NCERT Solutions for Class-XI Maths

Chapter-13 Exercise-13.1

NCERT Math Class 11

1. Evaluate the Given $\lim_{x \to 3} x+3$ limit:

1. $\lim_{x \to 3} x+3 = 3+3 = 6$

2. $\lim_{x \to \pi}\left(x - \frac{22}{7}\right)$

2. $\Rightarrow \lim_{x \to \pi}\left(x - \frac{22}{7}\right) = \left(\pi - \frac{22}{7}\right)$

3. Evaluate the Given limit: $\lim_{r \to 1} \pi r^2$

3. $\lim_{r \to 1} \pi r^2 = \pi(1)^2 = \pi$

4. $\lim_{x \to 4} \frac{4x+3}{x-2}$

4. $\Rightarrow \lim_{x \to 4} \frac{4x+3}{x-2} = \frac{4(4)+3}{4-2} = \frac{19}{2}$

5. Evaluate the Given limit: $\lim_{x \to -1} \frac{x^{10}+x^3+1}{x-1}$

5. $\lim_{x \to -1} \frac{x^{10}+x^5+1}{x-1} = \frac{(-1)^{10}+(-1)^5+1}{-1-1} = \frac{1-1+1}{-2} = -\frac{1}{2}$

6. $\lim_{x \to 0} \frac{(x+1)^5-1}{x}$

6. $\Rightarrow \lim_{x \to 0} \frac{(x+1)^5-1}{x} = \frac{(0+1)^5-1}{0} = \frac{0}{0}$

Let x + 1 = y.

So, x = y – 1

$\Rightarrow \lim_{y \to 1} \frac{(y)^5-1}{y-1} = \lim_{y \to 1} \frac{(y)^5-1^5}{y-1}$

We know that $\lim_{x \to a} \frac{x^n-a^n}{x-a} = na^{n-1}$

$$\Rightarrow \lim_{y\to 1} \frac{(y)^5 - 1^5}{y-1}$$

$=5(1)^{5-1} = 5(1)^4 = 5$

7. Evaluate the Given limit: $\lim_{x\to 2} \frac{3x^2 - x - 10}{x^2 - 4}$

7. At $x = 2$, the value of the given rational function takes the form. $\frac{0}{0}$

$$\therefore \lim_{x\to 2} \frac{3x^2 - x - 10}{x^2 - 4} = \lim_{x\to 2} \frac{(x-2)(3x+5)}{(x-2)(x+2)}$$

$$= \lim_{x\to 2} \frac{3x+5}{x+2}$$

$$= \frac{3(2)+5}{2+2}$$

$$= \frac{11}{4}$$

8. $\lim_{x\to 3} \frac{x^4 - 81}{2x^2 - 5x - 3}$

8. $\Rightarrow \lim_{x\to 2} \frac{x^4 - 81}{2x^2 - 5x - 3} = \frac{81 - 81}{18 - 15 - 3} = \frac{0}{0}$

So, $\lim_{x\to 2} \frac{x^4 - 81}{2x^2 - 5x - 3} = \lim_{x\to 2} \frac{(x^2 - 3^2)(x^2 + 3^2)}{2x^2 - 6x + x - 3} = \lim_{x\to 2} \frac{(x-3)(x+3)(x^2+9)}{(x-3)(2x+1)} = \lim_{x\to 2} \frac{(x+3)(x^2+9)}{(2x+1)}$

$$\Rightarrow \frac{(2+3)(2^2+9)}{(2(2)+1)} = \frac{5(13)}{(5)} = 13$$

9. Evaluate the Given limit: $\lim_{x\to 0} \frac{ax+b}{cx+1}$

9. $\lim_{x\to 0} \frac{ax+b}{cx+1} = \frac{a(0)+b}{c(0)+1} = b$

10. $\lim_{z\to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1}$

10. $\Rightarrow \lim_{z\to 1} \frac{z^{\frac{1}{3}} - 1}{z^{\frac{1}{6}} - 1} = \frac{1-1}{1-1} = \frac{0}{0}$

Let $z^{1/6} = x$

$\Rightarrow \lim_{x\to 1}\frac{x^2-1}{x-1}=\frac{x^2-1^2}{x-1}$

We know that $\lim_{x\to a}\frac{x^n-a^n}{x-a}$=na^{n-1}

$\Rightarrow \lim_{x\to 1}\frac{x^2-1^2}{x-1}=2(1)^{2-1}=2$

11. Evaluate the Given limit: $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}, a+b+c\neq 0$

11. $\lim_{x\to 1}\frac{ax^2+bx+c}{cx^2+bx+a}=\frac{a(1)^2+b(1)+c}{c(1)^2+b(1)+a}$

$=\frac{a+b+c}{a+b+c}$

$=1\ [a+b+c\neq 0]$

12. $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}$

12. $\Rightarrow \lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}=\frac{0}{0}$

So, $\lim_{x\to -2}\frac{\frac{1}{x}+\frac{1}{2}}{x+2}=\frac{\frac{2+x}{2x}}{x+2}=\frac{1}{2x}=\frac{1}{2(-2)}=\frac{-1}{4}$

13. Evaluate the Given limit: $\lim_{x\to 0}\frac{\sin ax}{bx}$

13. $\lim_{x\to 0}\frac{\sin ax}{bx}$

At $x=0$, the value of the given function takes the form $\frac{0}{0}$

Now, $\lim_{x\to 0}\frac{\sin ax}{bx}=\lim_{x\to 0}\frac{\sin ax}{ax}\times\frac{ax}{bx}$

$=\lim_{x\to 0}\left(\frac{\sin ax}{ax}\right)\times\left(\frac{a}{b}\right)$

$=\frac{a}{b}\lim_{x\to 0}\left(\frac{\sin ax}{ax}\right) \qquad [x\to 0\Rightarrow ax\to 0]$

$$= \frac{a}{b} \times 1 \qquad \left[\lim_{y \to 0} \frac{\sin y}{y} = 1 \right]$$

$$= \frac{a}{b}$$

14. $\lim_{x \to 0} \frac{\sin ax}{\sin bx}, a, b \neq 0$

14. $\Rightarrow \lim_{x \to 0} \frac{\sin ax}{\sin bx} = \frac{0}{0}$

So, $\lim_{x \to 0} \frac{\sin ax}{\sin bx} = \lim_{x \to 0} \frac{\frac{\sin ax}{ax} \times ax}{\frac{\sin bx}{bx} \times bx}$

$$\Rightarrow \frac{a}{b} \frac{\lim_{ax \to 0} \frac{\sin ax}{ax}}{\lim_{bx \to 0} \frac{\sin bx}{bx}}$$

We know that $\lim_{x \to 0} \frac{\sin x}{x} = 1$

$$\Rightarrow \frac{a}{b} \times 1 = \frac{a}{b}$$

15. Evaluate the Given limit: $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

15. $\lim_{x \to \pi} \frac{\sin(\pi - x)}{\pi(\pi - x)}$

It is seen that $x \to \pi \Rightarrow (\pi - x) \to 0$

$$\therefore \lim_{x \to =} \frac{\sin(\pi - x)}{\pi(\pi - x)} = \frac{1}{\pi} \lim_{(x - x) \to 0} \frac{\sin(\pi - x)}{(\pi - x)}$$

$$= \frac{1}{\pi} \times 1 \left[\lim_{y \to 0} \frac{\sin y}{y} = 1 \right]$$

$$= \frac{1}{\pi}$$

16. $\lim_{x \to 0} \frac{\cos x}{\pi - x}$

16. $\Rightarrow \lim_{x \to 0} \frac{\cos x}{\pi - x} = \frac{\cos 0}{\pi - 0} = \frac{1}{\pi}$

Ans. $\lim_{x\to 0}\frac{\cos x}{\pi - x} = \frac{1}{\pi}$

17. Evaluate the Given limit: $\lim_{x\to 0}\frac{\cos 2x - 1}{\cos x - 1}$

17. $\lim_{x\to 0}\frac{\cos 2x - 1}{\cos x - 1}$

At $x = 0$, the value of the given function takes the form. $\frac{0}{0}$ Now,

$$\lim_{x\to 0}\frac{\cos 2x - 1}{\cos x - 1} = \lim_{x\to 0}\frac{1 - 2\sin^2 x - 1}{1 - 2\sin^2\frac{x}{2} - 1}\left[\cos x = 1 - 2\sin^2\frac{x}{2}\right]$$

$$= \lim_{x\to 0}\frac{\sin^2 x}{\sin^2\frac{x}{2}} = \lim_{x\to 0}\frac{\left(\frac{\sin^2 x}{x^2}\right)\times x^2}{\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)\times\frac{x^2}{4}}$$

$$= 4\frac{\lim_{x\to 0}\left(\frac{\sin^2 x}{x^2}\right)}{\lim_{x\to 0}\left(\frac{\sin^2\frac{x}{2}}{\left(\frac{x}{2}\right)^2}\right)}$$

$$= 4\frac{\left(\lim_{x\to 0}\frac{\sin x}{x}\right)^2}{\left(\lim_{\frac{x}{2}\to 0}\frac{\sin\frac{x}{2}}{\frac{x}{2}}\right)^2}\left[x\to 0 \Rightarrow \frac{x}{2}\to 0\right]$$

$$= 4\frac{1^2}{1^2}\left[\lim_{y\to 0}\frac{\sin y}{y} = 1\right] = 4$$

18. $\lim_{x\to 0}\frac{ax + x\cos x}{b\sin x}$

18. $\Rightarrow \lim_{x\to 0}\frac{ax + x\cos x}{b\sin x} = \frac{0}{0}$

So, $\lim_{x\to 0}\frac{ax + x\cos x}{b\sin x} = \frac{1}{b}\lim_{x\to 0}\frac{x(a + \cos x)}{\sin x} = \frac{1}{b}\lim_{x\to 0}\times\lim_{x\to 0}(a + \cos x)$

$$=\frac{1}{b}\times\frac{1}{\lim_{x\to 0}\frac{\sin x}{x}}\times\lim_{x\to 0}(a+\cos x)$$

We know that $\lim_{x\to 0}\frac{\sin x}{x}=1$

$$=\frac{1}{b}\times(a+\cos 0)=\frac{a+1}{b}$$

Ans. $\lim_{x\to 0}\frac{ax+x\cos x}{b\sin x}=\frac{a+1}{b}$

19. Evaluate the Given limit: $\lim_{x\to 0} x\sec x$

19. $\lim_{x\to 0} x\sec x=\lim_{x\to 0}\frac{x}{\cos x}=\frac{0}{\cos 0}=\frac{0}{1}=0$

20. $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}\,a,b,a+b\neq 0$

20. $\Rightarrow \lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}=\frac{0}{0}$

So, $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}=\lim_{x\to 0}\frac{(\sin\frac{ax}{ax})ax+bx}{ax+(\sin\frac{bx}{bx})}$

$$=\frac{\left(\lim_{ax\to 0}\sin\frac{ax}{ax}\right)\times\lim_{x\to 0}ax+\lim_{x\to 0}bx}{\lim_{x\to 0}ax+\lim_{x\to 0}bx\times\left(\lim_{bx\to 0}\sin\frac{bx}{bx}\right)}$$

We know that $\lim_{x\to 0}\frac{\sin x}{x}=1$

$$=\frac{\lim_{x\to 0}ax+\lim_{x\to 0}bx}{\lim_{x\to 0}ax+\lim_{x\to 0}bx}$$

$$=\frac{\lim_{x\to 0}\left(ax+bx\right)}{\lim_{x\to 0}(ax+bx)}$$

=1

Ans. $\lim_{x\to 0}\frac{\sin ax+bx}{ax+\sin bx}=1$

21. Evaluate the Given limit: $\lim_{x\to 0}\left(\operatorname{cosec}x-\cot x\right)$

21. At $x=0$, the value of the given function takes the form. $\infty-\infty$

Now,

$$\lim_{x\to 0}(\operatorname{cosec}x-\cot x)$$

$$=\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)$$

$$=\lim_{x\to 0}\left(\frac{1-\cos x}{\sin x}\right)$$

$$=\lim_{x\to 0}\frac{\left(\frac{1-\cos x}{x}\right)}{\left(\frac{\sin x}{x}\right)}$$

$$=\frac{\lim_{x\to 0}\frac{1-\cos x}{x}}{\lim_{x\to 0}\frac{\sin x}{x}}\left[\lim_{x\to 0}\frac{1-\cos x}{x}=0\text{ and }\lim_{x\to 0}\frac{\sin x}{x}=1\right]$$

$$=\frac{0}{1}=0$$

22. $\lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}$

22. $\Rightarrow \lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}=\frac{0}{0}$

Let x – (π/2) = y

So, x→ (π/2) = y→ 0

Now, $\lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}=\lim_{y\to 0}\frac{\tan 2\left(y+\frac{\pi}{2}\right)}{y}$

$$=\lim_{y\to 0}\frac{\tan(2y+\pi)}{y}$$

$$=\lim_{y\to 0}\frac{\tan(2y)}{y}$$

We know that tan x = sin x/ cos x

$$=\lim_{y\to 0}\frac{\sin 2y}{y\cos 2y}$$

Multiply and divide by 2,

$$=\lim_{y\to 0}\frac{\sin 2y}{2y}\times\frac{2}{\cos 2y}$$

$$=\lim_{2y\to 0}\frac{\sin 2y}{2y}\times\lim_{y\to 0}\frac{2}{\cos 2y}$$

$$=1\times\frac{2}{\cos 0}$$

$$=1\times\frac{2}{1}$$

$=2$

Ans. $\lim_{x\to\frac{\pi}{2}}\frac{\tan 2x}{x-\frac{\pi}{2}}=2$

23. Find $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$, where $f^f(x)=\begin{cases}2x+3, & x\le 0\\ 3(x+1), & x>0\end{cases}$

23. The given function is $f(x)=\begin{cases}2x+3, & x\le 0\\ 3(x+1), & x>0\end{cases}$

$$\lim_{x\to 0^-} f(x)=\lim_{x\to 0}[2x+3]=2(0)+3=3$$

$$\lim_{x\to 0^+} f(x)=\lim_{x\to 0}3(x+1)=3(0+1)=3$$

$$\therefore \lim_{x\to 0^-} f(x)=\lim_{x\to 0^+} f(x)=\lim_{x\to 0} f(x)=3$$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1}3(x+1)=3(1+1)=6$$

$$\lim_{x\to 1^-} f(x)=\lim_{x\to 1}3(x+1)=3(1+1)=6$$

$$\therefore \lim_{x\to 1^-} f(x)=\lim_{x\to 1^+} f(x)=\lim_{x\to 1} f(x)=6$$

24. Find $\lim_{x\to 1} f(x)$, where

$$f(x)=\begin{cases}x^2-1 & x\le 1\\ -x^2-1 & x>1\end{cases}$$

24. Given function is f(x) = $x^2 - 1$, $x \le 1$

$-x^2 - 1$, $x > 1$

$\lim_{x\to 1} f(x)$:

$$\Rightarrow \lim_{x\to 1^-} f(x)=\lim_{x\to 1} x^2-1=1^2-1=1-1=0$$

$\Rightarrow \lim_{x\to 1^+} f(x) = \lim_{x\to 1}(-x^2-1) = (-1^2-1) = -1-1 = -2$

Here $\lim_{x\to 1^-} f(x) \neq \lim_{x\to 1^+} f(x)$

$\therefore \lim_{x\to 1} f(x)$ does not exist.

25. Evaluate $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

25. The given function is $f(x) = \begin{cases} \frac{|x|}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

$$\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-}\left[\frac{|x|}{x}\right]$$

$$= \lim_{x\to 0}\left(\frac{-x}{x}\right) \quad [\text{When } x \text{ is negaitve}, |x| = -x]$$

$$= \lim_{x\to 0}(-1)$$

$$= -1$$

$$\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+}\left[\frac{|x|}{x}\right]$$

$$= \lim_{x\to 0}\left[\frac{x}{x}\right] \quad [\text{When } x \text{ is positive}, |x| = x]$$

$$= \lim_{x\to 0}(1)$$

$$= 1$$

It is observed that $\lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x)$.

Hence, $\lim_{x\to 0} f(x)$ does not exist.

26. Find $\lim_{x\to 0} f(x)$, where $f(x) = \begin{cases} \frac{x}{|x|}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

26. Given function is f(x) = $\frac{x}{|x|}$, x ≠ 0

0, x = 0

$\lim_{x\to 0} f(x)$:

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} \frac{x}{|x|} = \lim_{x\to 0} \frac{x}{-x} = \lim_{x\to 0} \frac{1}{-1} = -1$

$\Rightarrow \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{x}{|x|} = \lim_{x \to 0} \frac{x}{x} = \lim_{x \to 0}(1) = 1$

Here $\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

$\therefore \lim_{x \to 0} f(x)$ does not exist.

27. Find $\lim_{x \to 5} f(x)$, where $f(x) = |x| - 5$

27. The aiven function is $f(x) = |x| - 5$.

$\lim_{x \to 5^-} f(x) = \lim_{x \to 5^-}[|x| - 5]$

$= \lim_{x \to 5}(x - 5)$ [When $x > 0, |x| = x$]

$= 5 - 5$

$= 0$

$\lim_{x \to 5^+} f(x) = \lim_{x \to 5^+}(|x| - 5)$

$= \lim_{x \to 5}(x - 5)$ [When $x > 0, |x| = x$]

$= 5 - 5$

$= 0$

$\therefore \lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x) = 0$

Hence, $\lim_{x \to 5} f(x) = 0$

28. Suppose $f(x) = \begin{cases} a + bx, x < 1 \\ 4, \quad x = 1 \\ b - ax \; x > 1 \end{cases}$ and if $\lim_{x \to 1} f(x) =$ f (1) what are possible values of a and b?

28. Given function is $f(x) = \begin{cases} a + bx, x < 1 \\ 4, x = 1 \\ b - ax, x > 1 \end{cases}$

And $\lim_{x \to 1} f(x) = f(1)$

$\Rightarrow \lim_{x \to 1^-} f(x) = \lim_{x \to 1} a + bx = a + b(1) = a + b$

$\Rightarrow \lim_{x \to 1^+} f(x) = \lim_{x \to 1} b - ax = b - a(1) = b - a$

Here, f(1) = 4

$\therefore \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1} f(x) = f(1)$

So, a + b = 4 and b – a = 4

Solving the above two equations, we get

⇒ a = 0 and b = 4

Ans. The possible values of a and b for the given function f(x) are 0 and 4 respectively.

29. Let $a_1, a_2, \ldots,$ an be fixed real numbers and define a function

$f(x)=(x-a_1)(x-a_2)\ldots(x-a_n)$.

What is $\lim_{x\to a_1} f(x)$? For some $a \neq a_1, a_2 \ldots a_n$, compute $\lim_{x\to a} f(x)$.

29. The aiven function is $f(x)=(x-a_1)(x-a_2)\ldots(x-a_n)$

$$\lim_{x\to a_1} f(x)=\lim_{x\to a_1}\left[(x-a_1)(x-a_2)\ldots(x-a_n)\right]$$

$$=\left[\lim_{x\to a_1}(x-a_1)\right]\left[\lim_{x\to a_1}(x-a_2)\right]\ldots\left[\lim_{x\to a_1}(x-a_n)\right]$$

$$=(a_1-a_1)(a_1-a_2)\ldots(a_1-a_n)=0$$

$$\therefore \lim_{x\to a_1} f(x)=0$$

Now, $\lim_{x\to a} f(x)=\lim_{x\to a}\left[(x-a_1)(x-a_2)\ldots(x-a_n)\right]$

$$=\left[\lim_{x\to a}(x-a_1)\right]\left[\lim_{x\to a}(x-a_2)\right]\ldots\left[\lim_{x\to a}(x-a_n)\right]$$

$$=(a-a_1)(a-a_2)\ldots\ldots(a-a_n)$$

$$\therefore \lim_{x\to a} f(x)=(a-a_1)(a-a_2)\ldots(a-a_n)$$

30. If $f(x)=\begin{cases}|x|+1, x<0\\ 0, \quad x=0\\ |x|-1, x>0\end{cases}$

For what value (s) of a does $\lim_{x\to a} f(x)$ exists?

30. Given function is f(x) = $|x|+1$, x < 0

0, x = 0

$|x|-1$, x > 0

There are three cases.

Case 1: When a = 0

$\lim_{x\to 0} f(x)$:

$$\Rightarrow \lim_{x\to 0^-} f(x)=\lim_{x\to 0^-}(|x|+1)=\lim_{x\to 0}(-x+1)=-0+1=1$$

$$\Rightarrow \lim_{x\to 0^+} f(x)=\lim_{x\to 0^+}(|x|-1)=\lim_{x\to 0}(x-1)=0-1=-1$$

Here $\lim_{x\to 0^-} f(x)\neq \lim_{x\to 0^+} f(x)$

$\therefore \lim_{x\to 0} f(x)$ does not exist.

Case 2: When a < 0

$\lim_{x\to a} f(x)$:

$\Rightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^-}(|x|+1) = \lim_{x\to a}(-x+1) = -a+1$

$\Rightarrow \lim_{x\to a^+} f(x) = \lim_{x\to a^+}(|x|+1) = \lim_{x\to a}(-x+1) = -a+1$

$\therefore \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a} f(x) = -a+1$

∴ lim(f(x)) exists at x = a when a < 0

Case 3: When a > 0

$\lim_{x\to a} f(x)$:

$\Rightarrow \lim_{x\to a^-} f(x) = \lim_{x\to a^-}(|x|-1) = \lim_{x\to a}(x-1) = a-1$

$\Rightarrow \lim_{x\to a^+} f(x) = \lim_{x\to a^+}(|x|-1) = \lim_{x\to a}(x-1) = a-1$

$\therefore \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = \lim_{x\to a} f(x) = a-1$

∴ lim(f(x)) exists at x = a when a > 0

Ans. $\lim_{x\to a} f(x)$ exists for all a ≠ 0

31. If the function $f(x)$ satisfies, $\lim_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$ evaluate $\lim_{x\to 1} f(x)$.

31. $\lim_{x\to 1}\frac{f(x)-2}{x^2-1} = \pi$

$\Rightarrow \frac{\lim_{x\to 1}(f(x)-2)}{\lim_{x\to 1}(x^2-1)} = \pi$

$\Rightarrow \lim_{x\to 1}(f(x)-2) = \pi \lim_{x\to 1}(x^2-1)$

$\Rightarrow \lim_{x\to 1}(f(x)-2) = \pi(1^2-1)$

$\Rightarrow \lim_{x\to 1}(f(x)-2) = 0$

$\Rightarrow \lim_{x\to 1} f(x) - \lim_{x\to 1} 2 = 0$

$\Rightarrow \lim_{x\to 1} f(x) - 2 = 0$

$\therefore \lim_{x\to 1} f(x) = 2$

32. If $f(x)=\begin{cases} mx^2+n, & x<0 \\ nx+m, & 0\le x\le 1 \\ nx^3+m. & x>1 \end{cases}$ For what integers m and n does both $\lim_{x\to 0} f(x)$ and $\lim_{x\to 1} f(x)$ exist?

32. Given function is f(x) $\begin{cases} = mx^2 + n, & x < 0 \\ nx + m, & 0 \le x \le 1 \\ nx^3 + m, & x > 1 \end{cases}$

$\lim_{x\to 0} f(x)$:

$\Rightarrow \lim_{x\to 0^-} f(x) = \lim_{x\to 0}(mx^2+n) = m(0)+n = 0+n = n$

$\Rightarrow \lim_{x\to 0^+} f(x) = \lim_{x\to 0}(nx+m) = n(0)+m = 0+m = m$

$\therefore \lim_{x\to 0} f(x)$ exists if n = m.

Now $\lim_{x\to 1} f(x)$:

$\Rightarrow \lim_{x\to 1^-} f(x) = \lim_{x\to 1}(nx+m) = n(1)+m = n+m$

$\Rightarrow \lim_{x\to 1^+} f(x) = \lim_{x\to 1}(nx^3+m) = n(1)^3+m = n(1)+m = n+m$

$\therefore \lim_{x\to 1^-} f(x) = \lim_{x\to 1^+} f(x) = \lim_{x\to 1} f(x)$

$\therefore \lim_{x\to 1} f(x)$ exists for any integral value of m and n.