

EXERCISE 18.4

1. Find the absolute maximum and the absolute minimum values of the following functions in the given intervals:

(i) $f(x) = 4x - \frac{x^2}{2}$ in $[-2, 9/2]$

Solution:

Given function is $f(x) = 4x - \frac{x^2}{2}$

On differentiation we get

$$\therefore f'(x) = 4 - x$$

Now, for local minima and local maxima we have $f'(x) = 0$

$$4 - x = 0$$

$$x = 4$$

Then, we evaluate of f at critical points $x = 4$ and at the interval $[-2, \frac{9}{2}]$

$$f(4) = 4(4) - \frac{(4)^2}{2} = 8$$

$$f(-2) = 4(-2) - \frac{(-2)^2}{2} = -10$$

$$f\left(\frac{9}{2}\right) = 4\left(\frac{9}{2}\right) - \frac{\left(\frac{9}{2}\right)^2}{2} = 18 - \frac{81}{8} = 7.875$$

Hence, we can conclude that the absolute maximum value of f on $[-2, 9/2]$ is 8 occurring at $x = 4$ and the absolute minimum value of f on $[-2, 9/2]$ is -10 occurring at $x = -2$

(ii) $f(x) = (x - 1)^2 + 3$, $x \in [-3, 1]$

Solution:

Given function is $f(x) = (x - 1)^2 + 3$

On differentiation we get

$$\Rightarrow f'(x) = 2(x - 1)$$

Now, for local minima and local maxima we have $f'(x) = 0$

$$2(x - 1) = 0$$

$$x = 1$$

Then, we evaluate of f at critical point $x = 1$ and at the interval $[-3, 1]$

$$f(1) = (1 - 1)^2 + 3 = 3$$

$$f(-3) = (-3 - 1)^2 + 3 = 19$$

Hence, we can conclude that the absolute maximum value of f on $[-3, 1]$ is 19 occurring at $x = -3$ and the minimum value of f on $[-3, 1]$ is 3 occurring at $x = 1$

(iii) $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$

Solution:

Given function is $f(x) = 3x^4 - 8x^3 + 12x^2 - 48x + 25$ on the interval $[0, 3]$

On differentiating we get

$$f'(x) = 12x^3 - 24x^2 + 24x - 48$$

$$f'(x) = 12(x^3 - 2x^2 + 2x - 4)$$

$$f'(x) = 12(x - 2)(x^2 + 2)$$

Now, for local minima and local maxima we have $f'(x) = 0$

$x = 2$ or $x^2 + 2 = 0$ for which there are no real roots.

Therefore, we consider only $x = 2 \in [0, 3]$.

Then, we evaluate of f at critical point $x = 2$ and at the interval $[0, 3]$

$$f(2) = 3(2)^4 - 8(2)^3 + 12(2)^2 - 48(2) + 25$$

$$f(2) = 48 - 64 + 48 - 96 + 25 = -39$$

$$f(0) = 3(0)^4 - 8(0)^3 + 12(0)^2 - 48(0) + 25 = 25$$

$$f(3) = 3(3)^4 - 8(3)^3 + 12(3)^2 - 48(3) + 25 = 16$$

Hence, we can conclude that the absolute maximum value of f on $[0, 3]$ is 25 occurring at $x = 0$ and the minimum value of f at $[0, 3]$ is -39 occurring at $x = 2$

(iv) $f(x) = (x - 2)\sqrt{x - 1}$ in $[1, 9]$

Solution:

$$\text{Given } f(x) = (x - 2)\sqrt{x - 1}$$

$$f'(x) = \sqrt{x - 1} + \frac{(x - 2)}{2\sqrt{x - 1}}$$

$$\text{Put } f'(x) = 0$$

$$\Rightarrow \sqrt{x-1} + \frac{(x-2)}{2\sqrt{x-1}} = 0$$

$$\Rightarrow \frac{2(x-1) + (x-2)}{2\sqrt{x-1}} = 0$$

$$\Rightarrow \frac{3x-4}{2\sqrt{x-1}} = 0$$

$$\Rightarrow x = \frac{4}{3}$$

Now, $f(1) = 0$

$$f(4/3) = \left(\frac{4}{3} - 2\right) \sqrt{\frac{4}{3} - 1} = -\frac{2}{3\sqrt{3}} = -\frac{2\sqrt{3}}{9}$$

And,

$$f(9) = (9 - 2) \sqrt{9 - 1} = 7\sqrt{8} = 14\sqrt{2}$$

Hence, we can conclude that the absolute maximum value of f is $14\sqrt{2}$ occurring at $x = 9$ and the minimum value of f is $-2\sqrt{3}/9$ occurring at $x = 4/3$.

2. Find the maximum value of $2x^3 - 24x + 107$ in the interval $[1, 3]$. Find the maximum value of the same function in $[-3, -1]$.

Solution:

$$\text{Let } f(x) = 2x^3 - 24x + 107$$

$$\therefore f'(x) = 6x^2 - 24 = 6(x^2 - 4)$$

Now, for local maxima and local minima we have $f'(x) = 0$

$$\Rightarrow 6(x^2 - 4) = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

We first consider the interval $[1, 3]$.

Then, we evaluate the value of f at the critical point $x = 2 \in [1, 3]$ and at the end points of the interval $[1, 3]$.

$$f(2) = 2(2^3) - 24(2) + 107 = 75$$

$$f(1) = 2(1)^3 - 24(1) + 107 = 85$$

$$f(3) = 2(3)^3 - 24(3) + 107 = 89$$

Hence, the absolute maximum value of $f(x)$ in the interval $[1, 3]$ is 89 occurring at $x = 3$,

Next, we consider the interval $[-3, -1]$.

Evaluate the value of f at the critical point $x = -2 \in [-3, -1]$

$$f(-3) = 2(-3)^3 - 24(-3) + 107 = 125$$

$$f(-2) = 2(-2)^3 - 24(-2) + 107 = 139$$

$$f(-1) = 2(-1)^3 - 24(-1) + 107 = 129$$

Hence, the absolute maximum value of f is 139 and occurs when $x = -2$.



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