NCERT Solutions for Class-XII Maths

Chapter-3.3

NCERT Math Class 12

1. Find the transpose of each of the following matrices:

(i) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$

(ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

(iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

1. (i) Let $A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$, then $A^T = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$

(ii) Let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$, then $A^T = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$

(iii) Let $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$, then $A^T = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$

2. If, $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$ then verify that

(i) (A + B)' = A' + B'

(ii) (A – B)' = A' – B'

2. $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

(i)(A+B)' = A'+B'

Explanation: We will first calculate L.H.S i.e. (A+B)' and then consecutively we will calculate R.H.S and verify that both are equal.

$$\text{So, } A+B=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}+\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}$$

$$\Rightarrow A+B=\begin{bmatrix}-1+(-4) & 2+1 & 3+(-5)\\ 5+1 & 7+2 & 9+0\\ -2+1 & 1+3 & 1+1\end{bmatrix}$$

$$\Rightarrow A+B=\begin{bmatrix}-5 & 3 & -2\\ 6 & 9 & 9\\ -1 & 4 & 2\end{bmatrix}$$

$$\text{Therefore, } (A+B)^{'}=\begin{bmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{bmatrix}\rightarrow \text{eqn1}$$

$$\text{Now, } A'=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix} \text{ and } B'=\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}$$

$$\text{So, } A'+B'=\begin{bmatrix}-1 & 5 & -2\\ 2 & 7 & 1\\ 3 & 9 & 1\end{bmatrix}+\begin{bmatrix}-4 & 1 & 1\\ 1 & 2 & 3\\ -5 & 0 & 1\end{bmatrix}$$

$$\Rightarrow A'+B'=\begin{bmatrix}-1+(-4) & 5+1 & -2+1\\ 2+1 & 7+2 & 1+3\\ 3+(-5) & 9+0 & 1+1\end{bmatrix}$$

$$\Rightarrow A'+B'=\begin{bmatrix}-5 & 6 & -1\\ 3 & 9 & 4\\ -2 & 9 & 2\end{bmatrix}\rightarrow eq^{n}2$$

From equation 1 & 2 we verify that

(A+B)' = A'+B'. Hence verified.

(ii)(A-B)' = A'-B'

Explanation: We will first calculate L.H.S i.e. (A+B)' and then consecutively we will calculate R.H.S and verify that both are equal.

$$\text{So, } A-B=\begin{bmatrix}-1 & 2 & 3\\ 5 & 7 & 9\\ -2 & 1 & 1\end{bmatrix}-\begin{bmatrix}-4 & 1 & -5\\ 1 & 2 & 0\\ 1 & 3 & 1\end{bmatrix}$$

$$\Rightarrow A-B=\begin{bmatrix} -1-(-4) & 2-1 & 3-(-5) \\ 5-1 & 7-2 & 9-0 \\ -2-1 & 1-3 & 1-1 \end{bmatrix}$$

$$\Rightarrow A-B=\begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$$

Therefore, $(A-B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \rightarrow \text{eqn1}$

Now, $A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ and $B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

So, $A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$$\Rightarrow A' - B' \begin{bmatrix} -1-(-4) & 5-1 & -2-1 \\ 2-1 & 7-2 & 1-3 \\ 3-(-5) & 9-0 & 1-1 \end{bmatrix}$$

$$\Rightarrow A' - B' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \rightarrow \text{eqn2}$$

From equation 1 & 2 we verify that
(A-B)' = A'-B'. Hence verified.

3. If , $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$ then verify that

(i) (A + B)' = A' + B'
(ii) (A – B)' = A' – B'

3. (i) It is known that A = (A')'
Therefore, we have:

$$A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$$

$$B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$$

$$A+B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}+\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}2 & 1 & 1\\ 5 & 4 & 4\end{bmatrix}$$

$$\therefore (A+B)'=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}$$

$$A'+B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}+\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}2 & 5\\ 1 & 4\\ 1 & 4\end{bmatrix}$$

Thus, we have verified that (A + B)' = A' + B'.

(ii) $A-B=\begin{bmatrix}3 & -1 & 0\\ 4 & 2 & 1\end{bmatrix}-\begin{bmatrix}-1 & 2 & 1\\ 1 & 2 & 3\end{bmatrix}=\begin{bmatrix}4 & -3 & -1\\ 3 & 0 & -2\end{bmatrix}$

$$\therefore (A-B)'=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}$$

$$A'-B'=\begin{bmatrix}3 & 4\\ -1 & 2\\ 0 & 1\end{bmatrix}-\begin{bmatrix}-1 & 1\\ 2 & 2\\ 1 & 3\end{bmatrix}=\begin{bmatrix}4 & 3\\ -3 & 0\\ -1 & -2\end{bmatrix}$$

Thus, we have verified that (A – B)' = A' – B'.

4. If $A'=\begin{bmatrix}-2 & 3\\ 1 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$, then find (A + 2B)'

4. $A=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}$ and $B=\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$

Explanation: Whenever a constant term is multiplied with a matrix then it implies that every elements in each rows and columns are to be multiplied with that constant term. So in order to solve this question we have to multiplied every element of the matrix B with constant term that is 2.

So, $A+2B=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}+2\begin{bmatrix}-1 & 0\\ 1 & 2\end{bmatrix}$

$$\Rightarrow A+2B=\begin{bmatrix}-2 & 1\\ 3 & 2\end{bmatrix}+\begin{bmatrix}-2 & 0\\ 2 & 4\end{bmatrix}$$

$$\Rightarrow A+2B=\begin{bmatrix}-4 & 1\\ 5 & 6\end{bmatrix}$$

Now, (A+2B)’ = transpose of A+2B

$$\Rightarrow (A+2B)'=\begin{bmatrix}-4 & 5\\ 1 & 6\end{bmatrix}$$

5. for the matrices A and B, verify that (AB)' = B'A' where

(i) $A=\begin{bmatrix}1\\-4\\3\end{bmatrix}, B=[-1 \quad 2 \quad 1]$

(ii) $A=\begin{bmatrix}0\\1\\2\end{bmatrix}, B=[1 \quad 5 \quad 7]$

5.

(i) $AB=\begin{bmatrix}1\\-4\\3\end{bmatrix}[-1 \quad 2 \quad 1]=\begin{bmatrix}-1 & 2 & 1\\4 & -8 & -4\\-3 & 6 & 3\end{bmatrix}$

$\therefore (AB)'=\begin{bmatrix}-1 & 4 & -3\\2 & -8 & 6\\1 & -4 & 3\end{bmatrix}$

Now, $A'=[1 \quad -4 \quad 3], B'=\begin{bmatrix}-1\\2\\1\end{bmatrix}$

$\therefore B'A'=\begin{bmatrix}-1\\2\\1\end{bmatrix}[1 \quad -4 \quad 3]\begin{bmatrix}-1 & 4 & -3\\2 & -8 & 6\\1 & -4 & 3\end{bmatrix}$

Hence, we have verified that (AB)' = B'A'.

(ii) $AB=\begin{bmatrix}0\\1\\2\end{bmatrix}[1 \quad 5 \quad 7]=\begin{bmatrix}0 & 0 & 0\\1 & 5 & 7\\2 & 10 & 14\end{bmatrix}$

$\therefore (AB)'=\begin{bmatrix}0 & 1 & 2\\0 & 5 & 10\\0 & 7 & 14\end{bmatrix}$

Now, $A'=[0 \quad 1 \quad 2], B'=\begin{bmatrix}1\\5\\7\end{bmatrix}$

$\therefore B'A'=\begin{bmatrix}1\\5\\7\end{bmatrix}[0 \quad 1 \quad 2]=\begin{bmatrix}0 & 1 & 2\\0 & 5 & 10\\0 & 7 & 14\end{bmatrix}$

Hence, we have verified that (AB)' = B'A'.

6. $A=\begin{bmatrix}\cos a & \sin a\\ -\sin a & \cos a\end{bmatrix}$ If (i), then verify that A'A = I

$A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$ (ii), then verify that A'A = I

6. (i) $A=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}$

We know A' can be calculated by taking the transpose of the given matrix A.

Therefore, $A'=\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

Now multiply A and A'. So,

$AA'=\begin{bmatrix}\cos\alpha & \sin\alpha\\ -\sin\alpha & \cos\alpha\end{bmatrix}\times\begin{bmatrix}\cos\alpha & -\sin\alpha\\ \sin\alpha & \cos\alpha\end{bmatrix}$

$\Rightarrow AA'=\begin{bmatrix}(\cos\alpha\times\cos\alpha)+(\sin\alpha)\times(\sin\alpha) & \cos\alpha\times(-\sin\alpha)+(\sin\alpha)\times\cos\alpha\\ -\sin\alpha\times\cos\alpha+\cos\alpha\times(\sin\alpha) & -\sin\alpha\times(-\sin\alpha)+\cos\alpha\times\cos\alpha\end{bmatrix}$

$\Rightarrow AA'=\begin{bmatrix}\cos^2\alpha+\sin^2\alpha & -\sin\alpha\cos\alpha+\cos\alpha\sin\alpha\\ -\sin\alpha\cos\alpha+\cos\alpha\sin\alpha & \sin^2\alpha+\cos^2\alpha\end{bmatrix}$

$\Rightarrow AA'=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ eqn1 $\left(\because \cos\alpha^2+\sin\alpha^2=1\right)$

And we know 'I' represents an identity matrix

Therefore, $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ eqn2

From equation 1 & 2 we can say that

AA' = I

Ans. AA' = I. Hence verified.

(ii) $A=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}$

Again, $A'=\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$

$AA'=\begin{bmatrix}\sin\alpha & \cos\alpha\\ -\cos\alpha & \sin\alpha\end{bmatrix}\times\begin{bmatrix}\sin\alpha & -\cos\alpha\\ \cos\alpha & \sin\alpha\end{bmatrix}$

$\Rightarrow AA'=\begin{bmatrix}\sin\alpha\times\sin\alpha+\cos\alpha\times\cos\alpha & \sin\alpha\times(-\cos\alpha)+\cos\alpha\times\sin\alpha\\ -\cos\alpha\times\sin\alpha+\sin\alpha\times\cos\alpha & -\cos\alpha\times(-\cos\alpha)+\sin\alpha\times\sin\alpha\end{bmatrix}$

$\Rightarrow AA'=\begin{bmatrix}\sin^2\alpha+\cos^2\alpha & -\sin\alpha\cos\alpha+\cos\alpha\sin\alpha\\ -\cos\alpha\sin\alpha+\sin\alpha\cos\alpha & \cos^2\alpha+\sin^2\alpha\end{bmatrix}$

$\Rightarrow AA'=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ eqn1 $\left(\because \cos^2\alpha+\sin^2\alpha=1\right)$

And we know ‘I’ represents an identity matrix

$$\text{Therefore, I} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{eqn2}$$

From equation 1 & 2 we can say that

AA’ = I

Ans. AA’ = I. Hence verified.

7. (i) Show that the $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ matrix is a symmetric matrix

(ii) show that the $A = \begin{bmatrix} 0 & 1 & 5 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ matrix is a skew symmetric matrix

7. (i) We have

$$A' - \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix} = A$$

$\therefore A' = A$

Hence, A is a symmetric matrix.

(ii) We have :

$$A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix} = -A$$

$\therefore A' = -A$

Hence, A is a skew -symmetric matrix.

8. For the matrix $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ Verify that

(i) (A + A') is a symmetric matrix

(ii) (A – A') is a skew symmetric matrix

8. $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$

(i)(A + A’) is a symmetric matrix.

So, $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ and $A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

On adding them we get,

$$A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 1+1 & 5+6 \\ 6+5 & 7+7 \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} \text{eqn1}$$

Explanation: Now to show that the matrix obtained i.e. (A + A') is symmetric we need to calculate its transpose and prove that the matrix (A + A') and its transpose are equal. This means that **(A + A') = (A + A')'.**

$$\text{Therefore,} (A + A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} \rightarrow \text{eq}^n\ 2$$

So, from equation 1 & 2 we get,

(A + A') = (A + A')', hence we can say that (A + A') is a symmetric matrix.

Ans. Hence proved.

(ii)(A – A') is a skew symmetric matrix.

So, $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$ and $A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

On subtracting A' from A, we get,

$$A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 1-1 & 5-6 \\ 6-5 & 7-7 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{eqn1}$$

Explanation: Now to show that the matrix obtained i.e. (A + A') is skew symmetric we need to calculate its transpose and prove that the matrix (A + A') is equal to the negative of its transpose are equal. This means that **(A + A') =- (A + A')'.**

$$\text{Therefore, } (A - A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

We can rewrite above equation as

$$(A - A')' = (-1)\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \text{ eqn2}$$

Also, (A – A')' = (-1)×(A – A') (from equation 1)

(A – A')' = - (A – A'), hence we can say that Matrix A is a skew symmetric matrix.

Ans. Hence proved.

9. Find $\frac{1}{2}(A+A')\ \frac{1}{2}(A-A')$, when $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$

9. The given matrix is $A=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$

$$\therefore A'=\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}$$

$$A+A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}+\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

$$\therefore \frac{1}{2}(A+A')=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$$

Now, $A-A'=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}-\begin{bmatrix}0 & -a & -b\\ a & 0 & -c\\ b & c & 0\end{bmatrix}=\begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}$

$$\therefore \frac{1}{2}(A+A')=\begin{bmatrix}0 & a & b\\ -a & 0 & c\\ -b & -c & 0\end{bmatrix}$$

10. Express the following matrices as the sum of a symmetric and a skew symmetric matrix:

(i) $\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}$

(ii) $\begin{bmatrix}6 & -2 & 2\\ -2 & 3 & -1\\ 2 & -1 & 3\end{bmatrix}$

(iii) $\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}$

(iv) $\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}$

10. (i) $\begin{bmatrix}3 & 5\\ 1 & -1\end{bmatrix}$

Explanation: As per Theorem 2 “Any square matrix can be expressed as the sum of a symmetric and skew symmetric matrix.” So in order to prove this we will be using Theorem 1 which states that “For any square matrix A with real number entries, A + A’ is a symmetric matrix and A – A’ is a skew symmetric matrix.”

$$\text{Now, Let } A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

$$\text{Therefore, } A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

Now, on adding A and A’ we will get,

$$A + A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 3+3 & 5+1 \\ 1+5 & -1+(-1) \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\text{Now, Let } M = \frac{1}{2}(A + A')$$

$$\text{Therefore, } M = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$\text{Now, } M' = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix}$$

$$\Rightarrow M' = M$$

Thus $M = \frac{1}{2}(A + A')$ is a symmetric matrix as $M' = M$

Now, on subtracting A’ from A we will get,

$$A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 3-3 & 5-1 \\ 1-5 & -1-(-1) \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$\text{Now, Let } N = \frac{1}{2}(A - A')$$

$$\text{Therefore, } N = \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$\Rightarrow N = \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\text{Now, } N' = \begin{bmatrix} 0 & -2 \\ 2 & 0 \end{bmatrix}$$

$$\Rightarrow N' = (-1)\begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\Rightarrow N' = -N$$

Thus $M = \frac{1}{2}(A + A')$ is a skew symmetric matrix as $N' = -N$

Now, Add M and N, we get,

$$M + N = \begin{bmatrix} 3 & 3 \\ 3 & -1 \end{bmatrix} + \begin{bmatrix} 0 & 2 \\ -2 & 0 \end{bmatrix}$$

$$\Rightarrow M + N = \begin{bmatrix} 3+0 & 3+2 \\ 3+(-2) & -1+0 \end{bmatrix}$$

$$\Rightarrow M + N = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$$

$$\text{So we see here, } M + N = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = A$$

Thus, A is represented as the sum of a symmetric matrix M and a skew symmetric matrix N.

Ans. Hence proved

$$\text{(ii)} \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Explanation: As per Theorem 2 "Any square matrix can be expressed as the sum of a symmetric and skew symmetric matrix." So in order to prove this we will be using Theorem 1 which states that "For any square matrix A with real number entries, A + A' is a symmetric matrix and A – A' is a skew symmetric matrix."

$$\text{Now, Let } A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\text{Therefore, } A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Now, on adding A and A' we will get,

$$A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 6+6 & -2+(-2) & 2+2 \\ -2+(-2) & 3+3 & -1+(-1) \\ 2+2 & -1+(-1) & 3+3 \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 2 & -2 & 6 \end{bmatrix}$$

Now, Let $M = \frac{1}{2}(A + A')$

Therefore, $M = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 2 & -2 & 6 \end{bmatrix}$

$$\Rightarrow M = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

Now, $M' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$\Rightarrow M' = M$

Thus $M = \frac{1}{2}(A + A')$ is a symmetric matrix as $M' = M$

Now, on subtracting A' from A we will get,

$$A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 6-6 & -2-(-2) & 2-2 \\ -2\_(-2) & 3-3 & -1-(-1) \\ 2-2 & -1-(-1) & 3-3 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, Let $N = \frac{1}{2}(A - A')$

Therefore, $N = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Rightarrow N = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

Now, $N' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Rightarrow N' = (-1)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$\Rightarrow$ N' = -N

Thus $M = \frac{1}{2}(A + A')$ is a skew symmetric matrix as $N' = -N$

Now, Add M and N, we get,

$$M + N = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow M + N = \begin{bmatrix} 6+0 & -2+0 & 2+0 \\ -2+0 & 3+0 & -1+0 \\ 2+0 & -1+0 & 3+0 \end{bmatrix}$$

$$\Rightarrow M + N = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

So we see here, $M + N = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = A$

Thus, A is represented as the sum of a symmetric matrix M and a skew symmetric matrix N.

(iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Explanation: As per Theorem 2 "Any square matrix can be expressed as the sum of a symmetric and skew symmetric matrix." So in order to prove this we will be using Theorem 1 which states that "For any square matrix A with real number entries, A + A' is a symmetric matrix and A – A' is a skew symmetric matrix."

Now, Let $A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Therefore, $A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$

Now, on adding A and A' we will get,

$$A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow A+A' = \begin{bmatrix} 3+3 & 3+(-2) & -1+(-4) \\ -2+3 & -2+(-2) & 1+(-5) \\ -4+(-1) & -5+1 & 2+2 \end{bmatrix}$$

$$\Rightarrow A+A' = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

Now, Let $M = \frac{1}{2}(A + A')$

Therefore, $M = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$

$$\Rightarrow M = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$$

Now, $M' = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix}$

$\Rightarrow M' = M$

Thus $M = \frac{1}{2}(A + A')$ is a symmetric matrix as $M' = M$

Now, on subtracting A' from A we will get,

$$A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 3-3 & 3-(-2) & -1-(-4) \\ -2-3 & -2-(-2) & 1-(-5) \\ -4-(-1) & -5-1 & 2-2 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

Now, Let $N = \frac{1}{2}(A - A')$

$$\text{Therefore, } N = \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -3 & -6 & 0 \end{bmatrix}$$

$$\Rightarrow N = \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix}$$

$$\text{Now, } N' = \begin{bmatrix} 0 & \frac{-5}{2} & \frac{-3}{2} \\ \frac{5}{2} & 0 & -3 \\ \frac{3}{2} & 3 & 0 \end{bmatrix}$$

$$\Rightarrow N' = (-1)\begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix}$$

$\Rightarrow N' = -N$

Thus $M = \frac{1}{2}(A + A')$ is a skew symmetric matrix as $N' = -N$

Now, Add M and N, we get,

$$M + N = \begin{bmatrix} 3 & \frac{1}{2} & \frac{-5}{2} \\ \frac{1}{2} & -2 & -2 \\ \frac{-5}{2} & -2 & 2 \end{bmatrix} + \begin{bmatrix} 0 & \frac{5}{2} & \frac{3}{2} \\ \frac{-5}{2} & 0 & 3 \\ \frac{-3}{2} & -3 & 0 \end{bmatrix}$$

$$\Rightarrow M + N = \begin{bmatrix} 3+0 & \frac{1}{2}+\frac{5}{2} & \frac{-5}{2}+\frac{3}{2} \\ \frac{1}{2}+\frac{-5}{2} & -2+0 & -2+3 \\ \frac{-5}{2}+\frac{-3}{2} & -2+(-3) & 0+2 \end{bmatrix}$$

$$\Rightarrow M+N=\begin{bmatrix}3 & \frac{6}{2} & \frac{-2}{2}\\ \frac{-4}{2} & -2 & 1\\ \frac{-8}{2} & -5 & 2\end{bmatrix}$$

So we see here, $M+N=\begin{bmatrix}3 & 3 & -1\\ -2 & -2 & 1\\ -4 & -5 & 2\end{bmatrix}=A$

Thus, A is represented as the sum of a symmetric matrix M and a skew symmetric matrix N.

(iv) $\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}$

Explanation: As per Theorem 2 "Any square matrix can be expressed as the sum of a symmetric and skew symmetric matrix." So in order to prove this we will be using Theorem 1 which states that "For any square matrix A with real number entries, A + A' is a symmetric matrix and A – A' is a skew symmetric matrix."

Now, Let $A=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}$

Therefore, $A'=\begin{bmatrix}1 & -1\\ 5 & 2\end{bmatrix}$

Now, on adding A and A' we will get,

$A+A'=\begin{bmatrix}1 & 5\\ -1 & 2\end{bmatrix}+\begin{bmatrix}1 & -1\\ 5 & 2\end{bmatrix}$

$\Rightarrow A+A'=\begin{bmatrix}1+1 & 5+(-1)\\ -1+5 & 2+2\end{bmatrix}$

$\Rightarrow A+A'=\begin{bmatrix}2 & 4\\ 4 & 4\end{bmatrix}$

Now, Let $M=\frac{1}{2}(A+A')$

Therefore, $M=\frac{1}{2}\begin{bmatrix}2 & 4\\ 4 & 4\end{bmatrix}$

$\Rightarrow M=\begin{bmatrix}1 & 2\\ 1 & 2\end{bmatrix}$

Now, $M'=\begin{bmatrix}1 & 2\\ 1 & 2\end{bmatrix}$

$\Rightarrow M'=M$

Thus $M=\frac{1}{2}(A+A')$ is a symmetric matrix as $M'=M$

Now, on subtracting A' from A we will get,

$$A - A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 1-1 & 5-(-1) \\ -1-5 & 2-2 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$$

Now, Let $N = \frac{1}{2}(A - A')$

Therefore, $N = \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$

$$\Rightarrow N = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

Now, $N' = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$

$$\Rightarrow N' = (-1)\begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

$\Rightarrow N' = -N$

Thus $M = \frac{1}{2}(A + A')$ is a skew symmetric matrix as $N' = -N$

Now, Add M and N, we get,

$$M + N = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} + \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$$

$$\Rightarrow M + N = \begin{bmatrix} 1+0 & 2+3 \\ 1+(-3) & 2+0 \end{bmatrix}$$

$$\Rightarrow M + N = \begin{bmatrix} 1 & 5 \\ -2 & 2 \end{bmatrix}$$

So we see here, $M + N = \begin{bmatrix} 1 & 5 \\ -2 & 2 \end{bmatrix} = A$

Thus, A is represented as the sum of a symmetric matrix M and a skew symmetric matrix N.

11. If A, B are symmetric matrices of same order, then AB − BA is a

A. Skew symmetric matrix B. Symmetric matrix

C. Zero matrix D. Identity matrix

11. The correct answer is A.

A and B are symmetric matrices; therefore, we have:

$A' = A$ and $B' = B$ …(1)

Consider $(AB - BA)' = (AB)' - (BA)'$ $[(A-B)' = A' - B']$

$= B'A' - A'B'$ $[(AB)' = B'A']$

$= BA - AB$ [by (1)]

$= -(AB - BA)$

$\therefore (AB - BA)' = -(AB - BA)$

Thus, (AB – BA) is as skew-symmetric matrix.

12. If $A = \begin{bmatrix} \cos a & -\sin a \\ \sin a & \cos a \end{bmatrix}$, then A + A' = I, if the value of a is

A. $\frac{\pi}{6}$

B. $\frac{\pi}{3}$

C. n

D. $\frac{3\pi}{2}$

12. The correct option is "(B) $\frac{\pi}{3}$ "

Given A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

Therefore, A' = $\begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

Also given that A + A' = I

(Putting the values in the above equation)

$$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} + \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} \cos\alpha + \cos\alpha & -\sin\alpha + \sin\alpha \\ \sin\alpha - \sin\alpha & \cos\alpha + \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2\cos\alpha & 0 \\ 0 & 2\cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

We know when two matrices are equal only when all their corresponding elements or entries are equal i.e. if A = B, then $a_{ij} = b_{ij}$ for all i and j.

This implies,

$2\cos\alpha = 1$

$\Rightarrow \cos\alpha = \frac{1}{2}$

$\Rightarrow \cos\alpha = \cos\frac{\pi}{3}$ $\left(\because \cos\frac{\pi}{3} = \frac{1}{2} \right)$

$\Rightarrow \alpha = \frac{\pi}{3}$