EXERCISE 19.12

1. $\int \sin^4 x \cos^3 x \, dx$

Solution:

Let

$\sin x = t$

We know the Differentiation of $\sin x = \cos x$

$dt = d(\sin x) = \cos x dx$

So, $dx = \frac{dt}{\cos x}$

Substitute all in above equation,

$\int \sin^4 x \cos^3 x \, dx = \int t^4 \cos^3 x \frac{dt}{\cos x}$

$= \int t^4 \cos^2 x \, dt$

$= \int t^4 (1 - \sin^2 x) \, dt$

$= \int t^4 (1 - t^2) \, dt$

$= \int (t^4 - t^6) \, dt$

We know, basic integration formula, $\int x^n \, dx = \frac{x^{n+1}}{n+1} + c$ for any $c \neq -1$

Hence, $\int (t^4 - t^6) dt = \frac{t^5}{5} - \frac{t^7}{7} + c$

Put back t = sin x

$\int \sin^4 x \cos^3 x \, dx = \frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x + c$

2. $\int \sin^5 x \, dx$

Solution:

The given equation can be written as

$\int \sin^5 x\, dx = \int \sin^3 x \sin^2 x\, dx$

$= \int \sin^3 x(1 - \cos^2 x)dx$ {since $\sin^2 x + \cos^2 x = 1$}

$= \int (\sin^3 x - \sin^3 x \cos^2 x)\, dx$

$= \int (\sin x\, (\sin^2 x) - \sin^3 x \cos^2 x)\, dx$

$= \int (\sin x\, (1 - \cos^2 x) - \sin^3 x \cos^2 x)\, dx$ {since $\sin^2 x + \cos^2 x = 1$}

$= \int (\sin x - \sin x \cos^2 x - \sin^3 x \cos^2 x)dx$

$= \int \sin x\, dx - \int \sin x \cos^2 x\, dx - \int \sin^3 x \cos^2 x\, dx$ (separate the integrals)

We know, d (cos x) = -sin x dx

So put cos x = t and dt = -sin x dx in above integrals

$= \int \sin x\, dx - \int \sin x \cos^2 x\, dx - \int \sin^3 x \cos^2 x\, dx$

$= \int \sin x\, dx - \int t^2 (-dt) - \int (\sin^2 x \sin x)\, t^2\, dx$

$= \int \sin x\, dx - \int t^2 (-dt) - \int (1 - \cos^2 x)t^2\, (-dt)$

$= \int \sin x\, dx + \int (t^2 dt) + \int (1 - t^2)t^2\, dt$

$= \int \sin x\, dx + \int (t^2 dt) + \int (t^2 - t^4)dt$

$= -\cos x + \frac{t^3}{3} + \frac{t^3}{3} - \frac{t^5}{5} + c$ (since $\int x^n\, dx = \frac{x^{n+1}}{n+1} + c$ for any $c \neq -1$)

Put back t = cos x

$= -\cos x + \frac{t^3}{3} + \frac{t^3}{3} - \frac{t^5}{5} + c$

$= -\cos x + \frac{\cos^3 x}{3} + \frac{\cos^3 x}{3} - \frac{\cos^5 x}{5} + c$

$= -\cos x + \frac{2}{3}\cos^3 x - \frac{1}{5}\cos^5 x + c$ $= -[\cos x - \frac{2}{3}\cos^3 x + \frac{1}{5}\cos^5 x] + c$

3. $\int \cos^5 x\, dx$

Solution:

The given question can be written as

$\int \cos^5 x\, dx = \int \cos^3 x \cos^2 x\, dx$

$= \int \cos^3 x(1 - \sin^2 x)dx$ {since $\sin^2 x + \cos^2 x = 1$}

$= \int (\cos^3 x - \cos^3 x \sin^2 x)\, dx$

$= \int (\cos x\, (\cos^2 x) - \cos^3 x \sin^2 x)\, dx$

$= (\cos x\, (1 - \sin^2 x) - \cos^3 x \sin^2 x)\, dx$ {since $\sin^2 x + \cos^2 x = 1$}

$= \int (\cos x - \cos x \sin^2 x - \cos^3 x \sin^2 x)dx$

$= \int \cos x\, dx - \int \cos x \sin^2 x\, dx - \int \cos^3 x \sin^2 x\, dx$ (separate the integrals)

We know, d (sin x) = cos x dx

So put sin x = t and dt = cos x dx in above integrals

$= \int \cos x\, dx - \int t^2\, dt - \int \cos x \cos^2 x \sin^2 x\, dx$

$= \int \cos x\, dx - \int t^2\, (dt) - \int (\cos^2 x \cos x)\, t^2\, dx$

$= \int \cos x\, dx - \int t^2\, (dt) - \int (1 - \sin^2 x)t^2\, (dt)$

$= \int \cos x\, dx - \int (t^2\, dt) - \int (1 - t^2)t^2\, dt$

$= \int \cos x\, dx - \int (t^2\, dt) - \int (t^2 - t^4)dt$

$= \sin x - \frac{t^3}{3} - \frac{t^3}{3} + \frac{t^5}{5} + c$

Put back t = sin x

$= \sin x - \frac{\sin^3 x}{3} - \frac{\sin^3 x}{3} + \frac{\cos^5 x}{5} + c$

$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$

4. $\int \sin^5 x \cos x \, dx$

Solution:

Let sin x = t

Then d (sin x) = dt = cos x dx

Put t = sin x and dt = cos x dx in given equation

$\int \sin^5 x \cos x \, dx = \int t^5 dt$

On integrating we get

$= \frac{t^6}{6} + c$

Substituting the value of t

$= \frac{\sin^6 x}{6} + c$

5. $\int \sin^3 x \cos^6 x \, dx$

Solution:

Since power of sin is odd, put cos x = t

Then dt = -sin x dx

Substitute these in above equation,

$\int \sin^3 x \cos^6 x \, dx = \int \sin x \sin^2 x \, t^6 \, dx$

$= \int (1 - \cos^2 x) \, t^6 \sin x \, dx$

$= \int -(1 - t^2)\, t^6 dt$

$= \int -(t^6 - t^8) dt$

On integrating we get

$= -\frac{t^7}{7} + \frac{t^9}{9} + c$

Put the value of t we get

$= -\frac{1}{7}\cos^7 x + \frac{1}{9}\cos^9 x + c$