$= \int e^x(\sec x + \sec x\tan x)dx$

$= \int e^x \sec x dx + \int e^x \sec x \tan x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sec x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$\int e^x \sec x dx + \int e^x \sec x \tan x dx$

$= \sec x \int e^x dx - \int\left[\frac{d}{dx}(\sec x)\int e^x dx\right]dx + \int e^x \sec x \tan x dx$

$= e^x \sec x - \int e^x \sec x \tan x dx + \int e^x \sec x \tan x dx + c$

$= \mathbf{e^x \sec x + c}$, where c is the integrating constant

66. Question

Evaluate the following integrals:

$$\int e^x\left(\frac{2-\sin 2x}{1-\cos 2x}\right)dx$$

Answer

$\int e^x\left(\frac{2-\sin 2x}{1-\cos 2x}\right)dx$

$= \int e^x\left(\frac{1-\sin x\cos x}{\sin^2 x}\right)dx$

$= \int e^x(\operatorname{cosec}^2 x - \cot x)dx$

$= \int e^x \operatorname{cosec}^2 x dx - \int e^x \cot x dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot x$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int e^x \operatorname{cosec}^2 x dx - \int e^x \cot x dx$

$= \int e^x \operatorname{cosec}^2 x dx - \cot x \int e^x dx + \int\left\{\frac{d}{dx}(\cot x)\int e^x dx\right\}dx$

$= \int e^x \operatorname{cosec}^2 x dx - e^x \cot x - \int e^x \operatorname{cosec}^2 x dx$

$= \mathbf{-e^x \cot x + c}$, where c is the integrating constant

67. Question

Evaluate the following integrals:

$\int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

Answer

$\left(\frac{1+\sin x}{1+\cos x}\right)$

$=\left(\frac{1+\frac{2\tan {}^x/_2}{1+\tan^2({}^x/_2)}}{1+\frac{1-\tan^2({}^x/_2)}{1+\tan^2({}^x/_2)}}\right)$

$=\frac{\left(1+\tan {}^x/_2\right)^2}{2}$

$\therefore \int e^x\left(\frac{1+\sin x}{1+\cos x}\right)dx$

$=\int e^x \times \frac{\left(1+\tan {}^x/_2\right)^2}{2}$

$=\int \frac{e^x\left(1+\tan^2 {}^x/_2+2\tan {}^x/_2\right)}{2}dx$

$=\int \frac{e^x\left(\sec^2 {}^x/_2+2\tan {}^x/_2\right)}{2}dx$

$=\int \frac{e^x\sec^2 {}^x/_2\,dx}{2}+\int e^x\tan {}^x/_2\,dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan(x/2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$\int \frac{e^x\sec^2 {}^x/_2\,dx}{2}+\int e^x\tan {}^x/_2\,dx$

$=\int \frac{e^x\sec^2 {}^x/_2\,dx}{2}+\tan {}^x/_2\int e^x dx-\int\left[\frac{d}{dx}(\tan {}^x/_2)\int e^x dx\right]dx$

$=\int \frac{e^x\sec^2 {}^x/_2\,dx}{2}+e^x\tan {}^x/_2-\int \frac{e^x\sec^2 {}^x/_2\,dx}{2}+c$

$=\mathbf{e^x\tan {}^x/_2+c}$, where c is the integrating constant

68. Question

Evaluate the following integrals:

$\int e^x\left(\frac{\sin 4x-4}{1-\cos 4x}\right)dx$

Answer

$\int e^x\left(\frac{\sin 4x-1}{1-\cos 4x}\right)dx$

$$= \int e^x \left(\frac{2\sin 2x \cos 2x - 4}{2\sin^2 2x}\right) dx$$

$$= \int e^x (\cot 2x - 2\operatorname{cosec}^2 2x) dx$$

$$= \int e^x \cot 2x dx - \int 2e^x \operatorname{cosec}^2 2x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \cot 2x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \cot 2x dx - \int 2e^x \operatorname{cosec}^2 2x dx$$

$$= \cot 2x \int e^x dx - \int \left\{\frac{d}{dx}(\cot 2x) \int e^x dx\right\} dx - \int 2e^x \operatorname{cosec}^2 2x dx$$

$$= e^x \cot 2x + \int 2e^x \operatorname{cosec}^2 2x dx - \int 2e^x \operatorname{cosec}^2 2x dx + c$$

$= \mathbf{e^x \cot 2x + c}$, where c is the integrating constant

69. Question

Evaluate the following integrals:

$$\int \frac{e^x \left[\sqrt{1-x^2} \sin^{-1} x + 1\right]}{\sqrt{1-x^2}} dx$$

Answer

$$\int \frac{e^x [\sqrt{1-x^2} \sin^{-1} x + 1]}{\sqrt{1-x^2}} dx$$

$$= \int e^x \left(\sin^{-1} x + \frac{1}{\sqrt{1-x^2}}\right) dx$$

$$= \int e^x \sin^{-1} x dx + \int \frac{e^x}{\sqrt{1-x^2}} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \sin^{-1}x$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int e^x \sin^{-1} x dx + \int \frac{e^x}{\sqrt{1-x^2}} dx$$

$$= \sin^{-1} x \int e^x dx - \int \left\{\frac{d}{dx}(\sin^{-1} x) \int e^x dx\right\} dx + \int \frac{e^x}{\sqrt{1-x^2}} dx$$

$$= e^x \sin^{-1} x - \int \frac{e^x}{\sqrt{1-x^2}} dx + \int \frac{e^x}{\sqrt{1-x^2}} dx + c$$

$= \mathbf{e^x \sin^{-1} x + c}$, where c is the integrating constant

70. Question

Evaluate the following integrals:

$$\int e^x\left(\frac{1+x\log x}{x}\right)dx$$

Answer

$$\int e^x\left(\frac{1+x\log x}{x}\right)dx$$

$$=\int e^x\left(\frac{1}{x}+\log x\right)dx$$

$$=\int \frac{e^x}{x}dx+\int e^x\log x dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = logx and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{x}dx+\int e^x\log x dx$$

$$=\int \frac{e^x}{x}dx+\log x\int e^x dx-\int\left[\frac{d}{dx}(\log x)\int e^x dx\right]dx$$

$$=\int \frac{e^x}{x}dx+e^x\log x-\int \frac{e^x}{x}dx+c$$

$= \mathbf{e^x\log x + c}$, where c is the integrating constant

71. Question

Evaluate the following integrals:

$$\int e^x\cdot\frac{x}{(1+x)^2}dx$$

Answer

$$\frac{x}{(1+x)^2}=\frac{A}{(1+x)}+\frac{B}{(1+x)^2}$$

$\Rightarrow x = A(1+x)+B$

For x=-1, equation: -1 = B i.e. B = -1

For x=0, equation: 0 = A-1 i.e. A = 1

$$\therefore \frac{x}{(1+x)^2}$$

$$=\frac{1}{(1+x)}-\frac{1}{(1+x)^2}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(1+x)}-\frac{1}{(1+x)^2}\right]dx$$

$$=\int e^x\times\frac{1}{(1+x)}dx-\int e^x\times\frac{1}{(1+x)^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(1+x)}dx - \int \frac{e^x}{(1+x)^2}dx$$

$$= \frac{1}{(1+x)}\int e^x dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+x}\right)\int e^x dx\right]dx - \int \frac{e^x}{(1+x)^2}dx$$

$$= \frac{e^x}{(1+x)} + \int \frac{e^x}{(1+x)^2}dx - \int \frac{e^x}{(1+x)^2}dx + c$$

$= \frac{e^x}{(1+x)} + c$, where c is the integrating constant

72. Question

Evaluate the following integrals:

$$\int e^x \frac{(x-1)}{(x+1)^3}dx$$

Answer

$$\frac{x-1}{(x+1)^3} = \frac{A}{(x+1)} + \frac{B}{(x+1)^2} + \frac{C}{(x+1)^3}$$

$\Rightarrow x - 1 = A(x+1)^2 + B(x+1) + C$

For x=-1, equation: -2 = C i.e. C = -2

For x=0, equation: -1 = A+B-2 i.e. A+B = 1

For x=1, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇒1+A = 1

⇒A = 0

And, B = 1

$\therefore \frac{x-1}{(x+1)^3}$

$$= \frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right]dx$$

$$= \int e^x \times \frac{1}{(x+1)^2}dx - \int e^x \times \frac{2}{(x+1)^3}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(x+1)^2}dx - \int \frac{2e^x}{(x+1)^3}dx$$

$$= \frac{1}{(x+1)^2}\int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{(x+1)^2}\right)\int e^x dx\right]dx - \int \frac{2e^x}{(x+1)^3}dx$$

$$= \frac{e^x}{(x+1)^2} + \int \frac{2e^x}{(x+1)^3}dx - \int \frac{2e^x}{(x+1)^3}dx + c$$

$= \frac{\mathbf{e^x}}{\mathbf{(x+1)^2}} + \mathbf{c}$, where c is the integrating constant

73. Question

Evaluate the following integrals:

$$\int e^x \frac{(2-x)}{(1-x)^2}dx$$

Answer

$$\frac{2-x}{(1-x)^2} = \frac{A}{(1-x)} + \frac{B}{(1-x)^2}$$

$\Rightarrow 2 - x = A(1-x) + B$

For x=1, equation: 1 = B i.e. B = 1

For x=2, equation: 0 = -A+1 i.e. A = 1

$$\therefore \frac{2-x}{(1-x)^2}$$

$$= \frac{1}{(1-x)} + \frac{1}{(1-x)^2}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(1-x)} + \frac{1}{(1-x)^2}\right]dx$$

$$= \int e^x \times \frac{1}{(1-x)^2}dx + \int e^x \times \frac{1}{1-x}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{(1-x)^2}dx + \int \frac{e^x}{1-x}dx$$

$$= \int \frac{e^x}{(1-x)^2}dx + \frac{1}{1-x}\int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{1-x}\right)\int e^x dx\right]dx$$

$$= \int \frac{e^x}{(1-x)^2}dx + \frac{e^x}{1-x} - \int \frac{e^x}{(1-x)^2}dx + c$$

$= \frac{\mathbf{e^x}}{\mathbf{1-x}} + \mathbf{c}$, where c is the integrating constant

74. Question

Evaluate the following integrals:

$$\int e^{x} \cdot \frac{(x-3)}{(x-1)^{3}} dx$$

Answer

$$\frac{x-3}{(x-1)^3} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3}$$

$$\Rightarrow x-3 = A(x-1)^2 + B(x-1) + C$$

For x=1, equation: -2 = C i.e. C = -2

For x=0, equation: -3 = A-B-2 i.e. B = A+1

For x=3, equation: 0 = 4A+2B-2

i.e. 2(A+B+A) = 2

⇒1+3A = 1

⇒A = 0

And, B = 1

$$\therefore \frac{x-3}{(x-1)^3}$$

$$= \frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}$$

The given equation becomes

$$\int e^x \left[\frac{1}{(x-1)^2} - \frac{2}{(x-1)^3}\right] dx$$

$$= \int e^x \times \frac{1}{(x-1)^2} dx - \int e^x \times \frac{2}{(x-1)^3} dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1-x)^2$ and $f_2(x) = e^x$ in the first integral and keeping the second integral intact,

$$\int \frac{e^x}{(x-1)^2} dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{1}{(x-1)^2} \int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{(x-1)^2}\right) \int e^x dx\right] dx - \int \frac{2e^x}{(x-1)^3} dx$$

$$= \frac{e^x}{(x-1)^2} + \int \frac{2e^x}{(x-1)^3} dx - \int \frac{2e^x}{(x-1)^3} dx + c$$

$= \frac{\mathbf{e^x}}{\mathbf{(x-1)^2}} + \mathbf{c}$, where c is the integrating constant

75. Question

Evaluate the following integrals:

$$\int e^{3x} \left(\frac{3x-1}{9x^2}\right) dx$$

Answer

$$\int e^{3x}\left(\frac{3x-1}{9x^2}\right)dx$$

$$=\int \frac{e^{3x}}{3x}dx-\int \frac{e^{3x}}{9x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/3x$and $f_2(x) = e^{3x}$ in the first integral and keeping the second integral intact,

$$\int \frac{e^{3x}}{3x}dx-\int \frac{e^{3x}}{9x^2}dx$$

$$=\frac{1}{3x}\int e^{3x}dx-\int\left[\frac{d}{dx}\left(\frac{1}{3x}\right)\int e^{3x}dx\right]dx-\int \frac{e^{3x}}{9x^2}dx$$

$$=\frac{e^{3x}}{9x}+\int \frac{e^{3x}}{9x^2}dx-\int \frac{e^{3x}}{9x^2}dx+c$$

$=\frac{e^{3x}}{9x}+c$, where c is the integrating constant

76. Question

Evaluate the following integrals:

$$\int\frac{(x+1)}{(x+2)^2}e^x dx$$

Answer

$$\frac{x+1}{(x+2)^2}=\frac{A}{(x+2)}+\frac{B}{(x+2)^2}$$

$\Rightarrow x+1=A(x+2)+B$

For x=-2, equation: -1 = B i.e. B = -1

For x=-1, equation: 0 = A-1 i.e. A = 1

$$\therefore \frac{x+1}{(x+2)^2}$$

$$=\frac{1}{(x+2)}-\frac{1}{(x+2)^2}$$

The given equation becomes

$$\int e^x\left[\frac{1}{(x+2)}-\frac{1}{(x+2)^2}\right]dx$$

$$=\int e^x\times\frac{1}{x+2}dx-\int e^x\times\frac{1}{(x+2)^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(x+2)$ and $f_2(x) = e^x$ in the second integral and keeping the first integral intact,

$$\int \frac{e^x}{x+2}dx - \int \frac{e^x}{(x+2)^2}dx$$

$$= \frac{1}{x+2}\int e^x dx - \int \left[\frac{d}{dx}\left(\frac{1}{x+2}\right)\int e^x dx\right]dx - \int \frac{e^x}{(x+2)^2}dx$$

$$= \frac{e^x}{x+2} + \int \frac{e^x}{(x+2)^2}dx - \int \frac{e^x}{(x+2)^2}dx + c$$

$= \mathbf{\frac{e^x}{x+2} + c}$, where c is the integrating constant

77. Question

Evaluate the following integrals:

$$\int \frac{x\,e^{2x}}{(1+2x)^2}dx$$

Answer

$$\frac{x}{(1+2x)^2} = \frac{A}{(1+2x)} + \frac{B}{(1+2x)^2}$$

$\Rightarrow x = A(1+2x) + B$

For x=-1/2, equation: -1/2 = B i.e. B = -1/2

For x=0, equation: 0 = A-1/2 i.e. A = 1/2

$$\therefore \frac{x}{(1+2x)^2}$$

$$= \frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}$$

The given equation becomes

$$\int e^{2x}\left[\frac{1}{2(1+2x)} - \frac{1}{2(1+2x)^2}\right]dx$$

$$= \int e^{2x} \times \frac{1}{2(1+2x)}dx - \int e^{2x} \times \frac{1}{2(1+2x)^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/(1+2x)$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$$\int e^{2x} \times \frac{1}{2(1+2x)}dx - \int e^{2x} \times \frac{1}{2(1+2x)^2}dx$$

$$= \frac{1}{2}\left[\frac{1}{1+2x}\int e^{2x}dx - \int\left[\frac{d}{dx}\left(\frac{1}{1+2x}\right)\int e^{2x}dx\right]dx - \int \frac{e^{2x}}{(1+2x)^2}dx\right]$$

$$= \frac{1}{2}\left[\frac{e^{2x}}{2(2x+1)} + \int \frac{e^{2x}}{(2x+1)^2}dx - \int \frac{e^{2x}}{(2x+1)^2}dx\right]$$

$= \mathbf{\frac{e^{2x}}{4(2x+1)} + c}$, where c is the integrating constant

78. Question

Evaluate the following integrals:

$$\int e^{2x}\left(\frac{2x-1}{4x^2}\right)dx$$

Answer

$$\int e^{2x}\left(\frac{2x-1}{4x^2}\right)dx$$

$$=\int \frac{e^{2x}}{2x}dx-\int \frac{e^{2x}}{4x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = 1/2x$ and $f_2(x) = e^{2x}$ in the first integral and keeping the second integral intact,

$$\int \frac{e^{2x}}{2x}dx-\int \frac{e^{2x}}{4x^2}dx$$

$$=\frac{1}{2x}\int e^{2x}dx-\int\left[\frac{d}{dx}\left(\frac{1}{2x}\right)\int e^{2x}dx\right]dx-\int \frac{e^{2x}}{4x^2}dx$$

$$=\frac{e^{2x}}{4x}+\int \frac{e^{2x}}{4x^2}dx-\int \frac{e^{2x}}{4x^2}dx+c$$

$=\frac{e^{2x}}{4x}+c$, where c is the integrating constant

79. Question

Evaluate the following integrals:

$$\int e^{x}\left(\log x+\frac{1}{x^2}\right)dx$$

Answer

$$\int e^{x}\left(\log x+\frac{1}{x^2}\right)dx$$

$$=\int e^{x}\log x dx-\int \frac{e^{x}}{x^2}dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \log x$ and $f_2(x) = e^{x}$ in the first integral and keeping the second integral intact,

$$\int e^{x}\log x dx-\int \frac{e^{x}}{x^2}dx$$

$$=\log x\int e^{x}dx-\int\left[\frac{d}{dx}(\log x)\int e^{x}dx\right]dx-\int \frac{e^{x}}{x^2}dx$$

$$=e^{x}\log x-\int \frac{e^{x}}{x}dx-\int \frac{e^{x}}{x^2}dx$$

$$=e^{x}\log x-\left[\frac{1}{x}\int e^{x}dx-\int\left[\frac{d}{dx}\left(\frac{1}{x}\right)\int e^{x}dx\right]dx\right]-\int \frac{e^{x}}{x^2}dx$$

$= e^x \log x - \frac{e^x}{x} + \int \frac{e^x}{x^2} dx - \int \frac{e^x}{x^2} dx + c$

$= \mathbf{e^x \left(\log x - \frac{1}{x}\right) + c}$, where c is the integrating constant

80. Question

Evaluate the following integrals:

$$\int \frac{\log x}{(1+\log x)^2} dx$$

Answer

$\frac{\log x}{(1+\log x)^2} = \frac{A}{(1+\log x)} + \frac{B}{(1+\log x)^2}$

$\Rightarrow \log x = A(1+\log x) + B$

For x=1, equation: 0 = A+B

For x=1/e, equation: -1 = B i.e. B = -1

So, A = 1

$\therefore \frac{\log x}{(1+\log x)^2}$

$= \frac{1}{(1+\log x)} - \frac{1}{(1+\log x)^2}$

The given equation becomes

$\int \left[\frac{1}{(1+\log x)} - \frac{1}{(1+\log x)^2}\right] dx$

$= \int \frac{1}{(1+\log x)} dx - \int \frac{1}{(1+\log x)^2} dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x) dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x) dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(1+logx) and $f_2(x)$ = 1in the second integral and keeping the first integral intact,

$\int \frac{1}{(1+\log x)} dx - \int \frac{1}{(1+\log x)^2} dx$

$= \frac{1}{(1+\log x)} \int dx - \int \left[\frac{d}{dx}\left(\frac{1}{(1+\log x)}\right) \int dx\right] dx - \int \frac{1}{(1+\log x)^2} dx$

$= \frac{x}{(1+\log x)} + \int \frac{1}{(1+\log x)^2} dx - \int \frac{1}{(1+\log x)^2} dx + c$

$= \mathbf{\frac{x}{(1+\log x)} + c}$, where c is the integrating constant

81. Question

Evaluate the following integrals:

$$\int \{\sin(\log x) + \cos(\log x)\} dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = sin(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \sin(\log x)dx + \int \cos(\log x)dx$$

$$= \sin(\log x) \int dx - \int \left[\frac{d}{dx}(\sin(\log x)) \int dx\right] dx + \int \cos(\log x)dx$$

$$= x\sin(\log x) - \int \cos(\log x)\,dx + \int \cos(\log x)\,dx + c$$

$= \mathbf{e^{\log x}\sin(\log x) + c}$, where c is the integrating constant

82. Question

Evaluate the following integrals:

$$\int \left\{ \frac{1}{\log x} - \frac{1}{(\log x)^2} \right\} dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = 1/(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \frac{1}{\log x} dx - \int \frac{1}{(\log x)^2} dx$$

$$= \frac{1}{\log x} \int dx - \int \left[\frac{d}{dx}\left(\frac{1}{\log x}\right) \int dx\right] dx - \int \frac{1}{(\log x)^2} dx$$

$$= \frac{x}{\log x} + \int \frac{1}{(\log x)^2} dx - \int \frac{1}{(\log x)^2} dx + c$$

$= \mathbf{\frac{x}{\log x} + c}$, where c is the integrating constant

83. Question

Evaluate the following integrals:

$$\int \left\{ \log(\log x) + \frac{1}{(\log x)^2} \right\} dx$$

Answer

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x) \int f_2(x)dx - \int \left\{\frac{d}{dx} f_1(x) \int f_2(x)dx\right\} dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = log(logx) and $f_2(x)$ = 1in the first integral and keeping the second integral intact,

$$\int \log(\log x)dx + \int \frac{1}{(\log x)^2} dx$$

$$= \log(\log x)\int dx - \int\left[\frac{d}{dx}(\log(\log x))\int dx\right]dx + \int\frac{1}{(\log x)^2}dx$$

$$= x\log(\log x) - \int\frac{1}{\log x}dx + \int\frac{1}{(\log x)^2}dx$$

$$= x\log(\log x) - \left[\frac{1}{\log x}\int dx - \int\left[\frac{d}{dx}\left(\frac{1}{\log x}\right)\int dx\right]dx\right] + \int\frac{1}{(\log x)^2}dx$$

$$= x\log(\log x) - \frac{x}{\log x} - \int\frac{1}{(\log x)^2}dx + \int\frac{1}{(\log x)^2}dx + c$$

$= \mathbf{x}\left[\mathbf{\log(\log x)} - \frac{\mathbf{1}}{\mathbf{\log x}}\right] + \mathbf{c}$, where c is the integrating constant

84. Question

Evaluate the following integrals:

$$\int\left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right)dx$$

Answer

It is know that $\sin^{-1}x+\cos^{-1}x = \pi/2$

$$\therefore \left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right)$$

$$= \frac{2}{\pi}\left(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\right)$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions , then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx - \int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Now, for the first term,

Taking $f_1(x) = \sin^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \sin^{-1}\sqrt{x}dx$$

$$= \sin^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}(\sin^{-1}\sqrt{x})\int dx\right\}dx$$

$$= x\sin^{-1}\sqrt{x} - \int\frac{1}{2\sqrt{x}\sqrt{1-x}}\times xdx$$

$$= x\sin^{-1}\sqrt{x} - \frac{1}{2}\int\frac{\sqrt{x}}{\sqrt{1-x}}dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int\frac{\sqrt{x}}{\sqrt{1-x}}dx$$

$$= \frac{1}{2}\int\frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= x\sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c'$, where c' is the integrating constant

For the second term,

Taking $f_1(x) = \cos^{-1}\sqrt{x}$ and $f_2(x) = 1$,

$$\therefore \int \cos^{-1}\sqrt{x}\,dx$$

$$= \cos^{-1}\sqrt{x}\int dx - \int\left\{\frac{d}{dx}\left(\cos^{-1}\sqrt{x}\right)\int dx\right\}dx$$

$$= x\cos^{-1}\sqrt{x} - \int \frac{-1}{2\sqrt{x}\sqrt{1-x}} \times x\,dx$$

$$= x\cos^{-1}\sqrt{x} + \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$$

Taking $(1-x)=a^2$,

$-dx=2ada$ i.e. $dx=-2ada$

Again, $x=1-a^2$

$$\therefore \frac{1}{2}\int \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$$

$$= \frac{1}{2}\int \frac{\sqrt{1-a^2}}{a}(-2ada)$$

$$= -\int \sqrt{1-a^2}\,da$$

$$= -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

Replacing the value of a, we get,

$$\therefore -\left[\frac{1}{2}a\sqrt{1-a^2} + \frac{1}{2}\sin^{-1} a\right]$$

$$= -\left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c$$

The total integration yields as

$= x\cos^{-1}\sqrt{x} - \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] + c''$, where c'' is the integrating constant

$$\therefore \int \left(\frac{\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}}{\sin^{-1}\sqrt{x} + \cos^{-1}\sqrt{x}}\right)dx$$

$= \frac{2}{\pi}\int\left(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\right)dx$

$= \frac{2}{\pi}\left[x\sin^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right] - x\cos^{-1}\sqrt{x} + \left[\frac{1}{2}x\sqrt{1-x} + \frac{1}{2}\sin^{-1}\sqrt{1-x}\right]\right] + c$

$= \frac{2}{\pi}\left[\sqrt{x - x^2} + x\left(\sin^{-1}\sqrt{x} - \cos^{-1}\sqrt{x}\right) + \sin^{-1}\sqrt{1-x}\right] + c$ where c is the integrating constant

85. Question

Evaluate the following integrals:

$\int 5^{5^{5^x}} \cdot 5^{5^x} \cdot 5^x \, dx$

Answer

Tip – 5^x is to be replaced by a

$\therefore 5^x = a$

$\Rightarrow 5^x \log 5 dx = da$

$\Rightarrow 5^x dx = \frac{da}{\log 5}$

The equation becomes as follows:

$\int 5^{5^a} \times 5^a \times \frac{1}{\log 5} da$

Tip – 5^a is to be replaced by k

$\therefore 5^a = k$

$\Rightarrow 5^a \log 5 da = dk$

$\Rightarrow 5^a da = \frac{dk}{\log 5}$

The equation becomes as follows:

$\int 5^k \times \frac{1}{(\log 5)^2} dk$

$= \frac{1}{(\log 5)^2}\int 5^k dk$

$= \frac{5^k}{(\log 5)^3} + c$

Re-replacing the value of k,

$\frac{5^{5^a}}{(\log 5)^3} + c$

Re-replacing the value of a,

$\frac{5^{5^{5^x}}}{(\log 5)^3} + c$, where c is the integrating constant

86. Question

Evaluate the following integrals:

$\int e^{2x}\left(\frac{1+\sin\ 2x}{1+\cos\ 2x}\right)dx$

Answer

$\left(\frac{1+\sin 2x}{1+\cos 2x}\right)$

$=\left(\frac{1+\frac{2\tan x}{1+\tan^2 x}}{1+\frac{1-\tan^2 x}{1+\tan^2 x}}\right)$

$=\frac{(1+\tan x)^2}{2}$

$\therefore \int e^{2x}\left(\frac{1+\sin 2x}{1+\cos 2x}\right)dx$

$=\int e^{2x}\times\frac{(1+\tan x)^2}{2}$

$=\int\frac{e^{2x}(1+\tan^2 x+2\tan x)}{2}dx$

$=\int\frac{e^{2x}(\sec^2 x+2\tan x)}{2}dx$

$=\int\frac{e^{2x}\sec^2 x dx}{2}+\int e^{2x}\tan x\, dx$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x) f_2(x) dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x) = \tan x$ and $f_2(x) = e^{2x}$ in the second integral and keeping the first integral intact,

$\int\frac{e^{2x}\sec^2 x dx}{2}+\int e^{2x}\tan x dx$

$=\int\frac{e^{2x}\sec^2 x dx}{2}+\tan x\int e^{2x}dx-\int\left[\frac{d}{dx}(\tan x)\int e^{2x}dx\right]dx$

$=\int\frac{e^{2x}\sec^2 x dx}{2}+\frac{1}{2}e^{2x}\tan x-\int\frac{e^{2x}\sec^2 x dx}{2}+c$

$=\frac{1}{2}e^x\tan{}^x/_2+c$, where c is the integrating constant

87. Question

Evaluate the following integrals:

$\int e^{2x}\left(\frac{1-\sin\ 2x}{1-\cos\ 2x}\right)dx$

Answer

$\left(\frac{1-\sin 2x}{1-\cos 2x}\right)$

$$=\left(\frac{1-\frac{2\tan x}{1+\tan^2 x}}{1-\frac{1-\tan^2 x}{1+\tan^2 x}}\right)$$

$$=\frac{(1-\tan x)^2}{2}$$

$$\therefore \int e^{2x}\left(\frac{1-\sin 2x}{1-\cos 2x}\right)dx$$

$$=\int e^{2x}\times\frac{(1-\tan x)^2}{2}$$

$$=\int \frac{e^{2x}(1+\tan^2 x-2\tan x)}{2}dx$$

$$=\int \frac{e^{2x}(\sec^2 x-2\tan x)}{2}dx$$

$$=\int \frac{e^{2x}\sec^2 x dx}{2}-\int e^{2x}\tan x\,dx$$

Tip – If $f_1(x)$ and $f_2(x)$ are two functions, then an integral of the form $\int f_1(x)f_2(x)dx$ can be INTEGRATED BY PARTS as

$f_1(x)\int f_2(x)dx-\int\left\{\frac{d}{dx}f_1(x)\int f_2(x)dx\right\}dx$ where $f_1(x)$ and $f_2(x)$ are the first and second functions respectively.

Taking $f_1(x)$ = tanx and $f_2(x)$ = e^{2x} in the second integral and keeping the first integral intact,

$$\int \frac{e^{2x}\sec^2 x dx}{2}-\int e^{2x}\tan x\,dx$$

$$=\int \frac{e^{2x}\sec^2 x dx}{2}-\tan x\int e^{2x}dx+\int\left[\frac{d}{dx}(\tan x)\int e^{2x}dx\right]dx$$

$$=\int \frac{e^{2x}\sec^2 x dx}{2}-\frac{1}{2}e^{2x}\tan x+\int \frac{e^{2x}\sec^2 x dx}{2}+c$$

$=-\frac{1}{2}e^x\tan {}^x/_2+c$, where c is the integrating constant

Objective Questions II

1. Question

Mark (√) against the correct answer in each of the following:

$$\int x\ e^x dx = ?$$

A. e^x (1 – x) + C

B. e^x (x – 1) + C

C. e^x (x – 1) + C

D. none of these

Answer

To find: Value of $\int x\,e^x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\, e^x dx$ ... (i)

$I = \int x\, e^x dx$

$\Rightarrow x \int e^x dx - \int \left[\frac{d(x)}{x} \int e^x dx\right] dx$

$\Rightarrow I = x\, e^x - \int 1.e^x dx$

$\Rightarrow I = x\, e^x - e^x + c$

$\therefore I = e^x (x-1) + c$

Ans) c $e^x (x-1) + c$

2. Question

Mark (√) against the correct answer in each of the following:

$\int x\, e^{2x} dx = ?$

A. $\frac{1}{2} x\, e^{2x} + \frac{1}{4} e^{2x} + C$

B. $\frac{1}{2} x\, e^{2x} - \frac{1}{4} e^{2x} + C$

C. $2x\, e^{2x} + 4e^{2x} + C$

D. none of these

Answer

To find: Value of $\int x\, e^{2x} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x) \int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int x\, e^{2x} dx$ ... (i)

$I = \int x\, e^{2x} dx$

$\Rightarrow x \int e^{2x} dx - \int \left[\frac{d(x)}{x} \int e^{2x} dx\right] dx$

$\Rightarrow I = x \frac{e^{2x}}{2} - \int 1. \frac{e^{2x}}{2} dx$

$\Rightarrow I = x \frac{e^{2x}}{2} - \frac{1}{2} \int \frac{e^{2x}}{2} dx$

$\Rightarrow I = \frac{x}{2} e^{2x} - \frac{1}{2} \int e^{2x} dx$

$\Rightarrow I = \frac{x}{2} e^{2x} - \frac{1}{2} \frac{e^{2x}}{2} + c$

$\Rightarrow I = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

$\therefore I = \frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

Ans) B $\frac{x}{2}e^{2x} - \frac{e^{2x}}{4} + c$

3. Question

Mark (√) against the correct answer in each of the following:

$\int x\cos 2x\,dx = ?$

A. $\frac{1}{2}x\sin 2x + \frac{1}{4}\cos 2x + C$

B. $\frac{1}{2}x\sin 2x - \frac{1}{4}\cos 2x + C$

C. 2x sin 2x + 4 cos 2x + C

D. none of these

Answer

To find: Value of $\int x\cos 2x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cos 2x dx$ ... (i)

Let 2x = t

$\Rightarrow x = \frac{t}{2}$

$\Rightarrow 2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2}$

$I = \int \frac{t}{2}\cos t\frac{dt}{2}$

$I = \frac{1}{4}\int t\cos t\, dt$

Taking 1^{st} function as t and second function as $\cos t$

$\Rightarrow I = \frac{1}{4}\left[t\int \cos t\, dt - \int\left(\frac{dt}{dt}\int \cos t\, dt\right)dt\right]$

$\Rightarrow I = \frac{1}{4}\left[t(\sin t) - \int(1(\sin t))dt\right]$

$\Rightarrow I = \frac{1}{4}[t(\sin t) - (-\cos t)] + c$

$\Rightarrow I = \frac{1}{4}[t \sin t + \cos t] + c$

$\Rightarrow I = \frac{1}{4}[2x \sin 2x + \cos 2x] + c$

$\Rightarrow I = \frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$

Ans) A $\frac{1}{2}x \sin 2x + \frac{1}{4}\cos 2x + c$

4. Question

Mark (√) against the correct answer in each of the following:

$\int x \sec^2 x \, dx = ?$

A. x tan x – log |cos x| + C

B. x tan x + log |cos x| + C

C. x tan x + log |sec x| + C

D. none of these

Answer

To find: Value of $\int x \sec^2 x \, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int x \sec^2 x \, dx$ ... (i)

Taking 1st function as x and second function as $\sec^2 x$

$\Rightarrow I = \left[x\int \sec^2 x \, dx - \int\left(\frac{dx}{dx} \int \sec^2 x \, dx\right)dx\right]$

$\Rightarrow I = \left[x \tan x - \int(1 \tan x)dx\right]$

$\Rightarrow I = \left[x \tan x - \int \tan x dx\right]$

$\Rightarrow I = [x \tan x - (-\log|\cos x|)] + c$

$\Rightarrow I = x \tan x + \log|\cos x| + c$

Ans) B $x \tan x + \log|\cos x| + c$

5. Question

Mark (√) against the correct answer in each of the following:

$\int x \sin 2x \, dx = ?$

A. $\frac{1}{2}x \cos 2x + \frac{1}{4}\sin 2x + C$

B. $-\frac{1}{2}x \cos 2x - \frac{1}{4}\sin 2x + C$

C. $-\frac{1}{2}x\cos 2x+\frac{1}{4}\sin 2x+C$

D. none of these

Answer

To find: Value of $\int x\sin 2x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\sin 2x dx$ ... (i)

Let 2x = t

$\Rightarrow x = \frac{t}{2}$

$\Rightarrow 2 = \frac{dt}{dx}$

$\Rightarrow dx = \frac{dt}{2}$

$I = \int \frac{t}{2}\sin t\frac{dt}{2}$

$I = \frac{1}{4}\int t\sin t\, dt$

Taking 1st function as t and second function as $\sin t$

$\Rightarrow I = \frac{1}{4}\left[t\int \sin t\, dt - \int\left(\frac{dt}{dt}\int \sin t\, dt\right)dt\right]$

$\Rightarrow I = \frac{1}{4}\left[t(-\cos t) - \int(1(-\cos t))dt\right]$

$\Rightarrow I = \frac{1}{4}\left[-t\cos t - \int -\cos t\, dt\right]$

$\Rightarrow I = \frac{1}{4}[-t\cos t + \sin t]+c$

$\Rightarrow I = \frac{1}{4}[-2x\cos 2x + \sin 2x]+c$

$\Rightarrow I = -\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x+c$

Ans) C $-\frac{1}{2}x\cos 2x + \frac{1}{4}\sin 2x+c$

6. Question

Mark (√) against the correct answer in each of the following:

$\int x \log x\, dx = ?$

A. $x\log x+\frac{1}{2}x^2+C$

B. $\frac{1}{2}x^2 \log x + \frac{1}{4}x^2 + C$

C. $\frac{1}{2}x^2 \log x - \frac{1}{4}x^2 + C$

D. none of these

Answer

To find: Value of $\int x\log x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int x\log x\, dx$ ... (i)

Taking 1st function as $\log x$ and second function as x

$$\Rightarrow I = \left[\log x \int x\, dx - \int\left(\frac{d\log x}{dx} \int x\, dx\right)dx\right]$$

$$\Rightarrow I = \left[\log x\frac{x^2}{2} - \int\left(\frac{1}{x}\ \frac{x^2}{2}\right)dx\right]$$

$$\Rightarrow I = \left[\log x\frac{x^2}{2} - \int\left(\frac{x}{2}\right)dx\right]$$

$$\Rightarrow I = \left[\log x\frac{x^2}{2} - \frac{1}{2}\int xdx\right]$$

$$\Rightarrow I = \left[\log x\frac{x^2}{2} - \frac{1}{2}\frac{x^2}{2}\right] + c$$

$$\Rightarrow I = \frac{1}{2}x^2\log x - \frac{1}{4}x^2 + c$$

Ans) C $\frac{1}{2}x^2\log x - \frac{1}{4}x^2 + c$

7. Question

Mark (√) against the correct answer in each of the following:

$\int x \operatorname{cosec}^2 x\, dx = ?$

A. x cot x – log |sin x| + C

B. - cot x + log |sin x| + C

C. x tan x – log |sec x| + C

D. none of these

Answer

To find: Value of $\int x \operatorname{cosec}^2 x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int x \operatorname{cosec}^2 x dx$ ... (i)

$$I = \int x \operatorname{cosec}^2 x dx$$

$$\Rightarrow x \int \operatorname{cosec}^2 x dx - \int \left[\frac{d(x)}{x} \int \operatorname{cosec}^2 x dx\right] dx$$

$$\Rightarrow I = x(-\cot x) - \int 1.(-\cot x) dx$$

$$\Rightarrow I = -x(\cot x) + \log|\sin x| + c$$

Ans) D None of these

8. Question

Mark (√) against the correct answer in each of the following:

$$\int x \sin x \cos x \, dx = ?$$

A. $-\frac{1}{4} x \sin 2x + \frac{1}{8} \cos 2x + C$

B. $\frac{1}{4} x \cos 2x - \frac{1}{8} \sin 2x + C$

C. $\frac{1}{2} x \sin 2x + \frac{1}{4} \cos 2x + C$

D. $-\frac{1}{4} x \cos 2x + \frac{1}{8} \sin 2x + C$

Answer

To find: Value of $\int x \sin x \cos x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right] dx$

We have, $I = \int x \sin x \cos x dx$ ... (i)

$$I = \frac{1}{2} \int x \, 2\sin x \cos x dx$$

$$I = \frac{1}{2} \int x \sin 2x dx$$

$$\Rightarrow \frac{1}{2}\left[x \int \sin 2x dx - \int \left[\frac{d(x)}{x} \int \sin 2x dx\right] dx\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{-x \cos 2x}{2} - \int \left[1 \frac{-\cos 2x}{2}\right] dx\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{-x \cos 2x}{2} + \frac{\sin 2x}{4}\right] + c$$

$$\Rightarrow \frac{-x \cos 2x}{4} + \frac{\sin 2x}{8} + c$$

Ans) D $\frac{-x\cos 2x}{4}+\frac{\sin 2x}{8}+c$

9. Question

Mark (√) against the correct answer in each of the following:

$\int x\cos^2 x\,dx = ?$

A. $\frac{x^2}{4}-\frac{x\sin 2x}{4}+\frac{\cos 2x}{8}+C$

B. $\frac{x^2}{4}+\frac{x\sin 2x}{4}+\frac{\cos 2x}{8}+C$

C. $\frac{x^2}{4}+\frac{x\sin 2x}{4}-\frac{\cos 2x}{8}+C$

D. none of these

Answer

To find: Value of $\int x\cos^2 x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cos^2 x\,dx$ ... (i)

$I = \int x\frac{1}{2}(1+\cos 2x)dx$

$I = \frac{1}{2}\int x\,dx+\frac{1}{2}\int x\cos 2x dx$

$I = \frac{1}{2}\frac{x^2}{2}+\frac{1}{2}\left[x\int \cos 2x dx - \int\left[\frac{d(x)}{x}\int \cos 2x dx\right]dx\right]$

$I = \frac{1}{2}\frac{x^2}{2}+\frac{1}{2}\left[x\frac{\sin 2x}{2}-\int 1.\frac{\sin 2x}{2}dx\right]$

$I = \frac{1}{2}\frac{x^2}{2}+\frac{1}{2}\left[x\frac{\sin 2x}{2}-\frac{1}{2}\int \sin 2x dx\right]$

$I = \frac{1}{2}\frac{x^2}{2}+\frac{1}{2}\left[x\frac{\sin 2x}{2}-\frac{1}{2}\left(-\frac{\cos 2x}{2}\right)+c\right]$

$I = \frac{1}{2}\frac{x^2}{2}+\frac{1}{2}\left[\frac{x\sin 2x}{2}+\frac{\cos 2x}{4}+c\right]$

$I = \frac{x^2}{4}+\frac{x\sin 2x}{4}+\frac{\cos 2x}{8}+c$

Ans) D $\frac{x^2}{4}+\frac{x\sin 2x}{4}+\frac{\cos 2x}{8}+c$

10. Question

Mark (√) against the correct answer in each of the following:

$\int \frac{\log x}{x^2} dx = ?$

A. $-\frac{1}{x}(\log x + 1) + C$

B. $\frac{1}{x}(\log x - 1) + C$

C. $\frac{1}{x}(\log x + 1) + C$

D. none of these

Answer

To find: Value of $\int \frac{logx}{x^2} dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx \right] dx$

We have, $I = \int \frac{\log x}{x^2} dx$ … (i)

$I = \int x^{-2} \log x dx$

$\Rightarrow \log x \int x^{-2}dx - \int \left[\frac{d(\log x)}{x} \int x^{-2}dx\right] dx$

$\Rightarrow \log x \frac{x^{-1}}{-1} - \int \left(\frac{1}{-x^2}\right) dx$

$\Rightarrow -\frac{\log x}{x} + \left(-\frac{1}{x}\right) + c$

$\Rightarrow -\frac{1}{x}(\log x + 1) + c$

Ans) A $-\frac{1}{x}(\log x + 1) + c$

11. Question

Mark (√) against the correct answer in each of the following:

$\int \log x \, dx = ?$

A. $\frac{1}{x} + C$

B. $\frac{1}{2}(\log x)^2 + C$

C. x (log x + 1) + C

D. x (log x – 1) + C

Answer

To find: Value of $\int \log x dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \log x\ .\ 1.\ dx$ ... (i)

Taking 1st function as $\log x$ and second function as 1

$$\Rightarrow I = \left[\log x\int 1\,dx - \int\left(\frac{d\log x}{dx}\int 1\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\log x\ .\ x - \int\left(\frac{1}{x}\int 1\,dx\right)dx\right]$$

$$\Rightarrow I = \left[\log x\ .\ x - \int\left(\frac{1}{x}x\right)dx\right]$$

$$\Rightarrow I = \left[\log x\ .\ x - \int 1dx\right]$$

$$\Rightarrow I = [\log x\ .\ x - x] + c$$

$$\Rightarrow I = x(\log x - 1) + c$$

Ans) D $x(\log x-1)+c$

12. Question

Mark (√) against the correct answer in each of the following:

$\int \log_{10} x\ dx = ?$

A. $\frac{1}{x}\log_e 10 + C$

B. $\frac{1}{x}\log_{10} e + C$

C. x (log x – 1) $\log_e$ 10 + C

D. x(log x – 1) $\log_{10}$ e + C

Answer

To find: Value of $\int \log_{10} x\ dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int \log_{10} x\ dx$ ... (i)

$$I = \int \log_{10} x\ dx = \int \frac{\log x}{\log 10}\ dx$$

$$I = \frac{1}{\log_e 10}\int \log x\ .\ 1\ dx$$

Taking 1st function as $\log x$ and second function as 1

$$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \int 1\, dx - \int \left(\frac{d\log x}{dx} \int 1\, dx\right) dx\right]$$

$$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int \left(\frac{1}{x} \int 1\, dx\right) dx\right]$$

$$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int \left(\frac{1}{x}\, x\right) dx\right]$$

$$\Rightarrow I = \frac{1}{\log_e 10}\left[\log x \,.\, x - \int 1 dx\right]$$

$$\Rightarrow I = \frac{1}{\log_e 10}[\log x \,.\, x - x] + c$$

$$\Rightarrow I = x(\log x - 1)\log_{10} e + c$$

Ans) D $x(\log x - 1)\log_{10} e + c$

13. Question

Mark (√) against the correct answer in each of the following:

$$\int (\log x)^2\, dx = ?$$

A. $\frac{2 \log x}{x} + C$

B. $\frac{1}{3}(\log x)^3 + C$

C. $x (\log x)^2 - 2x \log x + 2x + C$

D. $x (\log x)^2 + 2x \log x - 2x + C$

Answer

To find: Value of $\int (\log x)^2\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int (\log x)^2 \,.\, 1 \,.\, dx$ ... (i)

Taking 1^{st} function as $(\log x)^2$ and second function as 1

$$\Rightarrow I = \left[(\log x)^2 \int 1\, dx - \int \left(\frac{d(\log x)^2}{dx} \int 1\, dx\right) dx\right]$$

$$\Rightarrow I = \left[(\log x)^2 \int 1\, dx - \int \left(\frac{2(\log x)}{x} \int 1\, dx\right) dx\right]$$

$$\Rightarrow I = \left[(\log x)^2 .\, x - 2 \int \log x dx\right]$$

$$\Rightarrow I = [(\log x)^2 .\, x - 2(x\log x - x)] + c$$

$$\Rightarrow I = [(\log x)^2 .\, x - 2x\log x + 2x] + c$$

$\Rightarrow I = x(\log x)^2 - 2x\log x + 2x + c$

Ans) C $x(\log x)^2 - 2x\log x + 2x + c$

14. Question

Mark (√) against the correct answer in each of the following:

$\int e^{\sqrt{x}}\,dx = ?$

A. $e^{\sqrt{x}} + \sqrt{x} + C$

B. $\frac{1}{2}e^{\sqrt{x}}\left(\sqrt{x}+1\right) + C$

C. $2e^{\sqrt{x}}\left(\sqrt{x}-1\right) + C$

D. none of these

Answer

To find: Value of $\int e^{\sqrt{x}}dx$

Formula used: $\int \frac{1}{x}dx = \log|x| + c$

We have, $I = \int e^{\sqrt{x}}dx$ ... (i)

Putting $\sqrt{x} = t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx = 2\sqrt{x}\,dt$

$\Rightarrow dx = 2t\,dt$

$\Rightarrow I = 2\int t.e^t\,dt$

$\Rightarrow I = 2\left[t\int e^t\,dt - \int\left[\frac{d(t)}{dt}\int e^t dt\right]dt\right]$

$\Rightarrow I = 2\left[te^t - \int[1\ e^t]dt\right]$

$\Rightarrow I = 2[te^t - e^t]$

$\Rightarrow I = e^t.\ 2(t-1) + c$

$\therefore I = 2\,e^{\sqrt{x}}\,(\sqrt{x}-1) + c$

Ans) C $2\,e^{\sqrt{x}}\,(\sqrt{x}-1) + c$

15. Question

Mark (√) against the correct answer in each of the following:

$\int \cos\sqrt{x}\,dx = ?$

A. $\sin\sqrt{x} + \cos\sqrt{x} + C$

B. $\frac{1}{2}\left(\sqrt{x}\sin\sqrt{x}-\cos\sqrt{x}\right)+C$

C. $2\left[\sqrt{x}\sin\sqrt{x}+\cos\sqrt{x}\right]+C$

D. none of these

Answer

To find: Value of $\int \cos\sqrt{x}\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \cos\sqrt{x}\,dx$ … (i)

Putting $\sqrt{x}=t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx = 2\sqrt{x}\,dt$

$\Rightarrow$ dx = 2t dt

$\Rightarrow I = \int \cos t \,.\, 2t\,dt$

$\Rightarrow I = 2\int t.\cos t\,dt$

$\Rightarrow I = 2\left[t\int \cos t\;dt - \int\left[\frac{d(t)}{dt}\int \cos t\,dt\right]dt\right]$

$\Rightarrow I = 2\left[te^t - \int[1\;e^t]dt\right]$

$\Rightarrow I = 2[te^t - e^t]$

$\Rightarrow I = e^t.\,2(t-1)+c$

$\therefore I = 2\,e^{\sqrt{x}}\,(\sqrt{x}-1)+c$

Ans) C $2\,e^{\sqrt{x}}\,(\sqrt{x}-1)+c$

16. Question

Mark (√) against the correct answer in each of the following:

$\int \cos(\log\ x)dx = ?$

A. $\frac{x}{2}\left[\cos(\log x)-\sin(\log x)\right]+C$

B. $\frac{x}{2}\left[\cos(\log x)+\sin(\log x)\right]+C$

C. 2x [cos (log x) + sin (log x)] + C

D. 2x [cos (log x) – sin (log x)] + C

Answer

To find: Value of $\int \cos(log x)\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int \left[f'(x) \int g(x)dx\right]dx$

We have, $I = \int \cos(\log x)\, dx$ ... (i)

$I = \int 1.\ \cos(\log x)\, dx$

Taking cos(logx) as first function and 1 as second function.

$\Rightarrow I = \left[\cos\log x \int 1\, dx - \int \left[\frac{d[\cos(\log x)]}{dx} \int 1 dx\right] dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) - \int \left[-\sin(\log x)\frac{1}{x}\, x\right] dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \int [\sin(\log x)] dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \int [1.\sin(\log x)] dx\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \left\{\sin(\log x) \int 1\, dx - \left(\frac{d\sin(\log x)}{dx} \int 1.dx\right) dx\right\}\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \left\{x.\sin(\log x) - \left(\cos(\log x)\frac{1}{x}\, x\right) dx\right\}\right]$

$\Rightarrow I = \left[x.\cos(\log x) + \left\{x.\sin(\log x) - \left(\cos(\log x)\frac{1}{x}\, x\right) dx\right\}\right]$

$\Rightarrow I = [x.\cos(\log x) + \{x.\sin(\log x) - (\cos(\log x))dx\}]$

$\Rightarrow I = [x.\cos(\log x) + x.\sin(\log x) - I]$

$\Rightarrow 2I = [x.\cos(\log x) + x.\sin(\log x)]$

$\Rightarrow I = \frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$

Ans) B $\frac{x}{2}[\cos(\log x) + \sin(\log x)] + c$

17. Question

Mark (√) against the correct answer in each of the following:

$\int \sec^3 x\, dx = ?$

A. $\frac{1}{2}\{\sec x \tan x - \log|\sec x + \tan x|\} + C$

B. $\frac{1}{2}\{\sec x \tan x + \log|\sec x + \tan x|\} + C$

C. 2{sec x tan x + log |sec x + tan x|}+C

D. none of these

Answer

To find: Value of $\int \sec^3 x\, dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int \sec^3 x\, dx$ ... (i)

$I = \int \sec x \sec^2 x\, dx$

Taking secx as first function and $\sec^2 x$ as second function.

$\Rightarrow I = \left[\sec x \int \sec^2 x\, dx - \int \left[\frac{d[\sec x]}{dx} \int \sec^2 x\, dx\right] dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int [\sec x \tan x \tan x] dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int [\sec x \tan^2 x] dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int [\sec x(\sec^2 x - 1)] dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int (\sec^3 x - \sec x)\, dx\right]$

$\Rightarrow I = \left[\sec x \tan x - \int \sec^3 x\, dx + \int \sec x dx\right]$

$\Rightarrow I = [\sec x \tan x - I + \log|\sec x + \tan x| + c]$

$\Rightarrow 2I = [\sec x \tan x + \log|\sec x + \tan x| + c]$

$\Rightarrow I = \frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x| + c]$

Ans) B $\frac{1}{2}[\sec x \tan x + \log|\sec x + \tan x| + c]$

18. Question

Mark (√) against the correct answer in each of the following:

$\int\left\{\frac{1}{(\log x)} - \frac{1}{(\log x)^2}\right\}dx = ?$

A. x log x + C

B. $\frac{x}{\log x} + C$

C. $x + \frac{1}{\log x} + C$

D. none of these

Answer

To find: Value of $\int\left\{\frac{1}{(logx)}-\frac{1}{(logx)^2}\right\}dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int\left\{\frac{1}{(logx)}-\frac{1}{(logx)^2}\right\}dx$ … (i)

Put $t = logx$

$e^t = e^{logx} = x$

$\frac{dx}{dt} = e^t$

$\Rightarrow dx = e^t\, dt$

$\Rightarrow I = \int\left\{\frac{1}{t}-\frac{1}{t^2}\right\}dx$

We know $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x)$

$\Rightarrow I = \int\left\{\frac{1}{t}-\frac{1}{t^2}\right\}dx = e^t\frac{1}{t}$

$\Rightarrow \frac{x}{logx}+c$

Ans) B $\frac{x}{logx}+c$

19. Question

Mark (√) against the correct answer in each of the following:

$\int 2x^3 e^{x^2} dx = ?$

A. $e^{x^2}\left(x^2-1\right)+C$

B. $e^{x^2}\left(x^2+1\right)+C$

C. $e^{x^2}\left(x+1\right)+C$

D. none of these

Answer

To find: Value of $\int\left\{\frac{1}{(logx)}-\frac{1}{(logx)^2}\right\}dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int\left\{\frac{1}{(logx)}-\frac{1}{(logx)^2}\right\}dx$ … (i)

Put $t = logx$

$e^t = e^{\log x} = x$

$\frac{dx}{dt} = e^t$

$\Rightarrow dx = e^t\, dt$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx$

We know $\int e^x \left(f(x) + f'(x)\right) dx = e^x f(x)$

$\Rightarrow I = \int \left\{\frac{1}{t} - \frac{1}{t^2}\right\} dx = e^t \frac{1}{t}$

$\Rightarrow \frac{x}{\log x} + c$

Ans) B $\frac{x}{\log x} + c$

20. Question

Mark (√) against the correct answer in each of the following:

$\int \left(x 2^x\right) dx = ?$

A. $\frac{2^x}{(\log 2)}(x + \log 2) + C$

B. $\frac{2^x}{(\log 2)^2}(x + \log 2 - 1) + C$

C. $\frac{x \cdot 2^x}{(\log 2)} + \frac{2^x}{(\log 2)^2} + C$

D. none of these

Answer

To find: Value of $\int (x 2^x) dx$

Formula used:

(i) $\int f(x) g(x) dx = f(x) \int g(x) dx - \int \left[f'(x) \int g(x) dx\right] dx$

We have, $I = \int (x 2^x) dx$ ... (i)

$\Rightarrow I = x \int 2^x dx - \int \left(\frac{dx}{dx} \int 2^x dx\right) dx$

$\Rightarrow I = x \frac{2^x}{\log 2} - \int \left(\frac{2^x}{\log 2}\right) dx$

$\Rightarrow I = x \frac{2^x}{\log 2} - \frac{1}{\log 2} \int 2^x dx$

$\Rightarrow I = x\frac{2^x}{\log 2} - \frac{1}{\log 2}\frac{2^x}{\log 2}$

$\Rightarrow I = \frac{x.\ 2^x}{\log 2} - \frac{2^x}{(\log 2)^2} + c$

$\Rightarrow I = \frac{2^x}{(\log 2)^2}(x\log 2 - 1) + c$

Ans) D

21. Question

Mark (√) against the correct answer in each of the following:

$\int x\cot^2 x\,dx = ?$

A. $-x\cot x + \frac{x^2}{2} + \log|\sin x| + C$

B. $-x\cot x - \frac{x^2}{2} + \log|\sin x| + C$

C. $-x\cot x + \frac{x^2}{2} - \log|\sin x| + C$

D. none of these

Answer

To find: Value of $\int x\cot^2 x\,dx$

Formula used:

(i) $\int f(x)g(x)dx = f(x)\int g(x)dx - \int\left[f'(x)\int g(x)dx\right]dx$

We have, $I = \int x\cot^2 x\,dx$ ... (i)

$\Rightarrow I = x\int \cot^2 x\,dx - \int\left(\frac{dx}{dx}\int \cot^2 x\,dx\right)dx$

$\Rightarrow I = x\int(\operatorname{cosec}^2 x - 1)\,dx - \int\left(1.\int(\operatorname{cosec}^2 x - 1)dx\right)dx$

$\Rightarrow I = x(-\cot x - x) - \int(-\cot x - x)dx$

$\Rightarrow I = -x\cot x - x^2 + \log|\sin x| + \frac{x^2}{2}$

$\Rightarrow I = -x\cot x - \frac{x^2}{2} + \log|\sin x| + c$

Ans) B $-x\cot x - \frac{x^2}{2} + \log|\sin x| + c$

22. Question

Mark (√) against the correct answer in each of the following:

$\int \sin\sqrt{x}\,dx = ?$

A. $-\sqrt{x}\cos\sqrt{x}+C$

B. $-\sqrt{x}\cos\sqrt{x}-2\sin\sqrt{x}-C$

C. $-2\sqrt{x}\cos\sqrt{x}+2\sin\sqrt{x}-C$

D. none of these

Answer

To find: Value of $\int \sin\sqrt{x}\,dx$

Formula used: $\int \frac{1}{x}dx = \log|x|+c$

We have, $I = \int \sin\sqrt{x}\,dx$ ... (i)

$\sqrt{x}=t$

$\Rightarrow \frac{1}{2\sqrt{x}} = \frac{dt}{dx}$

$\Rightarrow dx = 2\sqrt{x}dt$

$\Rightarrow dx = 2tdt$

$I = \int \sin t.\ 2t\,dt$

$I = 2\int t.\ \sin t\,dt$

$\Rightarrow I = 2t\int \sin t\,dt - \int\left(\frac{dt}{dt}\int \sin t\,dt\right)dt$

$\Rightarrow I = 2t\,(-\cos t) - \int 1\,(-\cos t)dt$

$\Rightarrow I = 2t\,(-\cos t) + \int \cos t\,dt$

$\Rightarrow I = 2t\,(-\cos t) + \sin t + c$

$\Rightarrow I = -2\sqrt{x}\cos\sqrt{x} + \sin\sqrt{x} + c$

Ans) C $2\sqrt{x}\cos\sqrt{x}+\sin\sqrt{x}+c$

23. Question

Mark (√) against the correct answer in each of the following:

$\int e^{\sin x}\sin 2x\ dx = ?$

A. (2 sin x) $e^{\sin x}$ + C

B. (2 cos x) $e^{\sin x}$ + C

C. $2e^{\sin x}$ (sin x + 1) + C

D. $2e^{\sin x}$ (sin x – 1) + C

Answer

To find: Value of $\int e^{sinx}\sin 2x\,dx$