

## EXERCISE 23.8

**A line passes through a point A (1, 2) and makes an angle of  $60^\circ$  with the x-axis and intercepts the line  $x + y = 6$  at the point P. Find AP.**

**Solution:**

Given:

$$(x_1, y_1) = A (1, 2), \theta = 60^\circ$$

Let us find the distance AP.

By using the formula,

The equation of the line is given by:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

Now, substitute the values, we get

$$\frac{x-1}{\cos 60^\circ} = \frac{y-2}{\sin 60^\circ} = r$$

$$\frac{x-1}{\frac{1}{2}} = \frac{y-2}{\frac{\sqrt{3}}{2}} = r$$

Here, r represents the distance of any point on the line from point A (1, 2).

The coordinate of any point P on this line are  $(1 + r/2, 2 + \sqrt{3}r/2)$

It is clear that, P lies on the line  $x + y = 6$

So,

$$1 + \frac{r}{2} + 2 + \frac{\sqrt{3}}{2}r = 6$$

$$\frac{\sqrt{3}}{2}r + \frac{r}{2} = 3$$

$$r(\sqrt{3} + 1) = 6$$

$$r = \frac{6}{\sqrt{3} + 1} = 3(\sqrt{3} - 1)$$

$\therefore$  The value of AP is  $3(\sqrt{3} - 1)$

**1. If the straight line through the point P(3, 4) makes an angle  $\pi/6$  with the x-axis and meets the line  $12x + 5y + 10 = 0$  at Q, find the length PQ.**

**Solution:**

Given:

$$(x_1, y_1) = A (3, 4), \theta = \pi/6 = 30^\circ$$

Let us find the length PQ.

By using the formula,

The equation of the line is given by:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

Now, substitute the values, we get

$$\frac{x-3}{\cos 30^\circ} = \frac{y-4}{\sin 30^\circ} = r$$

$$\frac{x-3}{\frac{\sqrt{3}}{2}} - \frac{y-4}{\frac{1}{2}} = r$$

$$x - \sqrt{3}y + 4\sqrt{3} - 3 = 0$$

Let  $PQ = r$

Then, the coordinate of Q are given by

$$\frac{x-3}{\cos 30^\circ} = \frac{y-4}{\sin 30^\circ} = r$$

$$x = 3 + \frac{\sqrt{3}}{2}r, y = 4 + \frac{r}{2}$$

The coordinate of point Q is  $(3 + \frac{\sqrt{3}}{2}r, 4 + \frac{r}{2})$

It is clear that, Q lies on the line  $12x + 5y + 10 = 0$

So,

$$12\left(3 + \frac{\sqrt{3}}{2}r\right) + 5\left(4 + \frac{r}{2}\right) + 10 = 0$$

$$66 + \frac{12\sqrt{3}+5}{2}r = 0$$

$$r = -\frac{132}{5+12\sqrt{3}}$$

$$PQ = |r| = \frac{132}{5+12\sqrt{3}}$$

$\therefore$  The value of PQ is  $\frac{132}{5+12\sqrt{3}}$

**2. A straight line drawn through the point A (2, 1) making an angle  $\pi/4$  with positive x-axis intersects another line  $x + 2y + 1 = 0$  in the point B. Find length AB.**

**Solution:**

Given:

$$(x_1, y_1) = A(2, 1), \theta = \pi/4 = 45^\circ$$

Let us find the length AB.

By using the formula,

The equation of the line is given by:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

Now, substitute the values, we get

$$\frac{x-3}{\cos 30^\circ} = \frac{y-4}{\sin 30^\circ} = r$$

$$\frac{x-3}{\frac{\sqrt{3}}{2}} = \frac{y-4}{\frac{1}{2}} = r$$

$$x - y - 1 = 0$$

Let  $AB = r$

Then, the coordinate of B is given by

$$\frac{x-2}{\cos 45^\circ} = \frac{y-1}{\sin 45^\circ} = r$$

$$x = 2 + \frac{1}{\sqrt{2}}r, y = 1 + \frac{r}{\sqrt{2}}$$

The coordinate of point B is  $\left(2 + \frac{1}{\sqrt{2}}r, 1 + \frac{r}{\sqrt{2}}\right)$

It is clear that, B lies on the line  $x + 2y + 1 = 0$

$$2 + \frac{1}{\sqrt{2}}r + 2\left(1 + \frac{r}{\sqrt{2}}\right) + 1 = 0$$

$$5 + \frac{3r}{\sqrt{2}} = 0$$

$$r = \frac{5\sqrt{2}}{3}$$

$\therefore$  The value of AB is  $\frac{5\sqrt{2}}{3}$

**3. A line a drawn through A (4, -1) parallel to the line  $3x - 4y + 1 = 0$ . Find the coordinates of the two points on this line which are at a distance of 5 units from A.**

**Solution:**

Given:

$$(x_1, y_1) = A(4, -1)$$

Let us find Coordinates of the two points on this line which are at a distance of 5 units from A.

$$\text{Given: Line } 3x - 4y + 1 = 0$$

$$4y = 3x + 1$$

$$y = \frac{3x}{4} + \frac{1}{4}$$

$$\text{Slope } \tan \theta = \frac{3}{4}$$

So,

$$\sin \theta = \frac{3}{5}$$

$$\cos \theta = \frac{4}{5}$$

The equation of the line passing through A (4, -1) and having slope  $\frac{3}{4}$  is

By using the formula,

The equation of the line is given by:

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$$

Now, substitute the values, we get

$$\frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}}$$

$$3x - 4y = 16$$

Here, AP = r =  $\pm 5$

Thus, the coordinates of P are given by

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta} = r$$

$$\frac{x-4}{\frac{4}{5}} = \frac{y+1}{\frac{3}{5}} = r$$

$$x = \frac{4r}{5} + 4 \text{ and } y = \frac{3r}{5} - 1$$

$$x = \frac{4(\pm 5)}{5} + 4 \text{ and } y = \frac{3(\pm 5)}{5} - 1$$

$$x = \pm 4 + 4 \text{ and } y = \pm 3 - 1$$

$$x = 8, 0 \text{ and } y = 2, -4$$

$\therefore$  The coordinates of the two points at a distance of 5 units from A are (8, 2) and (0, -4).

**4. The straight line through  $P(x_1, y_1)$  inclined at an angle  $\theta$  with the x-axis meets the line  $ax + by + c = 0$  in Q. Find the length of PQ.**

**Solution:**

Given:

The equation of the line that passes through  $P(x_1, y_1)$  and makes an angle of  $\theta$  with the x-axis.

Let us find the length of PQ.

We know that,

$$\frac{x-x_1}{\cos\theta} = \frac{y-y_1}{\sin\theta}$$

Let PQ = r

Then, the coordinates of Q are given by

By using the formula,

The equation of the line is given by: