

EXERCISE 30.5

Differentiate the following functions with respect to x:

1. $\frac{x^2 + 1}{x + 1}$

Solution:

Let us consider

$$y = \frac{x^2 + 1}{x + 1}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$$u = x^2 + 1 \text{ and } v = x + 1$$

$$\therefore y = u/v$$

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \dots \text{Equation (1)}$$

$$\text{As, } u = x^2 + 1$$

$$\frac{du}{dx} = 2x^{2-1} + 0 = 2x \dots \text{Equation (2) } \left\{ \text{Since, } \frac{d}{dx} (x^n) = nx^{n-1} \right\}$$

$$\text{As, } v = x + 1$$

$$\frac{dv}{dx} = \frac{d}{dx} (x + 1) = 1 \dots \text{Equation (3) } \left\{ \text{Since, } \frac{d}{dx} (x^n) = nx^{n-1} \right\}$$

Now, from equation 1, we can find dy/dx

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{(x + 1)(2x) - (x^2 + 1)(1)}{(x + 1)^2} \quad \{ \text{Using equation 2 and 3} \} \end{aligned}$$

$$= \frac{2x^2 + 2x - x^2 - 1}{(x + 1)^2}$$

$$= \frac{x^2 + 2x - 1}{(x + 1)^2}$$

$$\therefore \frac{dy}{dx} = \frac{x^2 + 2x - 1}{(x + 1)^2}$$

2. $\frac{2x - 1}{x^2 + 1}$

Solution:

Let us consider

$$y = \frac{2x - 1}{x^2 + 1}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$$u = 2x - 1 \text{ and } v = x^2 + 1$$

$$\therefore y = u/v$$

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \dots \text{Equation (1)}$$

$$\text{As, } u = 2x - 1$$

$$\frac{du}{dx} = 2x^{1-1} - 0 = 2 \dots \text{Equation (2) } \left\{ \text{Since, } \frac{d}{dx} (x^n) = nx^{n-1} \right\}$$

$$\text{As, } v = x^2 + 1$$

$$\frac{dv}{dx} = \frac{d}{dx} (x^2 + 1) = 2x \dots \text{Equation (3) } \left\{ \text{Since, } \frac{d}{dx} (x^n) = nx^{n-1} \right\}$$

Now, from equation 1, we can find dy/dx

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{(x^2 + 1)(2) - (2x - 1)(2x)}{(x^2 + 1)^2} \quad \{ \text{Using equation 2 and 3} \} \\ &= \frac{2x^2 + 2 - 4x^2 + 2x}{(x^2 + 1)^2} \\ &= \frac{-2x^2 + 2x + 2}{(x^2 + 1)^2} \\ \therefore \frac{dy}{dx} &= \frac{2(1 + x - x^2)}{(x^2 + 1)^2} \end{aligned}$$

3. $\frac{x + e^x}{1 + \log x}$

Solution:

Let us consider

$$y = \frac{x + e^x}{1 + \log x}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$$u = x + e^x \text{ and } v = 1 + \log x$$

$$\therefore y = u/v$$

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \dots \text{Equation 1}$$

As, $u = x + e^x$

$$\frac{du}{dx} = \frac{d}{dx} (x + e^x) \left\{ \text{Since, } \frac{d}{dx} (x^n) = nx^{n-1} \ \& \ \frac{d}{dx} (e^x) = e^x \right\}$$

$$\frac{du}{dx} = \frac{d}{dx} (x) + \frac{d}{dx} (e^x) = 1 + e^x \dots \text{Equation 2}$$

As, $v = 1 + \log x$

$$\begin{aligned} \frac{dv}{dx} &= \frac{d}{dx} (\log x + 1) \\ &= \frac{d}{dx} (1) + \frac{d}{dx} (\log x) \end{aligned}$$

$$\frac{dv}{dx} = 0 + \frac{1}{x} = \frac{1}{x} \dots \text{Equation 3} \left\{ \text{Since, } \frac{d}{dx} (\log x) = \frac{1}{x} \right\}$$

Now, from equation 1, we can find dy/dx

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ \frac{dy}{dx} &= \frac{(1 + \log x)(1 + e^x) - (x + e^x) \left(\frac{1}{x} \right)}{(\log x + 1)^2} \left\{ \text{Using equation 2 and 3} \right\} \\ &= \frac{1 + e^x + \log x + e^x \log x - 1 - \frac{e^x}{x}}{(\log x + 1)^2} \\ &= \frac{x \log x (1 + e^x) + e^x (x - 1)}{x(\log x + 1)^2} \\ \therefore \frac{dy}{dx} &= \frac{x \log x (1 + e^x) - e^x (1 - x)}{x(1 + \log x)^2} \end{aligned}$$

4. $\frac{e^x - \tan x}{\cot x - x^n}$

Solution:

Let us consider

$$y = \frac{e^x - \tan x}{\cot x - x^n}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$$u = e^x - \tan x \text{ and } v = \cot x - x^n$$

$$\therefore y = u/v$$

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \dots \text{Equation (1)}$$

As, $u = e^x - \tan x$

$$\frac{du}{dx} = \frac{d}{dx} (e^x - \tan x) \quad \left\{ \text{Since, } \frac{d}{dx} (\tan x) = \sec^2 x \text{ \& } \frac{d}{dx} (e^x) = e^x \right\}$$

$$\frac{du}{dx} = -\frac{d}{dx} (\tan x) + \frac{d}{dx} (e^x) = \sec^2 x + e^x \dots \text{Equation (2)}$$

As, $v = \cot x - x^n$

$$\frac{dv}{dx} = \frac{d}{dx} (\cot x - x^n)$$

$$= \frac{d}{dx} (\cot x) - \frac{d}{dx} (x^n) \quad \left\{ \text{Since, } \frac{d}{dx} (\cot x) = -\operatorname{cosec}^2 x \text{ \& } \frac{d}{dx} (x^n) = nx^{n-1} \right\}$$

$$\frac{dv}{dx} = -\operatorname{cosec}^2 x - nx^{n-1} \dots \text{Equation (3)}$$

Now, from equation 1, we can find dy/dx

$$\frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \quad \left\{ \text{Using equation 2 and 3, we get} \right\}$$

$$\frac{dy}{dx} = \frac{(\cot x - x^n)(\sec^2 x + e^x) - (e^x - \tan x)(-\operatorname{cosec}^2 x - nx^{n-1})}{(\cot x - x^n)^2}$$

$$\therefore \frac{dy}{dx} = \frac{(\cot x - x^n)(e^x - \sec^2 x) + (e^x - \tan x)(\operatorname{cosec}^2 x + nx^{n-1})}{(\cot x - x^n)^2}$$

5. $\frac{ax^2 + bx + c}{px^2 + qx + r}$

Solution:

Let us consider

$$y = \frac{ax^2 + bx + c}{px^2 + qx + r}$$

We need to find dy/dx

We know that y is a fraction of two functions say u and v where,

$$u = ax^2 + bx + c \text{ and } v = px^2 + qx + r$$

$$\therefore y = u/v$$

Now let us apply quotient rule of differentiation.

By using quotient rule, we get

$$\frac{dy}{dx} = \frac{d}{dx} \left(\frac{u}{v} \right) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \dots \text{Equation (1)}$$

As, $u = ax^2 + bx + c$

$$\frac{du}{dx} = 2ax + b \dots \text{Equation (2) } \left\{ \text{Since, } \frac{d}{dx} (x^n) = nx^{n-1} \right\}$$

As, $v = px^2 + qx + r$

$$\frac{dv}{dx} = \frac{d}{dx} (px^2 + qx + r) = 2px + q \dots \text{Equation (3)}$$

Now, from equation 1, we can find dy/dx

$$\begin{aligned} \frac{dy}{dx} &= \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2} \\ &= \frac{(px^2 + qx + r)(2ax + b) - (ax^2 + bx + c)(2px + q)}{(px^2 + qx + r)^2} \quad \{\text{Using equation 2 and 3}\} \\ &= \frac{2apx^3 + bpx^2 + 2aqx^2 + bqx + 2arx + br - 2apx^3 - aqx^2 - 2bpx^2 - bqx - 2pcx - qc}{(px^2 + qx + r)^2} \\ &= \frac{aqx^2 - bpx^2 + 2arx + br - 2pcx - qc}{(px^2 + qx + r)^2} \\ &= \frac{x^2(aq - bp) + 2x(ar - pc) + br - qc}{(px^2 + qx + r)^2} \end{aligned}$$

$$\therefore \frac{dy}{dx} = \frac{x^2(aq - bp) + 2x(ar - pc) + br - qc}{(px^2 + qx + r)^2}$$

