

CIRCLES - CHAPTER- 12

EXERCISE 12A

ANSWER1 (by fig1)

Let the length of the chord be 16cm. AB

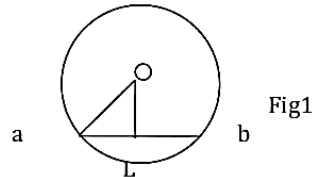
Radius of circle be 10cm

Then, OA = 10, from O draw OL⊥AB.

we know that perpendicular from the centre of a circle to a chord bisects the chord.

$$\therefore AL = \frac{1}{2}AB = \left(\frac{1}{2} \times 16\right) = 8\text{cm}$$

Because, the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.



From, $\triangle OLA$

$$OA^2 = OL^2 + AL^2$$

$$\Rightarrow OL^2 = OA^2 - AL^2 = [(10)^2 - (8)^2] = 100 - 64 = 36$$

$$OL = \sqrt{36} = 6\text{cm}$$

ANSWER2 (by fig1)

Given, distance centre from chord is 3cm. radius is given 5cm.

Then, in $\triangle AOL$

$$OA^2 = OL^2 + AL^2$$

$$(5)^2 = (3)^2 + AL^2$$

$$AL^2 = 25 - 9 = 16$$

$$AL = \sqrt{16} = 4\text{cm}$$

Hence, length of the chord be calculated and, $OL \perp AB$, $AL = LB$

$$AB = AL + LB = 4 + 4 = 8\text{cm}$$

ANSWER3 (by fig1.)

Given, length of the chord = 30cm

Then, OA = 30, from O draw $OL \perp AB$.

we know that perpendicular from the centre of a circle to a chord bisects the chord.

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$AL = \frac{1}{2}AB = \frac{1}{2} \times 30 = 15\text{cm}$$

And the distance from centre to the chord be 8cm

Then, in $\triangle AOL$

$$OA^2 = OL^2 + AL^2$$

$$OA^2 = (15)^2 + (8)^2$$

$$OA^2 = 225 + 64 = 289$$

$$OA = \sqrt{289} = 17\text{cm}$$

ANSWER4

Given, radius = 5cm

AB and CD are 2 parallel cords AB= 8cm, CD= 6cm

- (i) Same side of the centre
 Since, $OP \perp AB$, $OQ \perp CD$ and $AB \parallel CD$,
 The points O, P, Q are collinear.

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Bisects the chord on perpendicular lines

$$\therefore AP = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4\text{cm}$$

$$\text{And, } CQ = \frac{1}{2} CD = \frac{1}{2} \times 6 = 3\text{cm}$$

Join OA and OC.

Then, $OA = OC = 5\text{cm}$

In $\triangle OPA$, we have

$$OP^2 = OA^2 - AP^2 = [(5)^2 - (4)^2] = 9$$

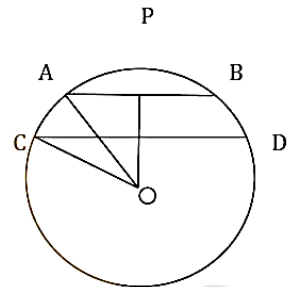
$$OP = \sqrt{9} = 3\text{cm}$$

From $\triangle OQC$, we have

$$OQ^2 = OC^2 - CQ^2 = [(5)^2 - (3)^2] = 16$$

$$OQ = \sqrt{16} = 4\text{cm}$$

$$\therefore PQ = OP - OQ = 4 - 3 = 1\text{cm}$$



- (ii) We know that \perp from the centre of a circle to a chord bisects the chord.
 As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\therefore AP = \frac{1}{2} AB = \frac{1}{2} \times 8 = 4\text{cm}$$

$$CQ = \frac{1}{2} CD = \frac{1}{2} \times 6 = 3\text{cm}$$

Join OA and OC

Then, $OA = OC = 5\text{cm}$

From the right - angles $\triangle OPA$, we have

$$OP^2 = OA^2 - AP^2 = [(5)^2 - (4)^2] = 9\text{cm}^2$$

$$OP = 3\text{cm}$$

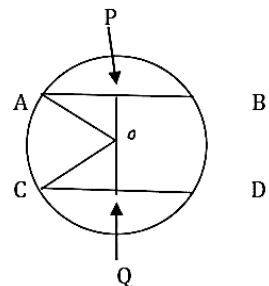
From the right angled $\triangle OCQ$, we have

$$OQ^2 = OC^2 - CQ^2 = [5^2 - 3^2] = 16\text{cm}^2$$

$$OQ = 4\text{cm}$$

Since, $OP \perp AB$, $OQ \perp CD$ and $AB \parallel CD$, the points P, O, Q are collinear.

$$\therefore PQ = OP + OQ = (3 + 4) = 7\text{cm}$$

**ANSWER5**

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RS Aggarwal solutions

Given, let $AB = 30$ and $CD = 16$ cm. and radius = 17 cm

We know that \perp from the centre of a circle to a chord bisects the chord.

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\therefore AP = \frac{1}{2}AB = \frac{1}{2} \times 30 = 15 \text{ cm}$$

$$CQ = \frac{1}{2}CD = \frac{1}{2} \times 16 = 8 \text{ cm}$$

Join OA and OC

Then, $OA = OC = 17$ cm

From the right-angles $\triangle OPA$, we have

$$OP^2 = OA^2 - AP^2 = [(17)^2 - (15)^2] = 64 \text{ cm}^2$$

$$OP = 8 \text{ cm}$$

From the right-angled $\triangle OCQ$, we have

$$OQ^2 = OC^2 - CQ^2 = [17^2 - 8^2] = 225 \text{ cm}^2$$

$$OQ = 15 \text{ cm}$$

Since, $OP \perp AB$, $OQ \perp CD$ and $AB \parallel CD$, the points P, O, Q are collinear.

$$\therefore PQ = OP + OQ = (8 + 15) = 23 \text{ cm}$$

ANSWER 6

Given, CD is diameter of a circle with centre O. $OE \perp AB$, $AB = 12$ cm and $CE = 3$ cm

If diameter of circle is CD then radius is OC (by fig)

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$AE = \frac{1}{2}AB = \frac{1}{2} \times 12 = 6 \text{ cm}$$

Let $OC = OA = r$

Then, in $\triangle AOE$

$$OA^2 = AE^2 + OE^2 = [(6)^2 + (r - 3)^2]$$

$$r^2 = 36 + r^2 - 6r + 9$$

$$6r = 36 + 9 = 45$$

$$r = 7.5 \text{ cm}$$

ANSWER 7

Given, CD is diameter of a circle with centre O. $OE \perp CD$, $CE = ED = 8$ cm, and $EB = 4$ cm

If diameter of circle is CD then radius is OC (by fig)

$$CE = 8 \text{ cm}$$

Let $OD = OB = r$

Then, in $\triangle OBE$

$$OD^2 = OE^2 + ED^2 = [(r - 4)^2 + (8)^2]$$

$$r^2 = r^2 - 8r + 16 + 64$$

$$8r = 16 + 64 = 80$$

$$r = 8 \text{ cm}$$

ANSWER8

Given, $OD \perp AB$, BC is diameter, so radius is $OB=OC$,

Here, $AD=DB$ meANSWER D is mid point of AB

And O is midpoint of BC

Join AC

In $\triangle ABC$

D is mid point of AB, and O is midpoint of BC

$$\therefore AC \parallel OD \text{ and } DO = \frac{1}{2} AC$$

Hence, $AC = 2 \times OD$

ANSWER9

Given, 2 chords AB and CD intersect each other at P. PO bisects $\angle BPD$

Let, draw $OE \perp AB$ and $OF \perp CD$

Now, $DC = DP + FC$, $\therefore DP = FC$

And $AB = AP + EA$, $\therefore AP = EA$

P is intersecting point

So, $\angle OEP = \angle OFP = 90^\circ$

Also, $OP = OP$, $\angle OPE = \angle OPF$

Hence, $\triangle OEP \cong \triangle OFP$, and hence $OE = OF$

Also, chords which are equidistant from the centre are equal

Hence, $AB = CD$

ANSWER10

Let the chords of the circle be AB and CD and both are parallel with centre O.

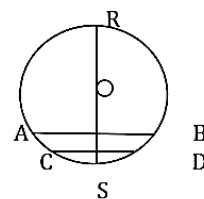
Let ROS be the diameter of the circle. Such that $\angle REB = 90^\circ$

Then, $\angle REB = \angle RFD = 90^\circ$ [corresponding angles]

Thus, we can say that

$RF \perp CD$, and so, $OF \perp CD$

Hence, $\Rightarrow CF = FD$

**ANSWER11**

Let suppose 2 different circles intersect at three points A, B, C. then, these points are not collinear. They cant fall on same line of bisection.

So, we have create unique circle to pass through above points. so these statement is contradict.

Hence, 2 different circles cannot intersect each other at more than 2 points.

ANSWER12

Given,

Radius of 1st circle is 10cm and 2nd circle be 8cm. chord length = 12cm for both of circle.

So, $AO = 10\text{cm}$ and $AO' = 8\text{cm}$
 Here, D is midpoint of chord 12cm
 $AD = DB$,

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$AD = \frac{1}{2}AB = \frac{1}{2} \times 12 = 6\text{cm}$$

In $\triangle AOD$,

$$\begin{aligned} AO^2 &= OD^2 + AD^2 \\ OD^2 &= AO^2 - AD^2 \\ OD^2 &= [10^2 - 6^2] = 100 - 36 = 64 \\ OD &= \sqrt{64} = 8\text{cm} \end{aligned}$$

In $\triangle AO'D$

$$\begin{aligned} AO'^2 &= O'D^2 + AD^2 \\ O'D^2 &= AO'^2 - AD^2 \\ O'D^2 &= [8^2 - 6^2] = 64 - 36 = 28 \\ O'D &= \sqrt{28} = 2\sqrt{7}\text{cm} \end{aligned}$$

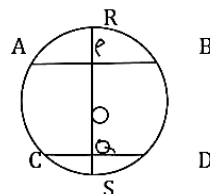
Hence, the distance between centre
 $OO' = OD + DO' = (8 + 2\sqrt{7})\text{cm}$

ANSWER13

Given,
 Equal circles intersect at P and Q.
 P is straight line meets the circles in A and B.
 Join PQ,
 So, PQ is common chord for both circles.
 So, arc PDQ = arc PCQ
 $QP \perp AB$ (by fig)
 Hence, $\angle QAP = \angle QBP$
 $AQ = QB$

ANSWER14

Let the chord of the circles be AB and CD with center O.
 Suppose the diameter of the circle be ROS bisects them at P and Q.
 Then the distance $OP = OQ$
 As, $OP \perp AB$ and $OQ \perp CD$
 Thus, $\angle APQ = \angle PQD \Rightarrow AB \parallel CD$.



ANSWER15 given,

Radius of bigger circle 5cm and smaller one 3cm.
 Join PA,

$$AB = (5 - 3) = 2\text{cm}$$

So, acc to fig,

$PQ \perp AB$, As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$AL = \frac{1}{2}AB = \frac{1}{2} \times 2 = 1 \text{ cm}$$

In $\triangle PAL$

$$\begin{aligned} PA^2 &= PL^2 + AL^2 \\ PL^2 &= PA^2 - AL^2 = [5^2 - 1^2] \\ PL &= \sqrt{25 - 1} = \sqrt{24} = 2\sqrt{6} \end{aligned}$$

So, $PL = 2 \times PL = 2 \times 2\sqrt{6} = 4\sqrt{6}$

ANSWER16

Given, $BC = OB$, $\angle ACD = y^\circ$, $\angle AOD = x^\circ$

If $OB = OC$, Then $\angle BOC = \angle BCO = y^\circ$

In $\triangle OBC$

$$\begin{aligned} \angle OBC &= \angle BOC + \angle OCB \\ &= y^\circ + y^\circ = 2y^\circ \end{aligned}$$

As fig, $OA = OB$ (radius of circle)

$$\angle OAC = \angle OBC = 2y^\circ$$

Then, acc to theorem,

If one side of a triangle is produced then the exterior angle so formed is equal to the sum of the two interior opposite angles.

$$\begin{aligned} \angle AOD &= \angle OAC + \angle ACO \\ &= \angle OAB + \angle BCO \\ x^\circ &= y^\circ + 2y^\circ = 3y^\circ \end{aligned}$$

ANSWER17

given, radius be r such that $AB = 2AC$

let $AC = x$. then $AB = 2x$.

acc to fig, $OL \perp AB$ and $OM \perp AC$.

Then, $OL = p$, $OM = q$ and $OA = r$.

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\text{Also, } AM = \frac{1}{2}AC = \frac{x}{2} \text{ and } AL = \frac{1}{2}AB = x$$

In $\triangle AOL$

$$\begin{aligned} OA^2 &= OL^2 + AL^2 \\ r^2 &= p^2 + x^2 \end{aligned}$$

In $\triangle AOM$

$$\begin{aligned} OA^2 &= OM^2 + AM^2 \\ r^2 &= q^2 + \left(\frac{x}{2}\right)^2 \end{aligned}$$

$$\begin{aligned} \therefore q^2 &= r^2 - \frac{x^2}{4} \\ 4q^2 &= 4r^2 - x^2 \\ \Rightarrow 4q^2 &= 4r^2 - (r^2 - p^2) \\ \text{Hence, } 4q^2 &= 3r^2 + p^2 \end{aligned}$$

ANSWER18

Given, $AB = AC$, $OP \perp AB$ and $OQ \perp AC$,

From centre O, $OP = OQ$ (radius)

As given 2 chords AB and AC

Then, $AB = AC$

$$\begin{aligned} \frac{1}{2} AB &= \frac{1}{2} AC \\ \Rightarrow MB &= NC \end{aligned}$$

Also, $\angle PMB = \angle QNC = 90^\circ$

Equal chords are equidistance from the centre $\Rightarrow OM = ON$

As we know

$OP = OQ \Rightarrow OP - OM = OQ - ON$

$\Rightarrow PM = QN$

$\therefore \triangle MPB \cong \triangle NQC$

$\Rightarrow PB = QC$

ANSWER19

Given, $AB \parallel AC$

Q and P is midpoint of both the chords CD and AB resp.

If Draw $OQ \perp CD$ and $OP \perp AB$, then

In $\triangle OPB \cong \triangle OQC$

Here, $\angle OBP = \angle OCP$

$OB = OC = r$ (radius)

$\angle OPB = \angle OQC = 90^\circ$

$\therefore OP = OQ$

But, the chords equidistance from the centre are equal

Hence, $AB = CD$

ANSWER20

Let suppose we make equilateral Δ of sides 9cm.

So, AD be the median of the Δ

Then, $AD \perp BC$ and

As D is midpoint of the chord BC

$$BD = \frac{1}{2} BC = \frac{1}{2} \times 9 = 4.5 \text{ cm}$$

In $\triangle ABD$

$$AB^2 = AD^2 + BD^2$$

$$AD^2 = AB^2 - BD^2$$

$$AD^2 = \left[9^2 - \left(\frac{9}{2}\right)^2 \right] = \frac{243}{4}$$

$$AD = \sqrt{\frac{243}{4}} = \frac{9\sqrt{3}}{2} \text{ cm}$$

In an equilateral Δ the centroid and circumcentre coincide and $AG:GD = 2:1$

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\therefore \text{radius } AG = \frac{2}{3} AD = \left(\frac{2}{3} \times \frac{9\sqrt{3}}{2}\right) = 3\sqrt{3} \text{ cm}$$

ANSWER21

given

$AB = AC$ are equal chords with centre O.

Lets join OC, OB, OA

In $\Delta OAB \cong OAC$

$OA = OA$

$AB = AC$ (given)

$OB = OC = r$

Hence, $\angle OAB = \angle OAC$

ANSWER22

Join OX and OY, so we get $r = OX = OY$

$\Delta OXP \cong OYR$

$\therefore OP = OR,$

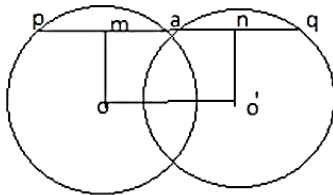
$OX = OY = r$

$\angle OPX = \angle ORY = 90^\circ$

$\Rightarrow PX = RY \Rightarrow PQ - PX = QR - RY$ [$PQ = QR$]

$\Rightarrow QX = QY$

ANSWER23



Draw $OM \perp PQ$ and $O'N \perp PQ$. then

$OM \perp$ chord PA

So, $PM = MA$

$\Rightarrow PM = 2PM = 2MA$

Similarly, $AQ = 2AN$.

\therefore

$$PQ = PA + AQ = 2MA + 2AN$$

$$2(MA + AN) = 2MN$$

$$= 2OO'$$

mc²⁴

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EXERCISE 12B

ANSWER1

- (i) Given O is the centre
 $AO = OC$, $\angle BAO = 40^\circ$, $\angle OCB = 30^\circ$
Join OB
Here, $OA = OB$
Then, $\angle BAO = \angle OBA = 40^\circ$
Also, $OC = OB$
 $\angle OCB = \angle BCO = 30^\circ$
 $\therefore \angle ABC = \angle ABO + \angle OBC$
 $= (40^\circ + 30^\circ) = 70^\circ$
Now, $\angle AOC = 2\angle ABC = 2 \times 70 = 140^\circ$

- (ii) Given,
 $\angle AOB = 90^\circ$, $\angle AOC = 110^\circ$
Here, $OB = OC = OA$ (radius)
As we know sum of all angles of circle be 360°
Then, by adding angles.
 $\angle AOB + \angle AOC + \angle BOC = 360$
 $90^\circ + 110^\circ + \angle BOC = 360^\circ$
 $\angle BOC = 360 - 110 - 90$
 $\angle BOC = 160^\circ$
Hence, $\angle BAC = \frac{1}{2} \angle BOC = \frac{1}{2} \times 160 = 80^\circ$

ANSWER2

Given,
 $\angle AOB = 70^\circ$
As we know that exterior angle is equal some of 2 angles then.
 $\angle AOB = \angle OCA + \angle OAC$
 $\Rightarrow OA = OC$ (radius)
 $\therefore \angle OCA = \angle OAC$
We can calculate, the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle OCA = \frac{1}{2} \angle AOB = \frac{1}{2} \times 70 = 35^\circ$$

Hence, $\angle OCA = \angle OAC = 35^\circ$

ANSWER3

Given, O is the centre
 $\angle APB = 110^\circ$, $\angle PBC = 25^\circ$
In liner APC,
 $180 = \angle APB + \angle BPC$
 $\angle BPC = 180 - 110 = 70^\circ$

So, $\angle ACB = \angle PCB$

Then, In $\triangle CPB$

$$\angle PCB = 180 - \angle PCB - \angle PBC$$

$$\angle PCB = 180 - 25 - 70 = 85^\circ$$

Hence, $\angle PCB = \angle ACB$

Angle with same segment .

$$\angle ACB = \angle ADB = 85^\circ$$

ANSWER4

Given, o is centre of the circle.

$$\angle ABD = 35^\circ, \angle BAC = 70^\circ$$

By fig, $AD \perp AB$, $\angle A = 90^\circ$

Then, In $\triangle ADB$

$$\angle DAB + \angle ADB + \angle DBA = 180^\circ$$

$$90^\circ + \angle ADB + 35^\circ = 180^\circ$$

$$\angle ADB = 180 - 90 - 35 = 55^\circ$$

Angle with same segment

$$\angle ADB = \angle ACB = 55^\circ$$

ANSWER5

Given, O is the centre of the circle.

$$\angle ACB = 50^\circ$$

$$\text{So, } \angle AOB = 2 \times \angle ACB = 2 \times 50 = 100^\circ$$

Then, let the radius be r,

$$\text{On the same segment } \angle OAB = \angle OBA = r^\circ$$

In $\triangle AOB$

$$\angle OAB + \angle OBA + \angle AOB = 180^\circ$$

$$r + r + 100 = 180$$

$$2r = 180 - 100 = 80$$

$$r = \frac{80}{2} = 40^\circ$$

ANSWER6

Given, $\angle ABD = 45^\circ, \angle BCD = 43^\circ$

(i) As we know that angle on same segment are equals

So, on chord AD

$$\angle ABD = \angle ACD = 54^\circ$$

(ii) On chord BD

$$\angle DCB = \angle BAD = 43^\circ$$

(iii) In $\triangle ABD$

$$\angle BAD = 43^\circ, \angle ABD = 54^\circ$$

Sum of all the angles be 180°

$$\angle BAD + \angle ABD + \angle ADB = 180$$

$$54 + 43 + \angle ADB = 180$$

$$\angle ADB = 180 - 54 - 43$$

$$\angle ADB = 83^\circ$$

ANSWER7

Given, $AC \parallel DE$, $\angle CBD = 60^\circ$

On same line segment chord CD

$$\angle DBC = \angle DAC = 60^\circ$$

And $\angle ADC = 90^\circ$ angle is in semi circle.

In $\triangle ADC$

As we know that sum of all the angles in the triangle 180

$$\angle ADC + \angle DAC + \angle ACD = 180^\circ$$

$$\angle ADC + 60 + 90 = 180$$

$$\angle ADC = 180 - 60 - 90$$

$$\text{Hence, } \angle ADC = 30^\circ$$

ANSWER8

Given, $AB \parallel CD$, $\angle ABC = 25^\circ$

Draw joining line OC and OD

Here, $\angle ABC = \angle BCD = 25^\circ$ [alternative int. angles]

Then, arc BD makes $\angle BOD$ at the centre and $\angle BCD$ at a point on the circle.

$$\angle BOD = 2\angle BCD = 50^\circ$$

Similarly,

$$\angle AOC = 2\angle ABC = 50^\circ$$

In liner segment AOB

Sum of all the angles on the line segment is 180°

Then,

$$\angle AOC + \angle COD + \angle BOD = 180^\circ$$

$$\angle COD + 50 + 50 = 180$$

$$\angle COD = 180 - 50 - 50 = 80^\circ$$

$$\text{Hence, similarly } \angle CED = \frac{1}{2} \angle COD = 40^\circ$$

ANSWER9

Given, $\angle AOC = 80^\circ$, $\angle CDE = 40^\circ$

(i) In $\triangle CDE$

Here, $\angle CDE = 90^\circ$ [with in semicircle make angle at 90°]

$$\text{So, } \angle CDE + \angle EDC + \angle DCE = 180$$

$$\angle DCE = 180 - 90 - 40 = 50^\circ$$

Hence, $\angle DCE = 50^\circ$

(ii) In line segment AOB

$$\angle BOC = 180 - \angle AOC$$

$$\angle BOC = 180 - 80 = 100^\circ$$

So, in $\triangle BOC$

$$\angle BOC + \angle CBO + \angle OCB = 180^\circ$$

$$\angle CBO = 180 - \angle BOC - \angle OCB$$

$$\angle CBO = 180 - 50 - 100 = 40^\circ$$

$$\text{Hence, } \angle ABC = \angle CBO = 30^\circ$$

ANSWER10

Given,

$$\angle AOB = 40^\circ, \angle BDC = 100^\circ$$

$$\text{Here, } \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 40 = 20^\circ$$

So, in $\triangle DBC$

$$\angle DCB + \angle DBC + \angle CDB = 180^\circ$$

$$\angle DCB = 180 - \angle DBC + \angle CDB$$

$$\angle DCB = 180 - 100 - 20 = 60^\circ$$

Hence, $\angle DCB = 60^\circ$

ANSWER11

Given $\angle OAB = 25^\circ$

Join OB we get radius of circle is same

$$OB = OA \text{ and } \angle OAB = \angle OBA = 25^\circ$$

Then, in $\triangle AOB$

$$\angle AOB + \angle OAB + \angle ABO = 180^\circ$$

$$\angle AOB + 25 + 25 = 180^\circ$$

$$\angle AOB = 180 - 25 - 25$$

$$\angle AOB = 130^\circ$$

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\text{Then, } \angle ACB = \frac{1}{2} \angle AOB = \frac{1}{2} \times 130 = 65^\circ$$

In $\triangle EBC$

So, $\angle CEB = 90^\circ$[by fig]

$$\angle CEB + \angle ECB + \angle CBE = 180^\circ$$

$$\angle EBC + 90 + 65 = 180^\circ$$

$$\angle EBC = 180 - 90 - 65$$

$$\angle EBC = 25^\circ$$

ANSWER12

Given, $\angle OAB = 20^\circ, \angle OCB = 55^\circ$

- (i) As we know that equal chords of a circle subtend equal angles at the centre

Here, $OC = OB$ (radius) and $\angle OCB = \angle OBC = 55^\circ$

In $\triangle OCB$

$$\angle OCB + \angle OBC + \angle BOC = 180^\circ$$

$$55^\circ + 55^\circ + \angle BOC = 180^\circ$$

$$\angle BOC = 180^\circ - 55 - 55 = 70^\circ$$

- (ii) In $\triangle AOB$

$OA = OB$ and $\angle OAB = \angle OBA$

Sum of all the angles is 180°

$$\angle AOB + \angle OAB + \angle OBA = 180^\circ$$

$$\angle AOB + 20 + 20 = 180$$

$$\angle AOB = 180 - 20 - 20 = 140^\circ$$

And $\angle AOB = \angle BOC + \angle AOC$

$$\angle AOC = \angle AOB - \angle BOC = 140 - 70$$

Hence, $\angle AOC = 70^\circ$

ANSWER13

Given $\angle BCO = 30^\circ$

And by fig, $\angle AOD = \angle OEC = 90^\circ$... [corresponding angles]

$OD \parallel BC$, OC is transversal

$\angle DOC = \angle OCE = 30^\circ$ [alternative int angles]

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Then, $\angle CBD = \frac{1}{2} \angle COD = \frac{1}{2} \times 30 = 15^\circ$

Hence, $y = 15^\circ$ centre

$\angle AOD = 90^\circ$ (given)

And $\angle ABC = \frac{1}{2} \angle AOD = \frac{1}{2} \times 90 = 45^\circ$

In $\triangle ABE$

$\angle A + \angle B + \angle E = 180^\circ$

$\angle A = 180 - \angle B - \angle E$

$\angle A = 180 - 90 - (45 + y) = 180 - 90 - (45 + 15)$

$\angle A = 180 - 90 - 60 = 30^\circ$

Hence, $\angle A = x = 30^\circ$

ANSWER14

Given, $BD = OD$, $CD \perp AB$

Join CA

By fig, $BD = OD$ and $OD = OB$ (radius of circle)

$BD = OD = OB$ [equilateral triangle]

Sum of angles will be 180° in equilateral so, each angles is divided into 60°

In $\triangle DBO$

$\angle DBO = \angle BDO = \angle BOD = 60^\circ$

Since altitudes of an angle of an equilateral \triangle bisects the vertical angle

So, $\angle BDE = \angle ODE = 30^\circ$

Angles on the segment will be equal, on segment of CB

$\angle CAB = \angle CDB = 30^\circ$

ANSWER15

Given PQ is diameter. $\angle PQR = 65^\circ$, $\angle SPR = 40^\circ$, $\angle PQM = 50^\circ$

In $\triangle QPR$, $\angle QRP = 90^\circ$ [angle in the semicircle is right angle]

$\angle QRP + \angle QPR + \angle PQR = 180^\circ$

$\angle QPR = 180 - \angle QRP - \angle PQR$

$\angle QPR = 180 - 65 - 90$

Hence, $\angle QPR = 25^\circ$

$\Rightarrow \angle QPR = \angle PRS = 25^\circ$ [alternative int angles]

Similarly, $\triangle QPM$

$\angle QPM + \angle PMQ + \angle PQM = 180^\circ$

$\angle QPM + 50 + 90 = 180$

$\angle QPM = 180 - 50 - 90 = 40^\circ$

Hence, $\angle QPM = 40^\circ$

ANSWER16

Given, $\angle APB = 150^\circ$, join BC which is common chord of the angles

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\therefore \angle ACB = \frac{1}{2} \angle APB = \frac{1}{2} \times 150 = 75^\circ$$

In linear segment ACD

$$\angle ACB + \angle CBD = 180^\circ$$

$$\angle CBD = 180 - \angle ACB$$

$$\angle CBD = 180 - 75 = 105^\circ$$

Similarly, in second circle

$$\angle BCD = \frac{1}{2} \text{reflex } \angle BQD \dots\dots\dots [\text{angle made by the major arc BFD at the centre } 2\angle BCD]$$

$$105^\circ = \frac{1}{2} (360 - x)$$

$$\Rightarrow 210^\circ = 360 - x^\circ$$

$$\Rightarrow x^\circ = 360 - 210 = 150^\circ$$

ANSWER17

Given, $\angle BAC = 30^\circ$

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle BOC = 2\angle BAC = 2 \times 30 = 60^\circ$$

Here, $OB = OC$ is radius of the circle

Then, from above in $\triangle OBC$

Since, $OB = OC$ (radius)

$$\angle OBC = \angle OCB$$

Sum of all the angles is 180°

$$\angle OBC + \angle OCB + \angle BOC = 180^\circ$$

$$\angle OBC + \angle OBC = 180 - \angle BOC$$

$$2\angle OBC = 180 - 60 = 120$$

$$\angle OBC = 120/2 = 60^\circ$$

Hence, $\triangle OBC$ is equilateral \triangle then, all the sides are equal too

BC is equal to radius of the circumcircle.

ANSWER18

Join AC, BC, BD

Given AB is the chord,

And angle subtended by an arc $CXA = \angle AOC$, Angle subtended by arc $DYB = \angle DOB$

As we know the angle made by an arc at the centre is twice the angle made by this arc at a point on the remaining part of the circle.

$$\angle AOC = 2\angle ABC \dots\dots\dots(1)$$

$$\text{Similarly, } \angle DOB = 2\angle DCB \dots\dots\dots(2)$$

Adding both equation

$$\therefore \angle AOC + \angle DOB = 2\angle ABC + 2\angle DCB = 2\angle AEC$$

$$\text{Hence, } \angle AEC = \frac{1}{2} (\angle AOC + \angle DOB)$$

EXERCISE12C

ANSWER1

Given, $\angle DBC = 60^\circ$, $\angle BAC = 40^\circ$

- (i) On same chord CB
 $\angle BAC = \angle BDC = 40^\circ$

In $\triangle DBC$

$$\angle DCB + \angle CBD + \angle CDB = 180^\circ$$

$$\angle DCB + 40^\circ + 60^\circ = 180^\circ$$

- (ii) On same chord CD
 $\angle CBD = \angle CAD = 60^\circ$

ANSWER2

Given, $\angle PSR = 150^\circ$

As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

$$\angle PSR + \angle PQR = 180^\circ$$

$$\angle PQR = 180 - 150 = 30^\circ$$

Here, angle in semicircle make 90°

$$\angle PRQ = 90^\circ$$

In $\triangle PQR$

$$\angle PQR + \angle RPQ + \angle QRP = 180^\circ$$

$$\angle RPQ + 90 + 30 = 180^\circ$$

$$\angle RPQ = 180 - 90 - 30 = 60^\circ$$

ANSWER3

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle PBC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 130 = 65^\circ$$

ANSWER4

Given $\angle FAE = 20^\circ$, $\angle ABC = 92^\circ$

As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

Then, $\angle ABC + \angle ADC = 180$

$$\angle ADC = 180 - 92 = 88^\circ$$

So, by the fig $CD \parallel AE$,

$$\angle ADC = \angle ADF \text{(alt int } \angle)$$

$$\angle ADF = \angle ADE + \angle FAE$$

$$\angle ADF = 88 + 20 = 108^\circ$$

As we know that exterior angle is equal to the int opposite angles

$$\angle BCD = \angle ADF = 108^\circ$$

Hence, $\angle BCD = 108^\circ$

ANSWER5

Given, $BD = DC$, $\angle CBD = 30^\circ$

$$\angle CBD = \angle DCB = 30^\circ$$

In $\triangle CBD$

$$\angle CBD + \angle CDB + \angle BCD = 180^\circ$$

$$\angle CDB + 30 + 30 = 180$$

$$\angle CDB = 180 - 30 - 30 = 120^\circ$$

As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

$$\angle BAC + \angle BDC = 180^\circ$$

$$\angle BAC + 120^\circ = 180^\circ$$

$$\text{Hence, } \angle BAC = 180 - 120 = 60^\circ$$

ANSWER6

$$\text{Given, } \angle AOC = 100^\circ$$

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle.

$$\angle ADC = \frac{1}{2} \angle AOC = \frac{1}{2} \times 100 = 50^\circ$$

So, As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

$$\text{Then, } \angle ADC + \angle ABC = 180^\circ$$

$$\angle ABC = 180 - \angle ADC = 180 - 50$$

$$\text{Hence, } \angle ABC = 130^\circ$$

ANSWER7

(i) Given $\triangle ABC$ is equilateral \triangle

$$\therefore AB = BC = AC$$

$$\text{And } \angle ABC = \angle BAC = \angle ACB = 60^\circ$$

On same segment of chord BC

(ii) So, As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

$$\angle BDC + \angle BEC = 180^\circ$$

$$\angle BEC = 180 - \angle BDC$$

$$\text{hence } \angle BEC = 180 - 60 = 120^\circ$$

ANSWER8

$$\text{Given, } \angle BCD = 100^\circ, \angle ABD = 50^\circ$$

As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

$$\angle BAD + \angle BCD = 180^\circ$$

$$\angle BAD + 100 = 180^\circ$$

$$\angle BAD = 180 - 100 = 80^\circ$$

So, in $\triangle ADB$

$$\angle ADB + \angle DBA + \angle BAD = 180^\circ$$

$$\angle ADB + 50 + 80 = 180^\circ$$

$$\angle ADB = 180 - 50 - 80$$

$$\text{Hence, } \angle ADB = 50^\circ$$

ANSWER9

$$\text{Given, } \angle BOD = 150^\circ$$

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle

$$\angle DCB = \frac{1}{2} \angle DOB = \frac{1}{2} \times 150 = 75^\circ$$

Hence, $y^\circ = 75$

Then, As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

$$\angle DCB + \angle BAD = 180^\circ$$

$$\angle BAD + 75 = 180^\circ$$

$$\text{Hence, } \angle BAD = 105^\circ$$

ANSWER10

Then, As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

$$\angle DCB + \angle BAD = 180^\circ$$

$$y + 50^\circ = 180^\circ$$

$$y = 180 - 50 = 130^\circ$$

In equilateral $\triangle OAB$

$$OA = OB, \angle OBA = \angle OAB = 50^\circ$$

As we know that exterior angle is equal to the int opposite angles

$$\angle BOD = 180 - (\angle OAB + \angle OBA)$$

$$\angle BOD = 180 - (50 + 50) = 80^\circ$$

ANSWER11

As we know that exterior angle is equal to the int opposite angles

$$\text{Then, } \angle CBF = 130^\circ \text{ (given)}$$

$$\begin{aligned} \text{Now, } \angle CDE &= 180 - \angle CBF \\ &= 180 - 130 = 50^\circ \end{aligned}$$

$$\text{Hence, } x = \angle CDE = 50^\circ$$

ANSWER12

Given, $DO \parallel CB$ and $\angle BCD = 120^\circ$

(i) As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

$$\angle BAD = 180 - \angle BCD$$

$$\angle BAD = 180 - 120 = 60^\circ$$

(ii) So, $\angle BDA = 90^\circ$

In $\triangle ADB$

$$\angle ADB + \angle BAD + \angle ABD = 180^\circ$$

$$\angle ABD + 60 + 90 = 180^\circ$$

$$\angle ABD = 180 - 60 - 90 = 30$$

$$\text{Hence, } \angle ABD = 30^\circ$$

(iii) $\triangle AOD$

$$OA = OD,$$

$$\text{And } \angle ODA = \angle BAD = \angle OAD = 60^\circ$$

In semicircle $\triangle ADB$

$$\therefore \angle ODB = 90 - \angle ODA$$

$$= 90 - 60$$

$$\angle ODB = 30^\circ$$

$OD \parallel CB$, DB is transversal

$$\angle ODB = \angle CBD = 30^\circ \dots\dots\dots [\text{alt int angles}]$$

- (iv) In $\triangle CBD$
 $\angle CBD + \angle BDC + \angle DCB = 180^\circ$
 $\angle BDC + 120 + 30 = 180^\circ$
 $\angle BDC = 180 - 120 - 30 = 30^\circ$
 Then, by fig
 $\angle ADC = \angle BDC + \angle BDA$ (Above values)
 Hence, $\angle ADC = 90 + 30 = 120^\circ$

$\triangle AOD$
 $OA = OD$ (radius)
 $\angle AOD = \angle ADO = \angle OAD = 60^\circ$
 Hence, it is proved equilateral \triangle

ANSWER13

As we know that AB and CD are the chords intersect at point then

$$AP \times BP = CP \times CD$$

$$(AB+BP) \times BP = (CD+DP) \times CD$$

$$(6+2) \times 2 = (CD+2.5) \times 2.5$$

$$\Rightarrow 8 \times 2 = 2.5CD + 6.25$$

$$\Rightarrow 2.5 CD = 16 - 6.25 = 9.75$$

$$\Rightarrow CD = \frac{9.75}{2.5} = 3.9 \text{ cm}$$

ANSWER14

Given, $\angle AOD = 140^\circ$, $\angle CAB = 50^\circ$

- (i) we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°
 So, $\angle BAC + \angle BDC = 180^\circ$
 $\angle BDC + 50 = 180^\circ$
 $\angle BDC = 180 - 50 = 130^\circ$
 Then, in line segment CDE
 $\angle BDC + \angle EDB = 180^\circ$
 $\angle EDB = 180 - 130 = 50^\circ$
 Hence, $\angle EDB = 50^\circ$
- (ii) here, $\angle BOD = 180 - \angle AOD$ (given)
 $\angle BOD = 180 - 140 = 40^\circ$
 Also, $OD = OB$ so, the angles $\angle OBD = \angle ODB$
 We can calculate by $\triangle ODB$
 $\angle ODB + \angle OBD + \angle BOD = 180$
 $2\angle ODB + 40 = 180$
 $2\angle ODB = 180 - 40 = 140^\circ$
 $\angle ODB = \frac{140}{2} = 70^\circ$
 $\angle ODB = \angle OBD = 70^\circ$
 In line segment OBE
 $180 = \angle OBD + \angle EBD$

$$\angle EBD = 180 - \angle OBD$$
$$\text{Hence, } \angle EBD = 180 - 70 = 110^\circ$$

ANSWER15

Given, In $\triangle ABC$

$$AB = AC$$

D is intersecting at AB and E is intersecting AC

$$\therefore \angle CBA = \angle BCA$$

As we know that exterior angle is equal to the int opposite angles

$$\text{And ext. } \angle ADE = \angle CBA = \angle BCA$$

$$\text{Hence, } \angle ADE = \angle ABC, DE \parallel BC$$

ANSWER16

As we know that exterior angle is equal to the int opposite angles

$$\text{Ext } \angle EDC = \angle A, \text{ Ext } \angle DCE = \angle B$$

$$AB \parallel CD \text{ (given)}$$

$$\text{So, } \angle A = \angle B$$

Hence, $\triangle AEB$ is isosceles

$$\angle A = \angle B, AE = BE$$

ANSWER17

$$\text{Given } \angle BAD = 75^\circ$$

we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

then in first Quadrilateral ABCD

$$\angle BAD + \angle BCD = 180^\circ$$

$$\angle BCD = 180 - \angle BAD$$

$$\angle BCD = 180 - 75 = 105^\circ$$

As we know that exterior angle is equal to the int opposite angles

$$\text{Then, } \angle BCD = \angle DEF = 105^\circ$$

$$y = 105^\circ$$

In line segment BCF

$$\angle BCD + \angle DCF = 180^\circ$$

$$\Rightarrow \angle DCF = 180 - \angle BCD = 180 - 105 = 75^\circ$$

$$\text{Hence, } \angle DCF = 75^\circ$$

$$x = 75^\circ$$

ANSWER18

Given, ABCD is quadrilateral $AD=BC$ and $\angle ADC = \angle BCD$

Draw \perp lines on AB such that $DE \perp AB$ and $CF \perp AB$

$$\text{So, } \angle DEA = \angle CFB = 90^\circ$$

In $\triangle ADE$ and $\triangle BCF$, we have

$$\angle ADE = \angle ADC - 90^\circ$$

$$\Rightarrow \angle BCD - 90^\circ = \angle BCF \text{ [} \angle ADC = \angle BCD \text{]}$$

$$AD = BC$$

$$\text{And } \angle ADE = \angle BFC = 90^\circ$$

$$\therefore \triangle ADE \cong \triangle BCF$$

$$\angle A = \angle B$$

$$\therefore \angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$2\angle B + 2\angle D = 360 \dots\dots [\angle ADC = \angle BCD \text{ (Given)}]$$

$$\text{Then, } \angle B + \angle D = 180^\circ$$

$$\text{Similarly, } \angle A + \angle C = 180^\circ$$

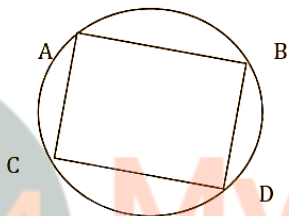
Because we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°
Hence proved ABCD is lie on circle

ANSWER19

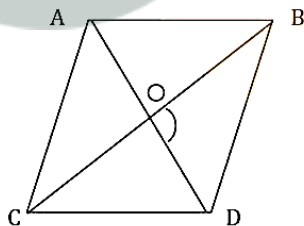
Suppose, ABCD be a cyclic quadrilaterals and O be the centre of the circle.

Then, AB, BC, CD, DA are the chords of the circle, and its bisector must pass through the centre of the circle, O.

Hence, we can say that right bisector of AB, BC, CD and DA pass through O so, it is concurrent.



ANSWER20



Let AD and BC is the diagonal of a rhombus ABCD intersect at O.

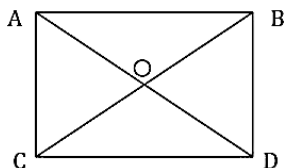
As we know that the diagonals of a rhombus bisects at 90° right angle Δ

$$\text{So, } \angle BOD = 90^\circ$$

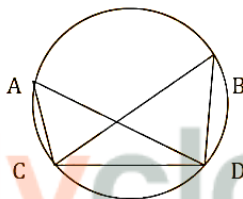
Also, $\angle BOD$ is lies in the semicircle.

Thus, the circle drawn with BD as diameter will pass through O

Similarly, AB, CD, AC are the diameter as pass through O.

ANSWER21

Let O be the intersection point of the diagonals AC and BD of rect, ABCD .
 Since the diagonals of a rectangles are equal and bisect each other , we have ,
 $OA = OB = OC = OD$.
 Hence, O is centre of the circle through A, B,C,D.

ANSWER22

Let ACD are the given points . with the B as an center radius equal to AD draw an arc. With D as centre and CB as radius draw another arc, intersecting the previous arc at B. then B is the desired point

PROOF join AD and BC

$\triangle ADC \cong \triangle BCD$ [$AC = BD$, $CB = AD$, $CD = CD$]

$\Rightarrow \angle DAC = \angle CBD$

Thus, CD subtends equal angles $\angle ACD$ and $\angle CBD$ on the same side of it.

$\therefore A , B, C, D$ are cyclic .

ANSWER23

Given, ABCD is cyclic equilateral , $(\angle B - \angle D) = 60^\circ$

As we know that the sum of either pair of the opposite angles of a cyclic quadrilateral is 180°

$$\angle B + \angle D = 180^\circ$$

\therefore by solving above equation

$$\Rightarrow \angle B - \angle D = 60^\circ$$

$$\underline{\angle B + \angle D = 180}$$

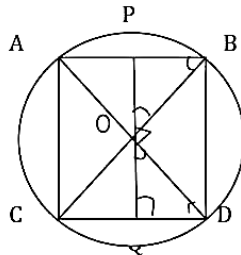
$$2\angle B = 240^\circ$$

$$\angle B = \frac{240}{2} = 120^\circ$$

$$\text{And } \angle D = 180 - \angle B$$

$$\Rightarrow \angle D = 180 - 120 = 60^\circ$$

ANSWER24



Let ABCD is the cyclic quadrilaterals whose diagonals AC and BD intersect at O at the right angles.
Let $OQ \perp CD$ such that OQ produced to P meet at AB chord

Then by fig,

We have to prove that $CM = MD$

Clearly, $\angle CBA = \angle ADC$ [same line segment]

$\angle QDO + \angle DOQ = 90^\circ$ [$\because \angle OQD = 90^\circ$]

$\angle QOD + \angle POB = 90^\circ$ [\because POQ is linear segment and $\angle BOD = 90^\circ$]

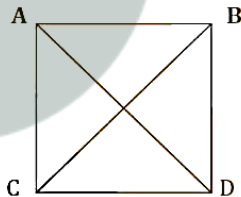
$\therefore \angle QDO + \angle DOQ = \angle QOD + \angle POB \Rightarrow \angle QDO = \angle POB$

Thus, $\angle CBA = \angle ADC$ and $\angle QDO = \angle POB \Rightarrow \angle CBA = \angle POB$

$\therefore OP = PB$ and $OP = PA$

Hence, $PB = PA$

ANSWER25



By fig, $\angle ACB = 90^\circ$ $\angle ADB = 90^\circ$

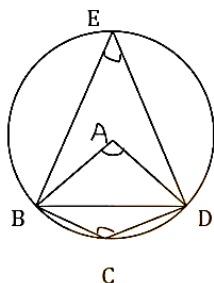
As we know that the opposite angles of quadrilateral ABCD are supplementary or 180°

Then, ABCD is cyclic quadrilateral

This meANSWER circles pass through the points A, B, C, D

$\therefore \angle BAC = \angle BDC$ [angles in the same segment]

ANSWER26



Given, ABCD is a quadrilateral such that A is the centre of the circle passing through B,C and D.

Take Point E on the circle outside arc BCD. Join BE, DE and BD

As we know the angle subtended by arc of a circle at the centre is double the angle subtended by it at any point on the remaining part of the circle

Clearly, $\angle BAD = 2\angle BED$

Now, EBCD is cyclic quadrilateral.

$$\therefore \angle BED + \angle BCD = 180^\circ$$

$$\Rightarrow \angle BCD = 180^\circ - \angle BED$$

$$\Rightarrow \angle BCD = 180^\circ - \frac{1}{2}\angle BAD \dots\dots[\angle BAD = 2\angle BED]$$

In $\triangle BCD$, we have

$$\angle BCD + \angle CBD + \angle BDC = 180^\circ$$

$$\angle CBD + \angle CDB = 180^\circ - \angle BCD$$

$$= 180^\circ - (180 - \frac{1}{2}\angle BAD) = \frac{1}{2}\angle BAD$$

$$\text{Hence } \angle CBD + \angle CDB = \frac{1}{2}\angle BAD .$$

MULTIPLE-CHOICE QUESTIONS

ANSWER 1:

(b) 12 cm

Let PQ be the chord of the given circle with centre O and a radius of 13 cm.

Then, PQ = 10 cm and OQ = 13 cm

From O, draw OX perpendicular to PQ.

We know that the perpendicular from the centre of a circle to a chord bisects the chord.

$$\therefore QX = \left(\frac{10}{2}\right)\text{cm} = 5\text{cm}$$

From the right $\triangle OXQ$, we have:

$$OQ^2 = OX^2 + XQ^2$$

$$\Rightarrow 13^2 = OX^2 + 5^2$$

$$\Rightarrow 169 = OX^2 + 25$$

$$\Rightarrow OX^2 = (169 - 25) = 144$$

$$\Rightarrow OX = \sqrt{144}\text{cm} = 12\text{cm}$$

Hence, the distance of the chord from the centre is 12 cm.

ANSWER 2:

(c) 30 cm

Let PQ be the chord of the given circle with centre O and a radius of 17 cm.

From O, draw OX perpendicular to PQ.

Then OX = 8 cm and OQ = 17 cm

From the right $\triangle OXQ$, we have:

$$OQ^2 = OX^2 + XQ^2$$

$$\Rightarrow 17^2 = 8^2 + XQ^2$$

$$\Rightarrow 289 = 64 + XQ^2$$

$$\Rightarrow XQ^2 = (289 - 64) = 225$$

$$\Rightarrow XQ = \sqrt{225}\text{cm} = 15\text{cm}$$

The perpendicular from the centre of a circle to a chord bisects the chord.

$$\therefore PQ = 2 \times XQ = (2 \times 15)\text{ cm} = 30\text{ cm}$$

Hence, the required length of the chord is 30 cm.

ANSWER 3:

(b) 45°

Since an angle in a semicircle is a right angle, $\angle BAC = 90^\circ$

$$\therefore \angle ABC + \angle ACB = 90^\circ$$

Now, $AB = AC$ (Given)

$$\Rightarrow \angle ABC = \angle ACB = 45^\circ$$

ANSWER 4:

(c) 60°

As the angle at the centre of a circle is twice the angle at any point on the remaining part of the circumference.

$$\text{Thus, } \angle AOB = (2 \times \angle ACB) = (2 \times 30^\circ) = 60^\circ$$

ANSWER 5:

(b) 50°

$OA = OB$

$$\Rightarrow \angle OBA = \angle OAB = 40^\circ$$

$$\text{Now, } \angle AOB = 180^\circ - (40^\circ + 40^\circ) = 100^\circ$$

$$\therefore \angle ACB = \frac{1}{2} \angle AOB = \left(\frac{1}{2} \times 100^\circ\right) = 50^\circ$$

ANSWER 6:

(a) 8 cm

Join OC. Then $OC = \text{radius} = 17 \text{ cm}$

$$CL = \frac{1}{2} CD = \left(\frac{1}{2} \times 30\right) \text{ cm} = 15 \text{ cm}$$

In right $\triangle OLC$, we have:

$$OL^2 = OC^2 - CL^2 = (17)^2 - (15)^2 = (289 - 225) = 64$$

$$\Rightarrow OL = \sqrt{64} = 8 \text{ cm}$$

\therefore Distance of CD from AB = 8 cm

ANSWER 7:

(b) 80°

Given: $AB = CD$

We know that equal chords of a circle subtend equal angles at the centre.

$$\therefore \angle COD = \angle AOB = 80^\circ$$

ANSWER8:

(c) 7.5 cm

Let $OA = OC = r$ cm.

Then $OE = (r - 3)$ cm and $AE = \frac{1}{2}AB = 6$ cm

Now, in right $\triangle OAE$, we have:

$$OA^2 = OE^2 + AE^2$$

$$\Rightarrow (r)^2 = (r - 3)^2 + 6^2$$

$$\Rightarrow r^2 = r^2 + 9 - 6r + 36$$

$$\Rightarrow 6r = 45$$

$$\Rightarrow r = \frac{45}{6} = 7.5 \text{ cm}$$

Hence, the required radius of the circle is 7.5 cm.

ANSWER9:

(a) 10 cm

Let the radius of the circle be r cm.

Let $OD = OB = r$ cm.

Then $OE = (r - 4)$ cm and $ED = 8$ cm

Now, in right $\triangle OED$, we have:

$$OD^2 = OE^2 + ED^2$$

$$\Rightarrow (r)^2 = (r - 4)^2 + 8^2$$

$$\Rightarrow r^2 = r^2 + 16 - 8r + 64$$

$$\Rightarrow 8r = 80$$

$$\Rightarrow r = 10 \text{ cm}$$

Hence, the required radius of the circle is 10 cm.

ANSWER 10:

(d) 10 cm

Draw $OM \perp AB$ and $ON \perp CD$.

In $\triangle OMB$ and $\triangle ONC$, we have:

$$OB = OC \quad (\text{Radius of a circle})$$

$$\angle BOM = \angle CON \quad (\text{Vertically opposite angles})$$

$$\angle OMB = \angle ONC \quad (90^\circ \text{ each})$$

$$\therefore \triangle OMB \cong \triangle ONC \quad (\text{By AAS congruency rule})$$

$$\therefore OM = ON$$

Chords equidistant from the centre are equal.

$$\therefore CD = AB = 10 \text{ cm}$$

ANSWER 11:

(b) 75°

OB = BC (Given)

$$\Rightarrow \angle BOC = \angle BCO = 25^\circ$$

$$\text{Exterior } \angle OBA = \angle BOC + \angle BCO = (25^\circ + 25^\circ) = 50^\circ$$

OA = OB (Radius of a circle)

$$\Rightarrow \angle OAB = \angle OBA = 50^\circ$$

In $\triangle AOC$, side CO has been produced to D.

$$\therefore \text{Exterior } \angle AOD = \angle OAC + \angle ACO$$

$$= \angle OAB + \angle BCO$$

$$= (50^\circ + 25^\circ) = 75^\circ$$

ANSWER 12:

(b) 12 cm

OD \perp AB

i.e., D is the mid point of AB.

Also, O is the mid point of BC.

Now, in $\triangle BAC$, D is the mid point of AB and O is the mid point of BC.

$$\therefore OD = \frac{1}{2}AC \quad (\text{By mid point theorem})$$

$$\Rightarrow AC = 2OD = (2 \times 6) \text{ cm} = 12 \text{ cm}$$

ANSWER 13:

(c) $3\sqrt{3}$ cm

Let $\triangle PQR$ be an equilateral triangle of side 9 cm.

Let PM be one of its medians.

Then PM \perp QR and QM = 4.5 cm

$$\therefore PM = \sqrt{PQ^2 - QM^2} = \sqrt{81 - \frac{81}{4}} = \frac{9\sqrt{3}}{2} \text{ cm}$$

Let O be the centroid of $\triangle PQR$

Then PO : OM = 2 : 1

$$\therefore \text{Radius} = PO = \frac{2}{3} PM = \left(\frac{2}{3} \times \frac{9\sqrt{3}}{2}\right) \text{ cm} = 3\sqrt{3} \text{ cm}$$

ANSWER 14:

(c) 90°

The angle in a semicircle measures 90° .

ANSWER 15:

(a) equal

The angles in the same segment of a circle are equal.

ANSWER 16:

(c) 70°

$\angle BDC = \angle BAC = 60^\circ$ (Angles in the same segment of a circle)

In $\triangle BDC$, we have:

$\angle DBC + \angle BDC + \angle BCD = 180^\circ$ (Angle sum property of a triangle)

$\therefore 50^\circ + 60^\circ + \angle BCD = 180^\circ$

$\Rightarrow \angle BCD = 180^\circ - (50^\circ + 60^\circ) = (180^\circ - 110^\circ) = 70^\circ$

ANSWER 17:

(c) 60°

Angles in a semi circle measure 90° .

$\therefore \angle BAC = 90^\circ$

In $\triangle ABC$, we have:

$\angle BAC + \angle ABC + \angle BCA = 180^\circ$ (Angle sum property of a triangle)

$\therefore 90^\circ + \angle ABC + 30^\circ = 180^\circ$

$\Rightarrow \angle ABC = (180^\circ - 120^\circ) = 60^\circ$

$\therefore \angle CDA = \angle ABC = 60^\circ$ (Angles in the same segment of a circle)

ANSWER 18:

(b) 50°

$\angle ODB = \angle OAC = 50^\circ$ (Angles in the same segment of a circle)

ANSWER 19:

(c) 100°

In $\triangle OAB$, we have:

$OA = OB$ (Radii of a circle)

$\Rightarrow \angle OAB = \angle OBA = 20^\circ$

In $\triangle OAC$, we have:

$OA = OC$ (Radii of a circle)

$\Rightarrow \angle OAC = \angle OCA = 30^\circ$

Now, $\angle BAC = (20^\circ + 30^\circ) = 50^\circ$

$\therefore \angle BOC = (2 \times \angle BAC) = (2 \times 50^\circ) = 100^\circ$

ANSWER 20:

(a) 85°

We have:

$$\angle BOC + \angle BOA + \angle AOC = 360^\circ$$

$$\Rightarrow \angle BOC + 100^\circ + 90^\circ = 360^\circ$$

$$\Rightarrow \angle BOC = (360^\circ - 190^\circ) = 170^\circ$$

$$\therefore \angle BAC = \left(\frac{1}{2} \times \angle BOC\right) = \left(\frac{1}{2} \times 170^\circ\right) = 85^\circ$$

ANSWER 21:

(d) 65°

We have:

OA = OB (Radii of a circle)

Let $\angle OAB = \angle OBA = x^\circ$

In ΔOAB , we have:

$$x^\circ + x^\circ + 50^\circ = 180^\circ \quad (\text{Angle sum property of a triangle})$$

$$\Rightarrow 2x^\circ = (180^\circ - 50^\circ) = 130^\circ$$

$$\Rightarrow x = \left(\frac{130}{2}\right)^\circ = 65^\circ$$

Hence, $\angle OAB = 65^\circ$

ANSWER 22:

(c) 30°

$$\angle COB = 180^\circ - 120^\circ = 60^\circ \quad (\text{Linear pair})$$

Now, arc BC subtends $\angle COB$ at the centre and $\angle BDC$ at the point D of the remaining part of the circle.

$$\therefore \angle COB = 2\angle BDC$$

$$\Rightarrow \angle BDC = \frac{1}{2}\angle COB = \left(\frac{1}{2} \times 60^\circ\right) = 30^\circ$$

ANSWER 23:

(b) 50°

We have:

OA = OB (Radii of a circle)

$$\Rightarrow \angle OBA = \angle OAB = 50^\circ$$

$$\therefore \angle CDA = \angle OBA = 50^\circ \quad (\text{Angles in the same segment of a circle})$$

ANSWER 24:

(b) 60°

We have:

$\angle CDB = \angle CAB = 40^\circ$ (Angles in the same segment of a circle)

In $\triangle CBD$, we have:

$\angle CDB + \angle BCD + \angle CBD = 180^\circ$ (Angle sum property of a triangle)

$\Rightarrow 40^\circ + 80^\circ + \angle CBD = 180^\circ$

$\Rightarrow \angle CBD = (180^\circ - 120^\circ) = 60^\circ$

ANSWER 25:

(c) 80°

We have:

$\angle AEB + \angle CEB = 180^\circ$ (Linear pair angles)

$\Rightarrow 110^\circ + \angle CEB = 180^\circ$

$\Rightarrow \angle CEB = (180^\circ - 110^\circ) = 70^\circ$

In $\triangle CEB$, we have:

$\angle CEB + \angle EBC + \angle ECB = 180^\circ$ (Angle sum property of a triangle)

$\Rightarrow 70^\circ + 30^\circ + \angle ECB = 180^\circ$

$\Rightarrow \angle ECB = (180^\circ - 100^\circ) = 80^\circ$

The angles in the same segment are equal.

Thus, $\angle ADB = \angle ECB = 80^\circ$

ANSWER 26:

(d) 60°

We have:

$OA = OB$ (Radii of a circle)

$\Rightarrow \angle OBA = \angle OAB = 20^\circ$

In $\triangle OAB$, we have:

$\angle OAB + \angle OBA + \angle AOB = 180^\circ$ (Angle sum property of a triangle)

$\Rightarrow 20^\circ + 20^\circ + \angle AOB = 180^\circ$

$\Rightarrow \angle AOB = (180^\circ - 40^\circ) = 140^\circ$

Again, we have:

$OB = OC$ (Radii of a circle)

$\Rightarrow \angle OBC = \angle OCB = 50^\circ$

In $\triangle OCB$, we have:

$\angle OCB + \angle OBC + \angle COB = 180^\circ$ (Angle sum property of a triangle)

$\Rightarrow 50^\circ + 50^\circ + \angle COB = 180^\circ$

$\Rightarrow \angle COB = (180^\circ - 100^\circ) = 80^\circ$

Since $\angle AOB = 140^\circ$, we have:

$\angle AOC + \angle COB = 140^\circ$

$$\Rightarrow \angle AOC + 80^\circ = 140^\circ$$
$$\Rightarrow \angle AOC = (180^\circ - 80^\circ) = 60^\circ$$

ANSWER 27:

(b) 30°

We have:

$$\angle ABC + \angle ADC = 180^\circ \quad (\text{Opposite angles of a cyclic quadrilateral})$$

$$\Rightarrow \angle ABC + 120^\circ = 180^\circ$$

$$\Rightarrow \angle ABC = (180^\circ - 120^\circ) = 60^\circ$$

Also, $\angle ACB = 90^\circ$ (Angle in a semicircle)

In $\triangle ABC$, we have:

$$\angle BAC + \angle ACB + \angle ABC = 180^\circ \quad (\text{Angle sum property of a triangle})$$

$$\Rightarrow \angle BAC + 90^\circ + 60^\circ = 180^\circ$$

$$\Rightarrow \angle BAC = (180^\circ - 150^\circ) = 30^\circ$$

ANSWER 28:

(b) 100°

Since ABCD is a cyclic quadrilateral, we have:

$$\angle BAD + \angle BCD = 180^\circ \quad (\text{Opposite angles of a cyclic quadrilateral})$$

$$\Rightarrow 100^\circ + \angle BCD = 180^\circ$$

$$\Rightarrow \angle BCD = (180^\circ - 100^\circ) = 80^\circ$$

Now, $AB \parallel DC$ and CB is the transversal.

$$\therefore \angle ABC + \angle BCD = 180^\circ$$

$$\Rightarrow \angle ABC + 80^\circ = 180^\circ$$

$$\Rightarrow \angle ABC = (180^\circ - 80^\circ) = 100^\circ$$

ANSWER 29:

(c) 115°

Take a point X on the remaining part of the circumference.

Join AX and CX.

$$\text{Then } \angle AXC = \frac{1}{2} \angle AOC = \left(\frac{1}{2} \times 130^\circ\right) = 65^\circ$$

In cyclic quadrilateral ABCX, we have:

$$\angle ABC + \angle AXC = 180^\circ \quad (\text{Opposite angles of a cyclic quadrilateral})$$

$$\Rightarrow \angle ABC + 65^\circ = 180^\circ$$

$$\Rightarrow \angle ABC = (180^\circ - 65^\circ) = 115^\circ$$

ANSWER 30:

(a) 30°

$\angle ADC = \angle BAD = 30^\circ$ (Alternate angles)

$\angle ADB = 90^\circ$ (Angle in semicircle)

$\therefore \angle CDB = (90^\circ + 30^\circ) = 120^\circ$

But ABCD being a cyclic quadrilateral, we have:

$\angle BAC + \angle CDB = 180^\circ$

$\Rightarrow \angle BAD + \angle CAD + \angle CDB = 180^\circ$

$\Rightarrow 30^\circ + \angle CAD + 120^\circ = 180^\circ$

$\Rightarrow \angle CAD = (180^\circ - 150^\circ) = 30^\circ$

ANSWER 31:

(a) 50°

Take a point X on the remaining part of the circumference.

Join AX and CX.

Then $\angle AXC = \frac{1}{2}\angle AOC = (\frac{1}{2} \times 100^\circ) = 50^\circ$

Now, side AB of the cyclic quadrilateral ABCX has been produced to D.

\therefore Exterior $\angle CBD = \angle AXC = 50^\circ$

ANSWER 32:

(c) 100°

OA = OB (Radii of a circle)

$\Rightarrow \angle OBA = \angle OAB = 50^\circ$

In ΔOAB , we have:

$\angle OAB + \angle OBA + \angle AOB = 180^\circ$ (Angle sum property of a triangle)

$\Rightarrow 50^\circ + 50^\circ + \angle AOB = 180^\circ$

$\Rightarrow \angle AOB = (180^\circ - 100^\circ) = 80^\circ$

Since $\angle AOB + \angle BOD = 180^\circ$ (Linear pair)

$\therefore \angle BOD = (180^\circ - 80^\circ) = 100^\circ$

ANSWER 33:

(b) 70°

BC = CD (given)

$\Rightarrow \angle BDC = \angle CBD = 35^\circ$

In ΔBCD , we have:

$\angle BCD + \angle BDC + \angle CBD = 180^\circ$ (Angle sum property of a triangle)

$\Rightarrow \angle BCD + 35^\circ + 35^\circ = 180^\circ$

$\Rightarrow \angle BCD = (180^\circ - 70^\circ) = 110^\circ$

In cyclic quadrilateral ABCD, we have:

$$\angle BAD + \angle BCD = 180^\circ$$

$$\Rightarrow \angle BAD + 110^\circ = 180^\circ$$

$$\therefore \angle BAD = (180^\circ - 110^\circ) = 70^\circ$$

ANSWER 34:

(c) 120°

Since $\triangle ABC$ is an equilateral triangle, each of its angle is 60° .

$$\therefore \angle BAC = 60^\circ$$

In a cyclic quadrilateral ABCD, we have:

$$\angle BAC + \angle BDC = 180^\circ$$

$$\Rightarrow 60^\circ + \angle BDC = 180^\circ$$

$$\Rightarrow \angle BDC = (180^\circ - 60^\circ) = 120^\circ$$

ANSWER 35:

(b) 80°

In a cyclic quadrilateral ABCD, we have:

Interior opposite angle, $\angle ADC = \text{exterior } \angle CBE = 100^\circ$

$$\therefore \angle CDF = (180^\circ - \angle ADC) = (180^\circ - 100^\circ) = 80^\circ \text{ (Linear pair)}$$

ANSWER 36:

(c) 110°

Join AB.

Then chord AB subtends $\angle AOB$ at the centre and $\angle AXB$ at a point X of the remaining parts of a circle.

$$\therefore \angle AOB = 2\angle AXB$$

$$\Rightarrow \angle AXB = \frac{1}{2}\angle AOB = \left(\frac{1}{2} \times 140^\circ\right) = 70^\circ$$

In the cyclic quadrilateral, we have:

$$\angle AXB + \angle ACB = 180^\circ$$

$$\Rightarrow 70^\circ + \angle ACB = 180^\circ$$

$$\therefore \angle ACB = (180^\circ - 70^\circ) = 110^\circ$$

ANSWER 37:

(c) 115°

Join AB.

Then chord AB subtends $\angle AOB$ at the centre and $\angle AXB$ at a point X of the remaining parts of a circle.

$$\therefore \angle AOB = 2\angle AXB$$

$$\Rightarrow \angle AXB = \frac{1}{2}\angle AOB = \left(\frac{1}{2} \times 130^\circ\right) = 65^\circ$$

In cyclic quadrilateral, we have:

$$\angle AXB + \angle ACB = 180^\circ$$

$$\Rightarrow 65^\circ + \angle ACB = 180^\circ$$

$$\therefore \angle ACB = (180^\circ - 65^\circ) = 115^\circ$$

ANSWER 38:

(d) 110°

Since ABCD is a cyclic quadrilateral, we have:

$$\angle BAD + \angle BCD = 180^\circ$$

$$\Rightarrow \angle BAD + 110^\circ = 180^\circ$$

$$\Rightarrow \angle BAD = (180^\circ - 110^\circ) = 70^\circ$$

Similarly in ABEF, we have:

$$\angle BAD + \angle BEF = 180^\circ$$

$$\Rightarrow 70^\circ + \angle BEF = 180^\circ$$

$$\Rightarrow \angle BEF = (180^\circ - 70^\circ) = 110^\circ$$

ANSWER 39:

(c) 105°

We have:

$$\angle ABC + \angle ADC = 180^\circ$$

$$\Rightarrow \angle ABC + 95^\circ = 180^\circ$$

$$\Rightarrow \angle ABC = (180^\circ - 95^\circ) = 85^\circ$$

Now, $CF \parallel AB$ and CB is the transversal.

$$\therefore \angle BCF = \angle ABC = 85^\circ \quad (\text{Alternate interior angles})$$

$$\Rightarrow \angle BCE = (85^\circ + 20^\circ) = 105^\circ$$

$$\Rightarrow \angle DCB = (180^\circ - 105^\circ) = 75^\circ$$

Now, $\angle BAD + \angle BCD = 180^\circ$

$$\Rightarrow \angle BAD + 75^\circ = 180^\circ$$

$$\Rightarrow \angle BAD = (180^\circ - 75^\circ) = 105^\circ$$

ANSWER 40:

(c) 8.5 cm

Join AC.

Then $AE : CE = DE : BE$ (Intersecting secant theorem)

$$\therefore AE \times BE = DE \times CE$$

Let $CD = y$ cm

Then $AE = (AB + BE) = (11 + 3)$ cm = 14 cm; $BE = 3$ cm; $CE = (y + 3.5)$ cm; $DE = 3.5$ cm

$$\therefore 14 \times 3 = (y + 3.5) \times 3.5$$

$$\Rightarrow y + 3.5 = \frac{14 \times 3}{3.5} = \frac{42}{3.5} = 12$$

$$\Rightarrow y = (12 - 3.5)$$
 cm = 8.5 cm

Hence, $CD = 8.5$ cm

ANSWER 41:

(b) 6 cm

We know that the line joining their centres is the perpendicular bisector of the common chord.

Join AP.

Then $AP = 5$ cm; $AB = 4$ cm

Also, $AP^2 = BP^2 + AB^2$

$$\text{Or } BP^2 = AP^2 - AB^2$$

$$\text{Or } BP^2 = 5^2 - 4^2$$

$$\text{Or } BP = 3$$
 cm

$\therefore \triangle ABP$ is a right angled and $PQ = 2 \times BP = (2 \times 3)$ cm = 6 cm

ANSWER 42:

(c) 60°

We have:

$$\angle AOB = 2\angle ACB$$

$$\Rightarrow \angle ACB = \frac{1}{2}\angle AOB = \left(\frac{1}{2} \times 90^\circ\right) = 45^\circ$$

$$\angle COA = 2\angle CBA = (2 \times 30^\circ) = 60^\circ$$

$$\therefore \angle COD = 180^\circ - \angle COA = (180^\circ - 60^\circ) = 120^\circ$$

$$\Rightarrow \angle CAO = \frac{1}{2}\angle COD = \left(\frac{1}{2} \times 120^\circ\right) = 60^\circ$$