – cos a sin b)

$$\frac{dy}{dx} = \frac{cos^2(y+a)}{\sin a}$$

24. Question

Find , when:

If $\cos^{-1}\left(\frac{x^2-y^2}{x^2+y^2}\right) = \tan^{-1} a$, prove that $\frac{dy}{dx} = \frac{y}{x}$.

Answer

Let us differentiate the whole equation w.r.t x

Formula : $\frac{d(\cos^{-1}x)}{dx} = -\frac{1}{\sqrt{1-x^2}}$

According to the chain rule of differentiation

$$\frac{d(y^2)}{dx} = \frac{d(y^2)}{dy} \times \frac{dy}{dx} = 2y \times \frac{dy}{dx}$$

Therefore ,

$$\frac{d(\cos^{-1}\frac{x^2-y^2}{x^2+y^2})}{dx} = \frac{d(\tan^{-1}a)}{dx}$$

$$-\frac{1}{\sqrt{1-\left(\frac{x^2-y^2}{x^2+y^2}\right)^2}} \times \frac{d\left(\frac{x^2-y^2}{x^2+y^2}\right)}{dx} = 0$$

$$\frac{d\left(\frac{x^2-y^2}{x^2+y^2}\right)}{dx} = 0$$

$$\frac{x^2+y^2\left[\frac{d(x^2-y^2)}{dx}\right]-(x^2-y^2)\left[\frac{d(x^2+y^2)}{dx}\right]}{(x^2+y^2)^2} = 0$$

$$x^2+y^2\left[\frac{d(x^2-y^2)}{dx}\right]-(x^2-y^2)\left[\frac{d(x^2+y^2)}{dx}\right] = 0$$

$$(x^2+y^2)\left(2x-2y\frac{dy}{dx}\right)-(x^2-y^2)\left(2x+2y\frac{dy}{dx}\right) = 0$$

$$(x^2+y^2)\left(x-y\frac{dy}{dx}\right) = (x^2-y^2)\left(x+y\frac{dy}{dx}\right)$$

$$\frac{dy}{dx}[-x^2y-y^3-x^2y+y^3] = x^3-xy^2-x^3-xy^2$$

$$\frac{dy}{dx}[-2x^2y] = -2xy^2$$

$$\frac{dy}{dx} = \frac{-2xy^2}{-2yx^2}$$

$$\frac{dy}{dx} = \frac{y}{x}$$

Exercise 10F

1. Question

Find $\frac{dy}{dx}$, when:

$y = x^{1/x}$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = \frac{lnx}{x}\ \{\ln(x^m) = m(lnx)\}$$

Now differentiating both sides by x we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{x\left(\frac{1}{x}\right) - lnx(1)}{x^2}$$

$$\frac{dy}{dx} = \frac{1 - lnx}{x^2} \times y \left\{ divide\ rule\ \frac{d\left(\frac{u}{v}\right)}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2} \right\}$$

$$\frac{dy}{dx} = \frac{1 - lnx}{x^2} \times x^{\frac{1}{x}}\ \{y = x^{\frac{1}{x}}\}$$

2. Question

Find $\frac{dy}{dx}$, when:

$$y = x^{\sqrt{x}}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = \sqrt{x}\,lnx$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \sqrt{x}\left(\frac{1}{x}\right) + lnx\left(\frac{1}{2\sqrt{x}}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}}\left(1 + \frac{lnx}{2}\right) \times y$$

$$\frac{dy}{dx} = \frac{1}{\sqrt{x}} \times \left(1 + \frac{lnx}{2}\right) \times \left(x^{\sqrt{x}}\right)$$

3. Question

Find $\frac{dy}{dx}$, when:

$$y = (\log x)^x$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = x\ln(lnx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx} = x\left(\frac{1}{lnx}\times\frac{1}{x}\right) + \ln(lnx)\ \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{1}{lnx} + \ln(lnx)\right)\times y$$

$$\frac{dy}{dx} = \left(\frac{1}{lnx} + \ln(lnx\ )\right)\times (lnx)^x$$

4. Question

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = \mathrm{x}^{\sin \mathrm{x}}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = sinx\ lnx$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx} = sinx\times\frac{1}{x} +\ lnx\times cosx\ \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(sinx\times\frac{1}{x} + lnx\times cosx\ \right)\times y$$

$$\frac{dy}{dx} = \left(\frac{sinx}{x} + cosx(lnx\ )\right)\times x^{sinx}$$

5. Question

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = \mathrm{x}^{(\cos^{-1}\mathrm{x})}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = \cos^{-1} x \, lnx$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \cos^{-1} x \times \left(\frac{1}{x}\right) + lnx \times \left(-\frac{1}{\sqrt{1-x^2}}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \cos^{-1} x \times \left(\frac{1}{x}\right) + lnx \times \left(-\frac{1}{\sqrt{1-x^2}}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\cos^{-1} x}{x} - \frac{lnx}{\sqrt{1-x^2}}\right) \times x^{(\cos^{-1} x)}$$

6. Question

Find $\frac{dy}{dx}$, when:

$$y = (\tan x)^{1/x}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = \left(\frac{1}{x}\right)\ln(tanx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \left(\frac{1}{x}\right) \times \left(\frac{1}{tanx} \times \sec^2 x\right) + \ln(tanx) \times \left(-\frac{1}{x^2}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\sec^2 x}{x \times tanx} - \frac{\ln(tanx)}{x^2}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\sec^2 x}{x \times tanx} - \frac{\ln(tanx)}{x^2}\right) \times tanx^{\frac{1}{x}}$$

7. Question

Find $\frac{dy}{dx}$, when:

$y = (\sin x)^{\cos x}$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (cosx)\ln(sinx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (cosx) \times \left(\frac{1}{sinx} \times cosx\right) + \ln(sinx) \times (-sinx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\cos^2 x}{sinx} - sinx(\ln(sinx))\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\cos^2 x}{sinx} - sinx(\ln(sinx))\right) \times sinx^{cosx}$$

8. Question

Find $\frac{dy}{dx}$, when:

$y = (\log x)^{\sin x}$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (sinx)\ln(lnx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (sinx) \times \left(\frac{1}{lnx} \times \frac{1}{x}\right) + \ln(lnx) \times (cosx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\sin x}{x \times lnx} - cosx(\ln(lnx))\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\sin x}{x \times lnx} - cosx(\ln(lnx))\right) \times lnx^{sinx}$$

9. Question

Find $\frac{dy}{dx}$, when:

$$y = (\cos x)^{\log x}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (lnx)\ln(cosx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (lnx) \times \left(\frac{1}{cosx} \times (-sinx)\right) + \ln(cosx) \times \left(\frac{1}{x}\right) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(-\frac{\sin x \times lnx}{cosx} + \frac{(\ln cosx)}{x}\right) \times y$$

$$\frac{dy}{dx} = \left(-\frac{\sin x \times lnx}{cosx} + \frac{(\ln cosx)}{x}\right) \times cosx^{lnx}$$

10. Question

Find $\frac{dy}{dx}$, when:

$$y = (\tan x)^{\sin x}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (sinx)\ln(tanx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (sinx) \times \left(\frac{1}{tanx} \times (\sec^2 x)\right) + \ln(tanx) \times (cosx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{\sin x \times \sec^2 x}{tanx} + \ln(tanx)\,cosx\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{\sin x \times \sec^2 x}{tanx} + \ln(tanx)\, cosx\right) \times tanx^{sinx}$$

11. Question

Find $\frac{dy}{dx}$, when:

$$y = (\cos x)^{\cos x}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (cosx)\ln(cosx)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (cosx) \times \left(\frac{1}{cosx} \times (-sinx)\right) + \ln(cosx) \times (-sinx) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = (-sinx - \ln(cosx)\, sinx) \times y$$

$$\frac{dy}{dx} = (-sinx - \ln(cosx)\, sinx) \times cosx^{cosx}$$

12. Question

Find $\frac{dy}{dx}$, when:

$$y = (\tan x)^{\cot x}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (\cot x)\ln(\tan x)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (cotx) \times \left(\frac{1}{tanx} \times (-\sec^2 x)\right) + \ln(tanx) \times (-cosec^2 x) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = (-cosec^2x - \ln(tanx)cosec^2x) \times y$$

$$\frac{dy}{dx} = -cosec^2x \times (1 + \ln(cosx)) \times tanx^{\cot x}$$

13. Question

Find $\frac{dy}{dx}$, when:

$$y = x^{\sin 2x}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (sin2x)\ln(x)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (sin2x) \times \left(\frac{1}{x}\right) + \ln(x) \times (cos2x \times 2) \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{sin2x}{x} + 2\cos 2x \times \ln x\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{sin2x}{x} + 2\cos 2x \times \ln x\right) \times x^{\sin 2x}$$

14. Question

Find $\frac{dy}{dx}$, when:

$$y = (\sin^{-1} x)^x$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (x)\ln(\sin^{-1} x)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=(x)\times\left(\frac{1}{\sin^{-1}x}\times\frac{1}{\sqrt{1-x^2}}\right)$$
$$+\ln(\sin^{-1}x)\left\{product\ rule,\frac{d(uv)}{dx}=u\frac{dv}{dx}+v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx}=\left(\frac{x}{\sin^{-1}x\times\sqrt{1-x^2}}\times\ln\sin^{-1}x\right)\times y$$

$$\frac{dy}{dx}=\left(\frac{x}{\sin^{-1}x\times\sqrt{1-x^2}}\times\ln\sin^{-1}x\right)\times\sin^{-1}x^x$$

15. Question

Find $\frac{dy}{dx}$, when:

$$y=\sin(x^x)$$

Answer

Here, the argument of the sinusoidal function has exponent as x itself.

For that, we will consider $x^x=u$ for simplicity.

$$y=\sin u$$

Differentiating both the sides,

$\frac{dy}{dx}=\cos u\times\frac{du}{dx}$. ....(1)

Now we have to find $\frac{du}{dx}$, where $u=x^x$

take log both the sides

$$\ln u=x\ln x$$

Now differentiating both sides by x, we get,

$$\frac{1}{u}\times\frac{du}{dx}=x\left(\frac{1}{x}\right)+\ln x$$

$$\frac{du}{dx}=(1+\ln x)\times u$$

$$\frac{du}{dx}=(1+\ln x)\times x^x$$

Substituting the value in equation 1,

$$\frac{dy}{dx} = cosx\ (1 + \ln x\ ) \times x^x$$

16. Question

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = (3\mathrm{x} + 5)^{(2\mathrm{x}-3)}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = (2x - 3)\ln(3x - 5)$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = (2x - 3) \times \left(\frac{1}{3x - 5} \times 3\right) + \ln(3x - 5) \times 2\ \left\{product\ rule, \frac{d(uv)}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}\right\}$$

$$\frac{dy}{dx} = \left(\frac{2x - 3}{3x - 5} \times 3 + \ln(3x - 5) \times 2\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2x - 3}{3x - 5} \times 3 + 2 \times \ln 3x - 5\right) \times (3x - 5)^{2x-3}$$

17. Question

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$\mathrm{y} = (\mathrm{x} + 1)^3 (\mathrm{x} + 2)^4 (\mathrm{x} + 3)^5$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$lny = 3\ln(x + 1) + \ 4\ln(x + 2) + 5\ln(x + 3)\ \{\ln(mn) = \ln n + \ln m\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{3}{x + 1} + \frac{4}{x + 2} + \frac{5}{x + 3}$$

$$\frac{dy}{dx} = \left(\frac{3}{x + 1} + \frac{4}{x + 2} + \frac{5}{x + 3}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{3}{x+1} + \frac{4}{x+2} + \frac{5}{x+3}\right) \times (x+1)^3(x+2)^4(x+3)^5$$

18. Question

Find $\frac{dy}{dx}$, when:

$$y = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \frac{1}{2}(\ln(x-1) + \ln(x-2) - \ln(x-3) - \ln(x-4) - \ln(x-5))$$

$$\{\ln(mn) = \ln n + \ln m\} \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right)$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right) \times y$$

$$\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{x-1} + \frac{1}{x-2} - \frac{1}{x-3} - \frac{1}{x-4} - \frac{1}{x-5}\right) \times \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

19. Question

Find $\frac{dy}{dx}$, when:

$$y = (2-x)^3(3+2x)^5$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 3\ln(2-x) + 5\ln(3+2x)$$

$$\{\ln(mn) = \ln n + \ln m\} \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = 3\left(\frac{-1}{2-x}\right) + 5\left(\frac{1}{3+2x} \times 2\right)$$

$$\frac{dy}{dx} = \left(\frac{3}{x-2} + \frac{10}{3+2x}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{3}{x-2} + \frac{10}{3+2x}\right) \times (2-x)^3(3+2x)^5$$

20. Question

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y = \cos x \cos 2x \cos 3x$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = \ln(\cos x) + \ln(\cos 2x) + \ln \cos 3x$$

$$\{\ln(mn) = \ln n + \ln m\} \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{1}{\cos x} \times (-\sin x) + \frac{1}{\cos 2x} \times (-2\sin 2x) + \frac{1}{\cos 3x}(-3\sin 3x)$$

$$\frac{dy}{dx} = \left(\frac{-\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{-\sin x}{\cos x} - \frac{2\sin 2x}{\cos 2x} - \frac{3\sin 3x}{\cos 3x}\right) \times \cos x \cos 2x \cos 3x$$

$$\frac{dy}{dx} = (-\tan x - 2\tan 2x - 3\tan 3x) \times \cos x \cos 2x \cos 3x$$

21. Question

Find $\frac{\mathrm{dy}}{\mathrm{dx}}$, when:

$$y = \frac{x^5\sqrt{x+4}}{(2x+3)^2}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 5\ln(x) + \frac{1}{2}\ln(x+4) - 2\ln(2x+3)$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{5}{x} + \frac{1}{2(x+4)} - \frac{4}{2x+3}$$

$$\frac{dy}{dx} = \left(\frac{5}{x} + \frac{1}{2(x+4)} - \frac{4}{2x+3}\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{5}{x} + \frac{1}{2(x+4)} - \frac{4}{2x+3}\right) \times \frac{x^5\sqrt{x+4}}{(2x+3)^2}$$

22. Question

Find $\frac{dy}{dx}$, when:

$$y = \frac{(x+1)^2\sqrt{x-1}}{(x+4)^3.e^x}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 2\ln(x+1) + \frac{1}{2}\ln(x-1) - 3\ln(x+4) - x$$

$$\{\ln(mn) = \ln n + \ln m\}\ \{\ln\left(\frac{m}{n}\right) = \ln m - \ln n\}\ \{\ln e = 1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y} \times \frac{dy}{dx} = \frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1$$

$$\frac{dy}{dx} = \left(\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1\right) \times y$$

$$\frac{dy}{dx} = \left(\frac{2}{x+1} + \frac{1}{2(x-1)} - \frac{3}{x+4} - 1\right) \times \frac{(x+1)^2\sqrt{x-1}}{(x+4)^3.e^x}$$

23. Question

Find $\frac{dy}{dx}$, when:

$$y=\frac{\sqrt{x}(3x+5)^2}{\sqrt{x+1}}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 2\ln(3x+5)+\frac{1}{2}\ln(x)-\frac{1}{2}\ln(x+1)$$

$$\{\ln(mn)=\ln n+\ln m\}\ \{\ln\left(\frac{m}{n}\right)=\ln m-\ln n\}\ \{\ln e=1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=\frac{2}{3x+5}+\frac{1}{2x}-\frac{1}{2(x+1)}$$

$$\frac{dy}{dx}=\left(\frac{2}{3x+5}+\frac{1}{2x}-\frac{1}{2(x+1)}\right)\times y$$

$$\frac{dy}{dx}=\left(\frac{2}{3x+5}+\frac{1}{2x}-\frac{1}{2(x+1)}\right)\times\frac{(3x+5)^2\sqrt{x}}{\sqrt{x+1}}$$

24. Question

Find $\frac{dy}{dx}$, when:

$$y=\frac{x^2\sqrt{1+x}}{(1+x^2)^{3/2}}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y = 2\ln(x)+\frac{1}{2}\ln(x+1)-\frac{3}{2}\ln(x^2+1)$$

$$\{\ln(mn)=\ln n+\ln m\}\ \{\ln\left(\frac{m}{n}\right)=\ln m-\ln n\}\ \{\ln e=1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=\frac{2}{x}+\frac{1}{2(x+1)}-\frac{3}{2(x^2+1)}\times 2x$$

$$\frac{dy}{dx}=\left(\frac{2}{x}+\frac{1}{2(x+1)}-\frac{6x}{2(x^2+1)}\right)\times y$$

$$\frac{dy}{dx}=\left(\frac{2}{x}+\frac{1}{2(x+1)}-\frac{6x}{2(x^2+1)}\right)\times\frac{(x)^2\sqrt{x+1}}{(1+x^2)^{\frac{3}{2}}}$$

25. Question

Find $\frac{dy}{dx}$, when:

$$y=\sqrt{(x-2)(2x-3)(3x-4)}$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y=\frac{1}{2}(\ln(x-2)+\ln(2x-3)+\ln(3x-4))$$

$$\{\ln(mn)=\ln n+\ln m\}\ \{\ln\left(\frac{m}{n}\right)=\ln m-\ln n\}\ \{\ln e=1\}$$

Now differentiating both sides by x, we get,

$$\frac{1}{y}\times\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x-2}+\frac{2}{2x-3}+\frac{3}{3x-4}\right)$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x-2}+\frac{2}{2x-3}+\frac{3}{3x-4}\right)\times y$$

$$\frac{dy}{dx}=\frac{1}{2}\left(\frac{1}{x-2}+\frac{2}{2x-3}+\frac{3}{3x-4}\right)\times\sqrt{(x-2)(2x-3)(3x-4)}$$

26. Question

Find $\frac{dy}{dx}$, when:

$$y=\sin 2x\sin 3x\sin 4x$$

Answer

Here, we need to take log both the sides to get that differentiation simple.

$$\ln y=\ (\ln(\sin 2x)+\ln(\sin 3x)+\ln(\sin 4x))$$