

Chapter 1

Real numbers

1. Use Euclid's division algorithm to find the HCF of :

(i) 135 and 225

Solution:

We know that,
 $225 > 135$

Applying Euclid's division algorithm

$$225 = 135 \times 1 + 90 \quad (\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder})$$

Here remainder = 90,

So, Again Applying Euclid's division algorithm

$$135 = 90 \times 1 + 45$$

Here remainder = 45,

So, Again Applying Euclid's division algorithm

$$90 = 45 \times 2 + 0$$

Remainder = 0,

Hence ,

$$\text{HCF of } (135, 225) = 45$$

(ii) 196 and 38220

Solution:

We know that,
 $38220 > 196$

So, Applying Euclid's division algorithm

$$38220 = 196 \times 195 + 0 \quad (\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder})$$

Remainder = 0

Hence,

$$\text{HCF of } (196, 38220) = 196$$

(iii) 867 and 255

Solution:

We know that,
 $867 > 255$

So, Applying Euclid's division algorithm

$$867 = 255 \times 3 + 102 \quad (\text{Dividend} = \text{Divisor} \times \text{Quotient} + \text{Remainder})$$

Remainder = 102

So, Again Applying Euclid's division algorithm

$$255 = 102 \times 2 + 51$$

Remainder = 51

So, Again Applying Euclid's division algorithm

$$102 = 51 \times 2 + 0$$

$$\text{Remainder} = 0$$

Hence,

$$(\text{HCF of } 867 \text{ and } 255) = 51$$

2. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Solution:

By taking, 'a' as any positive integer and $b = 6$.

Applying Euclid's algorithm

$$a = 6q + r$$

Here, $r = \text{remainder} = 0, 1, 2, 3, 4, 5$ and $q \geq 0$

So, total possible forms are $6q + 0, 6q + 1, 6q + 2, 6q + 3, 6q + 4, 6q + 5$

$6q + 0$, (6 is divisible by 2, its an even number)

$6q + 1$, (6 is divisible by 2 but 1 is not divisible by 2, its an odd number)

$6q + 2$, (6 and 2 both are divisible by 2, its an even number)

$6q + 3$, (6 is divisible by 2 but 3 is not divisible by 2, its an odd number)

$6q + 4$, (6 and 4 both are divisible by 2, its an even number)

$6q + 5$, (6 is divisible by 2 but 5 is not divisible by 2, its an odd number)

Therefore,

So, odd numbers will be in the form $6q + 1$, or $6q + 3$, or $6q + 5$

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

By using, Euclid's division algorithm

$$616 = 32 \times 19 + 8$$

$$\text{Remainder} \neq 0$$

So, again Applying Euclid's division algorithm

$$32 = 8 \times 4 + 0$$

HCF of (616, 32) is 8.

So,

They can march in 8 columns each.

4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m .

Solution:

By taking, 'a' as any positive integer and $b = 3$.

Applying Euclid's algorithm

$$a = 3q + r$$

Here, $r = \text{remainder} = 0, 1, 2$ and $q \geq 0$

So, $a = 3q$ or $3q+1$ or $3q+2$

And,

$$a^2 = (3q)^2 \text{ or } (3q+1)^2 \text{ or } (3q+2)^2$$

$$a^2 = (9q^2) \text{ or } 9q^2+6q+1 \text{ or } 9q^2+12q+4$$

$$a^2 = 3(3q^2) \text{ or } (3q^2 + 2q)+1 \text{ or } 3(3q^2+4q+1)+1$$

$$a^2 = 3k_1 \text{ or } 3k_2+1 \text{ or } 3k_3+1$$

Where k_1, k_2 and k_3 are some positive integers

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m+1$.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m, 9m + 1$ or $9m + 8$.

Solution:

By taking, 'a' as any positive integer and $b = 3$.

Applying Euclid's algorithm

$$a = 3q+r$$

Here, $r = \text{remainder} = 0, 1, 2$ and $q \geq 0$

So, total possible forms are

$$\therefore a = 3q \text{ or } 3q + 1 \text{ or } 3q + 2$$

Therefore, every number can be represented as these three forms.

when $a = 3q$,

$$a^3 = (3q)^3 = 27q^3 = 9(3q^3) = 9m,$$

where m is an integer such that $m = 3q^3$

when $a = 3q + 1$,

$$a^3 = (3q+1)^3$$

$$a^3 = 27q^3+27q^2+9q+1$$

$$a^3 = 9(3q^3+3q^2+q)+1$$

$$a^3 = 9m+1$$

where m is an integer such that $m = (3q^3+3q^2+q)$

when $a = 3q+2$,

$$a^3 = (3q+2)^3$$

$$a^3 = 27q^3 + 54q^2 + 36q + 8$$

$$a^3 = 9(3q^3 + 6q^2 + 4q) + 8$$

$$a^3 = 9m + 8$$

Hence,

The cube of any positive integer is of the form $9m$, $9m+1$, $9m+8$

