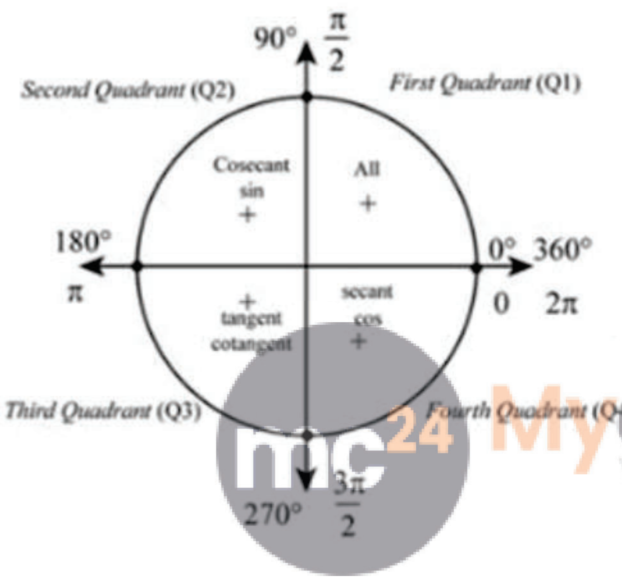
Trigonometric, Or Circular, Functions

Exercise 15A

Q. 1. If $\cos\theta = \frac{-\sqrt{3}}{2}$ and θ lies in Quadrant III, find the value of all the other five trigonometric functions.

Answer : Given: $\cos\theta = \frac{-\sqrt{3}}{2}$



Since, θ is in IIIrd Quadrant. So, sin and cos will be negative but tan will be positive.

We know that,

$\cos^2\theta + \sin^2\theta = 1$

Putting the values, we get

$\left(-\frac{\sqrt{3}}{2}\right)^2 + \sin^2\theta = 1$ [given]

$\Rightarrow \frac{3}{4} + \sin^2\theta = 1$

$\Rightarrow \sin^2\theta = 1 - \frac{3}{4}$

$\Rightarrow \sin^2\theta = \frac{4-3}{4}$

$\Rightarrow \sin^2\theta = \frac{1}{4}$

$\Rightarrow \sin\theta = \sqrt{\frac{1}{4}}$

$\Rightarrow \sin\theta = \pm\frac{1}{2}$

Since, θ in IIIrd quadrant and sinθ is negative in IIIrd quadrant

$\therefore \sin\theta = -\frac{1}{2}$

Now,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Putting the values, we get

$\tan\theta = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}}$

$= -\frac{1}{2} \times \left(-\frac{2}{\sqrt{3}}\right)$

$= \frac{1}{\sqrt{3}}$

Now,

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Putting the values, we get

$\operatorname{cosec}\theta = \frac{1}{-\frac{1}{2}}$

= -2

Now,

$\sec\theta = \frac{1}{\cos\theta}$

Putting the values, we get

$\sec\theta = \frac{1}{-\frac{\sqrt{3}}{2}}$

$= -\frac{2}{\sqrt{3}}$

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{\frac{1}{\sqrt{3}}}$

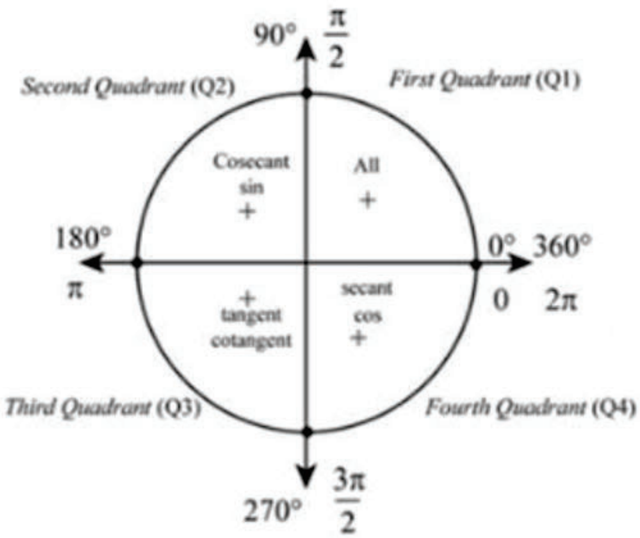
= $\sqrt{3}$

Hence, the values of other trigonometric Functions are:

Cos θ	Sin θ	Tan θ	Cosec θ	Sec θ	Cot θ
$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{3}}$	-2	$-\frac{2}{\sqrt{3}}$	$\sqrt{3}$

Q. 2. If $\sin\theta = \frac{-1}{2}$ and θ lies in Quadrant IV, find the values of all the other five trigonometric functions.

Answer : Given: $\sin\theta = \frac{-1}{2}$



Since, θ is in IVth Quadrant. So, sin and tan will be negative but cos will be positive.

We know that,

$\sin^2 \theta + \cos^2 \theta = 1$

Putting the values, we get

$\left(-\frac{1}{2}\right)^2 + \cos^2\theta = 1$ [given]

$\Rightarrow \frac{1}{4} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{1}{4}$

$\Rightarrow \cos^2\theta = \frac{4-1}{4}$

$\Rightarrow \cos^2\theta = \frac{3}{4}$

$\Rightarrow \cos\theta = \sqrt{\frac{3}{4}}$

$\Rightarrow \cos\theta = \pm\frac{\sqrt{3}}{2}$

Since, θ in IVth quadrant and cosθ is positive in IVth quadrant

$\therefore \cos\theta = \frac{\sqrt{3}}{2}$

Now,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Putting the values, we get

$\tan\theta = \frac{-\frac{1}{2}}{\frac{\sqrt{3}}{2}}$

$= -\frac{1}{2} \times \left(\frac{2}{\sqrt{3}}\right)$

$= -\frac{1}{\sqrt{3}}$

Now,

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Putting the values, we get

$\operatorname{cosec}\theta = \frac{1}{-\frac{1}{2}}$

= -2

Now,

$\sec\theta = \frac{1}{\cos\theta}$

Putting the values, we get

$\sec\theta = \frac{1}{\frac{\sqrt{3}}{2}}$

$= \frac{2}{\sqrt{3}}$

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{-\frac{1}{\sqrt{3}}}$

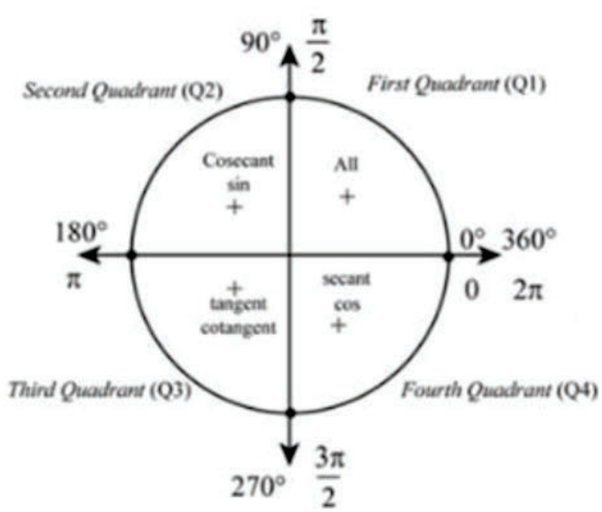
$= -\sqrt{3}$

Hence, the values of other trigonometric Functions are:

Cos θ	Sin θ	Tan θ	Cosec θ	Sec θ	Cot θ
$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\frac{1}{\sqrt{3}}$	-2	$\frac{2}{\sqrt{3}}$	$-\sqrt{3}$

Q. 3. If $cosec\ \theta = \frac{5}{3}$ and θ lies in Quadrant II, find the values of all the other five trigonometric functions.

Answer : Given: $cosec\ \theta = \frac{5}{3}$



Since, θ is in IInd Quadrant. So, cos and tan will be negative but sin will be positive.

Now, we know that

$\sin\theta = \frac{1}{\operatorname{cosec}\theta}$

Putting the values, we get

$\sin\theta = \frac{1}{\frac{5}{3}}$

$\sin\theta = \frac{3}{5}$ ...(i)

We know that,

$\sin^2\theta + \cos^2\theta = 1$

Putting the values, we get

$\left(\frac{3}{5}\right)^2 + \cos^2\theta = 1$ [from (i)]

$\Rightarrow \frac{9}{25} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \frac{9}{25}$

$\Rightarrow \cos^2\theta = \frac{25-9}{25}$

$\Rightarrow \cos^2\theta = \frac{16}{25}$

$\Rightarrow \cos\theta = \sqrt{\frac{16}{25}}$

$\Rightarrow \cos\theta = \pm\frac{4}{5}$

Since, θ in II^{nd} quadrant and cosθ is negative in II^{nd} quadrant

$\therefore \cos\theta = -\frac{4}{5}$

Now,

$\tan\theta = \frac{\sin\theta}{\cos\theta}$

Putting the values, we get

$\tan\theta = \frac{\frac{3}{5}}{-\frac{4}{5}}$

$= \frac{3}{5} \times \left(-\frac{5}{4}\right)$

$= -\frac{3}{4}$

Now,

$\sec\theta = \frac{1}{\cos\theta}$

Putting the values, we get

$\sec\theta = \frac{1}{-\frac{4}{5}}$

$= -\frac{5}{4}$

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{-\frac{3}{4}}$

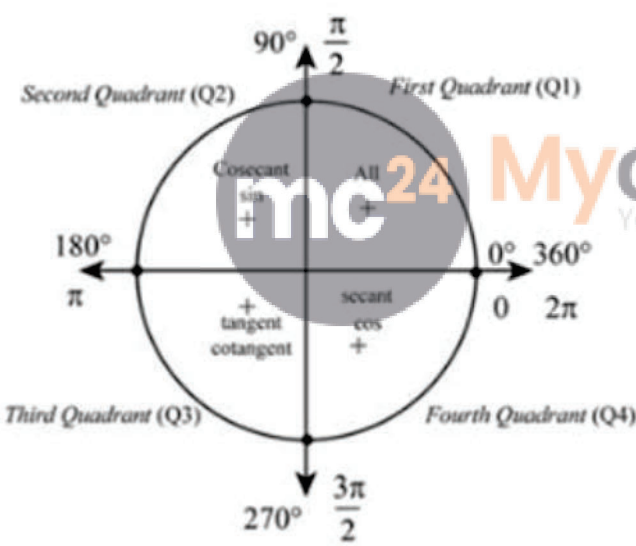
$= -\frac{4}{3}$

Hence, the values of other trigonometric Functions are:

Cos θ	Sin θ	Tan θ	Cosec θ	Sec θ	Cot θ
$-\frac{4}{5}$	$\frac{3}{5}$	$-\frac{3}{4}$	$\frac{5}{3}$	$-\frac{5}{4}$	$-\frac{4}{3}$

Q. 4. If sec θ $\sqrt{2}$ and θ lies in Quadrant IV, find the values of all the other five trigonometric functions.

Answer : Given: sec θ = $\sqrt{2}$



Since, θ is in IVth Quadrant. So, sin and tan will be negative but cos will be positive.

Now, we know that

$$\cos\theta = \frac{1}{\sec\theta}$$

Putting the values, we get

$$\cos\theta = \frac{1}{\sqrt{2}} \text{ ...(i)}$$

We know that,

$\cos^2 \theta + \sin^2 \theta = 1$

Putting the values, we get

$\left(\frac{1}{\sqrt{2}}\right)^2 + \sin^2 \theta = 1$ [Given]

$\Rightarrow \frac{1}{2} + \sin^2 \theta = 1$

$\Rightarrow \sin^2 \theta = 1 - \frac{1}{2}$

$\Rightarrow \sin^2 \theta = \frac{2-1}{2}$

$\Rightarrow \sin^2 \theta = \frac{1}{2}$

$\Rightarrow \sin \theta = \sqrt{\frac{1}{2}}$

$\Rightarrow \sin \theta = \pm \frac{1}{\sqrt{2}}$

Since, θ in IVth quadrant and sinθ is negative in IVth quadrant

$\therefore \sin \theta = -\frac{1}{\sqrt{2}}$

Now,

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

Putting the values, we get

$\tan \theta = \frac{-\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$

$= -\frac{1}{\sqrt{2}} \times \left(\sqrt{2}\right)$

= – 1

Now,

$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$

Putting the values, we get

$\operatorname{cosec}\theta = \frac{1}{-\frac{1}{\sqrt{2}}}$

= $-\sqrt{2}$

Now,

$\cot\theta = \frac{1}{\tan\theta}$

Putting the values, we get

$\cot\theta = \frac{1}{-1}$

= – 1

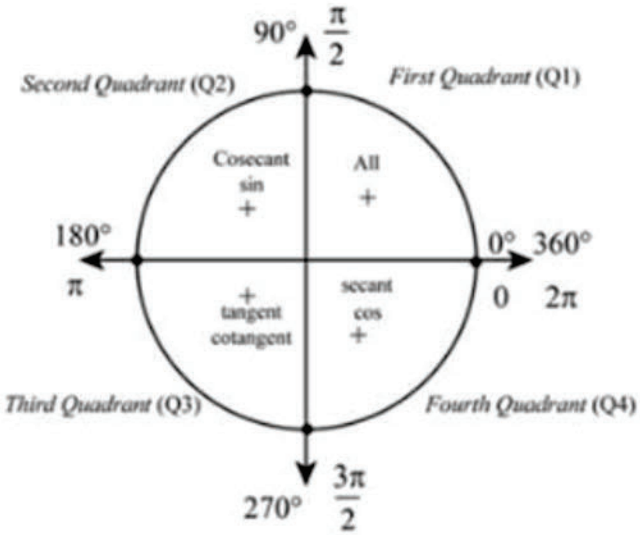
Hence, the values of other trigonometric Functions are:

Cos θ	Sin θ	Tan θ	Cosec θ	Sec θ	Cot θ
$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$	-1	-√2	√2	-1

Q. 5. If $\sin x = -\frac{2\sqrt{6}}{5}$ and x lies in Quadrant III, find the values of cos x and cot x.

Answer : Given: $\sin x = -\frac{2\sqrt{6}}{5}$

To find: cos x and cot x



Since, x is in IIIrd Quadrant. So, sin and cos will be negative but tan will be positive.

We know that,

$\sin^2 x + \cos^2 x = 1$

Putting the values, we get

$\left(-\frac{2\sqrt{6}}{5}\right)^2 + \cos^2 x = 1$ [Given]

$\Rightarrow \frac{24}{25} + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \frac{24}{25}$

$\Rightarrow \cos^2 x = \frac{25-24}{25}$

$\Rightarrow \cos^2 x = \frac{1}{25}$

$\Rightarrow \cos x = \sqrt{\frac{1}{25}}$

$\Rightarrow \cos x = \pm\frac{1}{5}$

Since, x in III[rd] quadrant and cos x is negative in III[rd] quadrant

$\therefore \cos x = -\frac{1}{5}$

Now,

$\tan x = \frac{\sin x}{\cos x}$

Putting the values, we get

$\tan x = \frac{-\frac{2\sqrt{6}}{5}}{-\frac{1}{5}}$

$= -\frac{2\sqrt{6}}{5} \times (-5)$

= $2\sqrt{6}$

Now,

$\cot x = \frac{1}{\tan x}$

Putting the values, we get

$\cot x = \frac{1}{2\sqrt{6}}$

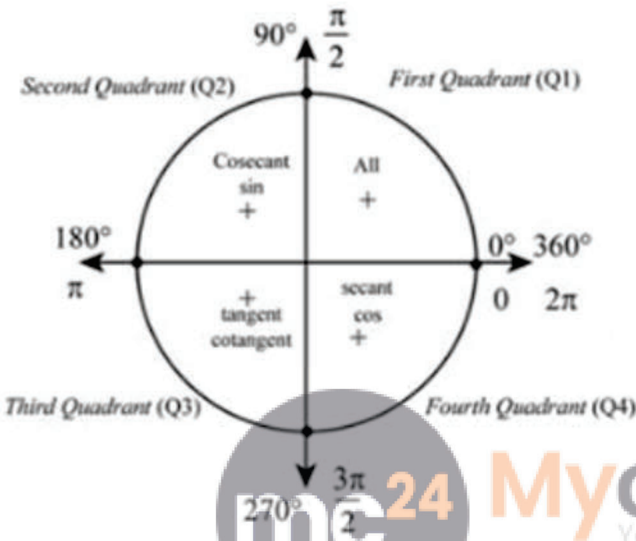
Hence, the values of other trigonometric Functions are:

Cos x	Sin x	Cot x
$-\frac{1}{5}$	$-\frac{2\sqrt{6}}{5}$	$\frac{1}{2\sqrt{6}}$

Q. 6. If $\cos x = \frac{-\sqrt{15}}{4}$ and $\frac{\pi}{2} < x < \pi$**, find the value of sin x.**

Answer : Given: $\cos x = -\frac{\sqrt{15}}{4}$

To find: value of sinx



Given that: $\frac{\pi}{2} < x < \pi$

So, x lies in II[nd] quadrant and sin will be positive.

We know that,

$\cos^2 \theta + \sin^2 \theta = 1$

Putting the values, we get

$\left(-\frac{\sqrt{15}}{4}\right)^2 + \sin^2 \theta = 1$ [Given]

$\Rightarrow \frac{15}{16} + \sin^2 \theta = 1$

$\Rightarrow \sin^2 \theta = 1 - \frac{15}{16}$

$\Rightarrow \sin^2\theta = \frac{16-15}{16}$

$\Rightarrow \sin^2\theta = \frac{1}{16}$

$\Rightarrow \sin\theta = \sqrt{\frac{1}{16}}$

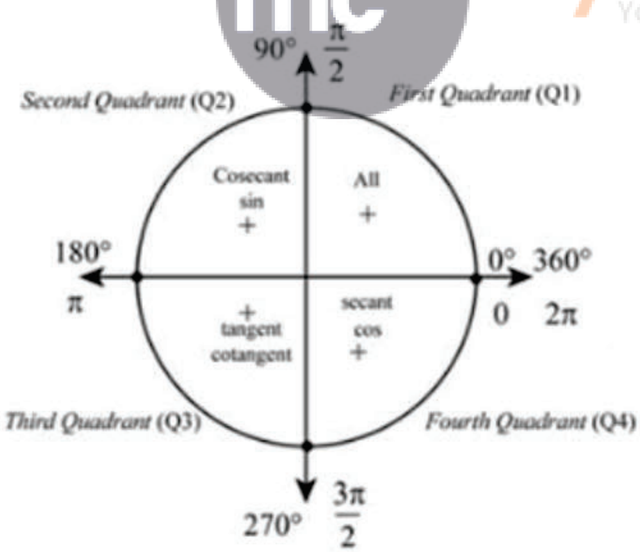
$\Rightarrow \sin\theta = \pm\frac{1}{4}$

Since, x in IInd quadrant and sinθ is positive in IInd quadrant

$\therefore \sin\theta = \frac{1}{4}$

Q. 7. If $\sec x = -2 \text{ and } \pi < x < \frac{3\pi}{2}$ **, find the values of all the other five trigonometric functions.**

Answer : Given: sec x = -2



Given that: $\pi < x < \frac{3\pi}{2}$

So, x lies in IIIrd Quadrant. So, sin and cos will be negative but tan will be positive.

Now, we know that

$\cos x = \frac{1}{\sec x}$

Putting the values, we get

$\cos x = \frac{1}{-2}$ ...(i)

We know that,

$\cos^2 x + \sin^2 x = 1$

Putting the values, we get

$\left(-\frac{1}{2}\right)^2 + \sin^2 x = 1$ [Given]

$\Rightarrow \frac{1}{4} + \sin^2 x = 1$

$\Rightarrow \sin^2 x = 1 - \frac{1}{4}$

$\Rightarrow \sin^2 x = \frac{4-1}{4}$

$\Rightarrow \sin^2 x = \frac{3}{4}$

$\Rightarrow \sin x = \sqrt{\frac{3}{4}}$

$\Rightarrow \sin x = \pm\frac{\sqrt{3}}{2}$

Since, x in IIIrd quadrant and sinx is negative in IIIrd quadrant

$\therefore \sin x = -\frac{\sqrt{3}}{2}$

Now,

$\tan x = \frac{\sin x}{\cos x}$

Putting the values, we get

$\tan x = \frac{-\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$= -\frac{\sqrt{3}}{2} \times (-2)$

= √3

Now,

$\text{cosec} x = \frac{1}{\sin x}$

Putting the values, we get

$\text{cosec} x = \frac{1}{-\frac{\sqrt{3}}{2}}$

$= -\frac{2}{\sqrt{3}}$

Now,

$\cot x = \frac{1}{\tan x}$

Putting the values, we get

$\cot x = \frac{1}{\sqrt{3}}$

Hence, the values of other trigonometric Functions are:

Cos x	Sin x	Tan x	Cosec x	Sec x	Cot x
$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	√3	$-\frac{2}{\sqrt{3}}$	-2	$\frac{1}{\sqrt{3}}$

Q. 8. A. Find the value of

$$\sin\left(\frac{31\pi}{3}\right)$$

Answer :

```
    10
3 ) 31
    30
    --
     1
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To find: Value of sin $\frac{31\pi}{3}$

$\sin\frac{31\pi}{3} = \sin\left(10\pi + \frac{1}{3}\pi\right)$

$= \sin\left(5 \times (2\pi) + \frac{1}{3}\pi\right)$

Value of sin x repeats after an interval of 2π, hence ignoring 5 × (2π)

$= \sin\left(\frac{1}{3}\pi\right)$

$= \sin\left(\frac{1}{3} \times 180°\right)$

= sin 60°

$= \frac{\sqrt{3}}{2}\left[\because \sin 60° = \frac{\sqrt{3}}{2}\right]$

Q. 8. B. Find the value of

$$\cos\left(\frac{17\pi}{2}\right)$$

Answer :

$$\begin{array}{r|l} & 8 \\ 2 & 17 \\ & 16 \\ \hline & 1 \end{array}$$

To find: Value of cos $\frac{17\,n}{2}$

$\cos\frac{17\pi}{2} = \cos\left(8\pi + \frac{1}{2}\pi\right)$

$= \cos\left(4 \times (2\pi) + \frac{1}{2}\pi\right)$

Value of cos x repeats after an interval of 2π, hence ignoring 4 × (2π)

$= \cos\left(\frac{1}{2}\pi\right)$

$= \cos\left(\frac{1}{2} \times 180°\right)$

= cos 90°

= 0 [∵ cos 90° = 1]

Q. 8. C. Find the value of

$\tan\left(\frac{-25\pi}{3}\right)$

Answer :

$$\begin{array}{r|l} & 8 \\ 3 & 25 \\ & 24 \\ \hline & 1 \end{array}$$

To find: Value of tan $\frac{-25\pi}{3}$

We know that,

tan(-θ) = - tan θ

$$\therefore \tan\left(-\frac{25\pi}{3}\right) = -\tan\left(\frac{25\pi}{3}\right)$$

$$\tan\left(-\frac{25\pi}{3}\right) = -\tan\left(\frac{25\pi}{3}\right) = -\tan\left(8\pi + \frac{1}{3}\pi\right)$$

$$= -\tan\left(4 \times (2\pi) + \frac{1}{3}\pi\right)$$

Value of tan x repeats after an interval of 2π, hence ignoring 4 × (2π)

$$= -\tan\left(\frac{1}{3}\pi\right)$$

$$= -\tan\left(\frac{1}{3} \times 180°\right)$$

= - tan 60°

= -√3

[∵ tan 60° = √3]

Q. 8. D. Find the value of

$$\cot\left(\frac{13\pi}{4}\right)$$

Answer : To find: Value of cot $\frac{13\pi}{4}$

We have,

$$\cot\frac{13\pi}{4}$$

Putting π = 180°

$$= \cot\left(\frac{13 \times 180°}{4}\right)$$

= cot (13 × 45°)

= cot (585°)

= cot [90° × 6 + 45°]

= cot 45°

[Clearly, 585° is in IIIrd Quadrant and the multiple of 90° is even]

= 1 [∵ cot 45° = 1]

Q. 8. E. Find the value of

$$\sec\left(\frac{-25\pi}{3}\right)$$

Answer : To find: Value of $\sec\left(-\frac{25\pi}{3}\right)$

We have,

$\sec\left(-\frac{25\pi}{3}\right) = \sec\frac{25\pi}{3}$

[∵ sec(-θ) = sec θ]

Putting π = 180°

$= \sec\frac{25\times180}{3}$

= sec[25 × 60°]

= sec[1500°]

= sec [90° × 16 + 60°]

Clearly, 1500° is in Ist Quadrant and the multiple of 90° is even

= sec 60°

= 2 [∵ $\sec 60° = 2$]

Q. 8. F. Find the value of

$$\operatorname{cosec}\left(\frac{-41\pi}{4}\right)$$

Answer : To find: Value of $\operatorname{cosec}\left(-\frac{41\pi}{4}\right)$

We have,

$\operatorname{cosec}\left(-\frac{41\pi}{4}\right) = -\operatorname{cosec}\frac{41\pi}{4}$

[∵ cosec(-θ) = -cosec θ]

Putting π = 180°

$= -\operatorname{cosec}\frac{41\times180}{4}$

= -cosec[41 × 45°]

= -cosec[1845°]

= -cosec [90° × 20 + 45°]

Clearly, 1845° is in I^{st} Quadrant and the multiple of 90° is even

= -cosec 45°

= -√2 [$\because \operatorname{cosec} 45° = \sqrt{2}$]

Q. 9. A. Find the value of

sin 405°

Answer : To find: Value of sin 405°

We have,

sin 405° = sin [90° × 4 + 45°]

= sin 45°

[Clearly, 405° is in Ist Quadrant and the multiple of 90° is even]

$= \frac{1}{\sqrt{2}}$ [$\because$ $\sin 45° = \frac{1}{\sqrt{2}}$]

Q. 9. B. Find the value of

sec (-1470^0)

Answer : To find: Value of sec (-1470°)

We have,

sec (-1470°) = sec (1470°)

[$\because$ sec(-θ) = sec θ]

= sec [90° × 16 + 30°]

Clearly, 1470° is in Ist Quadrant and the multiple of 90° is even

= sec 30°

$= \frac{2}{\sqrt{3}}$ [$\because$ $\sec 30° = \frac{2}{\sqrt{3}}$]

Q. 9. C. Find the value of

tan (-300^0)

Answer : To find: Value of tan (-300°)

We have,

tan (-300°) = - tan (300°)

[$\because$ tan(-θ) = -tan θ]

= - tan [90° × 3 + 30°]

Clearly, 300° is in IVth Quadrant and the multiple of 90° is odd

= - cot 30°

= -√3 [$\because$ $\cot 30° = \sqrt{3}$]

Q. 9. D. Find the value of

cot (585^0)

Answer : To find: Value of cot $\frac{13\pi}{4}$

We have,

cot (585°) = cot [90° × 6 + 45°]

= cot 45°

[Clearly, 585° is in IIIrd Quadrant and the multiple of 90° is even]

= 1 [∵ cot 45° = 1]

Q. 9. E. Find the value of

cosec (-750^0)

Answer : To find: Value of cosec (-750°)

We have,

cosec (-750°) = - cosec(750°)

[∵ cosec(-θ) = -cosec θ]

= - cosec [90° × 8 + 30°]

Clearly, 405° is in Ist Quadrant and the multiple of 90° is even

= - cosec 30°

= -2 [∵ cosec 30° = 2]

Q. 9. F. Find the value of

cos (-2220^0)

Answer : To find: Value of cos 2220°

We have,

cos (-2220°) = cos 2220°

[∵ cos(-θ) = cos θ]

= cos [2160 + 60°]

= cos [360° × 6 + 60°]

= cos 60°

[Clearly, 2220° is in Ist Quadrant and the multiple of 360° is even]

$= \frac{1}{2}$ [∵ $\cos 60° = \frac{1}{2}$]

Q. 10. A. Prove that

$$\tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2} = 8$$

Answer :

To prove: $\tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2} = 8$

Taking LHS,

$= \tan^2\frac{\pi}{3} + 2\cos^2\frac{\pi}{4} + 3\sec^2\frac{\pi}{6} + 4\cos^2\frac{\pi}{2}$

Putting π = 180°

$= \tan^2\frac{180}{3} + 2\cos^2\frac{180}{4} + 3\sec^2\frac{180}{6} + 4\cos^2\frac{180}{2}$

= $\tan^2 60° + 2\cos^2 45° + 3\sec^2 30° + 4\cos^2 90°$

Now, we know that,

$\tan 60° = \sqrt{3}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$\sec 30° = \frac{2}{\sqrt{3}}$

$\cos 90° = 0$

Putting the values, we get

$= (\sqrt{3})^2 + 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 3 \times \left(\frac{2}{\sqrt{3}}\right)^2 + 4(0)^2$

$= 3 + 2 \times \frac{1}{2} + 3 \times \frac{4}{3}$

= 3 + 1 + 4

= 8

= RHS

∴ LHS = RHS

Hence Proved

Q. 10. B. Prove that

$$\sin\frac{\pi}{6}\cos 0 + \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cos\frac{\pi}{6} = \frac{7}{4}$$

Answer :

To prove: $\sin\frac{\pi}{6}\cos 0 + \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cos\frac{\pi}{6} = \frac{7}{4}$

Taking LHS,

$= \sin\frac{\pi}{6}\cos 0 + \sin\frac{\pi}{4}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\cos\frac{\pi}{6}$

Putting π = 180°

$= \sin\frac{180}{6}\cos 0 + \sin\frac{180}{4}\cos\frac{180}{4} + \sin\frac{180}{3}\cos\frac{180}{6}$

= sin 30° cos 0° + sin 45° cos 45° + sin 60° cos 30°

Now, we know that,

$\sin 30° = \frac{1}{2}$

$\cos 0° = 1$

$\sin 45° = \frac{1}{\sqrt{2}}$

$\cos 45° = \frac{1}{\sqrt{2}}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 30° = \frac{\sqrt{3}}{2}$

Putting the values, we get

$= \frac{1}{2} \times 1 + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$

$= \frac{1}{2} + \frac{1}{2} + \frac{3}{4}$

$= \frac{2+2+3}{4}$

$= \frac{7}{4}$

= RHS

∴ LHS = RHS

Hence Proved

Q. 10. C. Prove that

$$4\sin\frac{\pi}{6}\sin^2\frac{\pi}{3} + 3\cos\frac{\pi}{3}\tan\frac{\pi}{4} + \operatorname{cosec}^2\frac{\pi}{2} = 4$$

Answer : To prove: $4\sin\frac{\pi}{6}\sin^2\frac{\pi}{3} + 3\cos\frac{\pi}{3}\tan\frac{\pi}{4} + \operatorname{cosec}^2\frac{\pi}{2} = 4$

Taking LHS,

$= 4\sin\frac{\pi}{6}\sin^2\frac{\pi}{3} + 3\cos\frac{\pi}{3}\tan\frac{\pi}{4} + \text{cosec}^2\frac{\pi}{2}$

Putting π = 180°

$= 4\sin\frac{180}{6}\sin^2\frac{180}{3} + 3\cos\frac{180}{3}\tan\frac{180}{4} + \text{cosec}^2\frac{180}{2}$

= 4 sin 30° $\sin^2$ 60° + 3 cos 60° tan 45° + cosec^2 90°

Now, we know that,

$\sin 30° = \frac{1}{2}$

$\sin 60° = \frac{\sqrt{3}}{2}$

$\cos 60° = \frac{1}{2}$

tan 45° = 1

cosec 90° = 1

Putting the values, we get

$= 4 \times \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 + 3 \times \frac{1}{2} \times 1 + (1)^2$

$= 2 \times \frac{3}{4} + \frac{3}{2} + 1$

$= \frac{3}{2} + \frac{3}{2} + 1$

$= \frac{3+3+2}{2}$

= 4

= RHS

∴ LHS = RHS

Hence Proved

Exercise 15B

Q. 1. Find the value of

(i) cos 840^0
(ii) sin 870^0
(iii) tan (- 120^0)
(iv) sec (- 420^0)
(v) cosec (- 690^0)
(vi) tan (225^0)
(vii) cot (- 315^0)
(viii) sin (- 1230^0)
(ix) cos (495^0)

Answer : (i) Cos840° = Cos(2.360° + 120°) …………(using Cos(2ϖ + x) = Cosx)

= Cos(120°)

= Cos(180° - 60°)

= - Cos60° ……………(using Cos(ϖ - x) = - Cosx)

$$= -\frac{1}{2}$$

(ii) sin870° = sin(2.360° + 150°) …………(using sin(2ϖ + x) = sinx)

= sin150°

= sin(180° - 30°) ………(using sin(ϖ - x) = sinx)

= sin30°

$$= \frac{1}{2}$$

(iii) tan(- 120°) = - tan12 …….(tan(- x) = tanx)

= - tan(180° - 60°) ……. (in II quadrant tanx is negative)

= - (- tan60°)

= tan60°

$= \sqrt{3}$

(iv) $\sec(-420°) = \frac{1}{\cos(-420°)}$

$= \frac{1}{-\cos 420°}$ ....(using cos(- x) = - cosx)

$= \frac{-1}{-\cos(360° + 60)}$ ....(using cos(2ϖ + x) = cosx)

$= \frac{-1}{\cos 60°} \Rightarrow \frac{-1}{1/2} = -2$

(v) $\text{cosec}(690°) = \frac{1}{\sin(-690°)} \Rightarrow \frac{1}{-\sin(690°)} = \frac{1}{-\sin(2.360 - 30°)}$

........(IV quadrant sinx is negative)

$= \frac{1}{-(-\sin 30°)} \Rightarrow \frac{1}{1/2} = 2$

(vi) tan225° = tan(180° + 45°) ....(in III quadrant tanx is positive)

$\Rightarrow \tan 45° = 1$

(vii) $\cot(-315°) = \frac{1}{\tan(-315)°} \Rightarrow \frac{1}{-\tan(315°)} = \frac{1}{-\tan(360° - 45°)}$

.....(tan(- x) = - tanx)

$= \frac{1}{-(-\tan 45°)} \Rightarrow 1$ .....(in IV quadrant tanx is negative)

(viii) sin(- 1230°) = sin1230° ....(using sin(- x) = sinx)

= sin(3.360° + 150°)

= sin150°

= sin(180° - 30°) ....(using sin(180° - x) = sinx)

= sin30°

$$= \frac{1}{2}$$

(ix) cos495° = cos(360° + 135°) ....(using cos(360° + x) = cosx)

= cos135°

= cos(180° - 45°) ....(using cos(180° - x) = - cosx)

= - cos45°

$$= -\frac{1}{\sqrt{2}}$$

Q. 2. Find the values of all trigonometric functions of 135^0

Answer : Sin135° = sin(180° - 45°) ....(using sin(180° - x) = sinx)

$$= \sin 45° \Rightarrow \frac{1}{\sqrt{2}}$$

Cos135° = cos(180° - 45°) ....(using cos(180° - x) = - cosx)

$= \cos45° \Rightarrow -\frac{1}{\sqrt{2}}$

$\text{Tan}135° = \frac{\sin135°}{\cos135°} \Rightarrow \frac{1/\sqrt{2}}{-1/\sqrt{2}} = -1$

$\text{Cosec}135° = \frac{1}{\sin135°} \Rightarrow \sqrt{2}$

$\text{Sec}135° = \frac{1}{\cos135°} \Rightarrow -\sqrt{2}$

$\text{Cot}135° = \frac{1}{\tan135°} \Rightarrow -1$

Q. 3. Prove that

(i) $\sin 80^0 \cos 20^0 - \cos 80^0 \sin 20^0 = \frac{\sqrt{3}}{2}$

(ii) $\cos 45^0 \cos 15^0 - \sin 45^0 \sin 15^0 = \frac{1}{2}$

(iii) $\cos 75^0 \cos 15^0 + \sin 75^0 \sin 15^0 = \frac{1}{2}$

(iv) $\sin 40^0 \cos 20^0 + \cos 40^0 \sin 20^0 = \frac{\sqrt{3}}{2}$

(v) $\cos 130^0 \cos 40^0 + \sin 130^0 \sin 40^0 = 0$

Answer : (i) sin80°cos20° - cos80°sin20° = sin(80° - 20°)

(using sin(A - B) = sinAcosB - cosAsinB)

$= \sin 60°$

$$= \frac{\sqrt{3}}{2}$$

(ii) $\cos 45° \cos 15° - \sin 45° \sin 15° = \cos(45° + 15°)$

(Using $\cos(A + B) = \cos A \cos B - \sin A \sin B$)

$= \cos 60°$

$$= \frac{1}{2}$$

(iii) $\cos 75° \cos 15° + \sin 75° \sin 15° = \cos(75° - 15°)$

(using $\cos(A - B) = \cos A \cos B + \sin A \sin B$)

$= \cos 60°$

$$= \frac{1}{2}$$

(iv) $\sin 40° \cos 20° + \cos 40° \sin 20° = \sin(40° + 20°)$

(using $\sin(A + B) = \sin A \cos B + \cos A \sin B$)

$= \sin 60°$

$$= \frac{\sqrt{3}}{2}$$

(v) $\cos 130° \cos 40° + \sin 130° \sin 40° = \cos(130° - 40°)$

(using $\cos(A - B) = \cos A \cos B + \sin A \sin B$)

$= \cos 90°$

$= 0$

Q. 4. Prove that

(i) $\sin(50^0+\theta)\cos(20^0+\theta)-\cos(50^0+\theta)\sin(20^0+\theta)=\frac{1}{2}$

(ii) $\cos(70^0+\theta)\cos(10^0+\theta)+\sin(70^0+\theta)\sin(10^0+\theta)=\frac{1}{2}$

Answer : (i) sin(50° + θ)cos(20° + θ) - cos(50° + θ)sin(20° + θ)

= sin(50° + θ - (20° + θ))(using sin(A - B) = sinAcosB - cosAsinB)

= sin(50° + θ - 20° - θ)

= sin30°

$= \frac{1}{2}$

(ii) cos(70° + θ)cos(10° + θ) + sin(70° + θ)sin(10° + θ)

= cos(70° + θ - (10° + θ))(using cos(A - B) = cosAcosB + sinAsinB)

= cos(70° + θ - 10° - θ)

= cos60°

$= \frac{1}{2}$

Q. 5. Prove that

(i) $\cos(n+2)x\cos(n+1)x+\sin(n+2)x\sin(n+1)x=\cos x$

(ii) $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)=\sin(x+y)$

Answer : (i)cos(n + 2)x.cos(n + 1)x + sin(n + 2)x.sin(n + 1)x

= sin((n + 2)x + (n + 1)x)(using cos(A - B) = cosAcosB + sinAsinB)

= cos(nx + 2x - (nx + x))

= cos(nx + 2x - nx - x)

= cosx

(ii) $\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)-\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

$= \cos\left(\frac{\pi}{4}-x+\frac{\pi}{4}-y\right)$(using cos(A + B) = cosAcosB - sinAsinB)

$= \cos\left(\frac{2\pi}{4}-x-y\right)$

$= \cos\left(\frac{\pi}{2}-(x+y)\right)$ (using $\cos\left(\frac{\pi}{2}-x\right) = \sin x$)

= sin(x + y)

Q. 6.

Prove that $\frac{\tan\left(\frac{\pi}{4}+x\right)}{\tan\left(\frac{\pi}{4}-x\right)} = \left(\frac{1+\tan x}{1-\tan x}\right)^2$

Answer :