

Exercise 24.1

1.

(i) Solution:

Given,

$$\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$$

$$\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$$

$$\vec{a} \cdot \vec{b} = 1 \times 4 + (-2) \times (-4) + 1 \times 7$$

$$\vec{a} \cdot \vec{b} = 19$$

Hence, the dot product is 19.

(ii) Solution:

Given,

$$\vec{a} = \hat{j} + 2\hat{k}$$

$$\vec{b} = 2\hat{i} + \hat{k}$$

$$\vec{a} \cdot \vec{b} = 0 \times 2 + 0 \times 2 + 1 \times 2$$

$$\vec{a} \cdot \vec{b} = 2$$

Hence, the dot product is 2.

(iii) Solution:

Given,

$$\vec{a} = \hat{j} - \hat{k}$$

$$\vec{b} = 2\hat{i} + 3\hat{j} - 2\hat{k}$$

$$\vec{a} \cdot \vec{b} = 0 \times 2 + 1 \times 3 + (-1) \times (-2)$$

$$\vec{a} \cdot \vec{b} = 5$$

Hence, the dot product is 5.

2.

(i) Solution:

Given,

$$\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{b} = 4\hat{i} - 9\hat{j} + 2\hat{k}$$

If \vec{a} and \vec{b} are \perp to each other then $\vec{a} \cdot \vec{b} = 0$

Now

$$\vec{a} \cdot \vec{b} = 0$$

$$\lambda(4) + 2(-9) + 1(2) = 0$$

$$4\lambda - 18 + 2 = 0$$

$$4\lambda - 16 = 0$$

$$4\lambda = 16$$

$$\lambda = 16/4$$

$$\therefore \lambda = 4$$

(ii) Solution:

Given,

$$\vec{a} = \lambda\hat{i} + 2\hat{j} + \hat{k}$$

$$\vec{b} = 5\hat{i} - 9\hat{j} + 2\hat{k}$$

If \vec{a} and \vec{b} are \perp to each other then $\vec{a} \cdot \vec{b} = 0$

Now,

$$\vec{a} \cdot \vec{b} = 0$$

$$\lambda(5) + 2(-9) + 1(2) = 0$$

$$5\lambda - 18 + 2 = 0$$

$$5\lambda - 16 = 0$$

$$5\lambda = 16$$

$$\therefore \lambda = 16/5$$

(iii) Solution:

Given,

$$\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$$\vec{b} = 3\hat{i} + 2\hat{j} - \lambda\hat{k}$$

If \vec{a} and \vec{b} are \perp to each other then $\vec{a} \cdot \vec{b} = 0$

Now,

$$\vec{a} \cdot \vec{b} = 0$$

$$2(3) + 3(2) + 4(-\lambda) = 0$$

$$-4\lambda + 6 + 6 = 0$$

$$-4\lambda + 12 = 0$$

$$-4\lambda = -12$$

$$\lambda = -12/-4$$

$$\therefore \lambda = 3$$

(iv) Solution:

$$\vec{a} = \hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{b} = \hat{i} - \hat{j} + 3\hat{k}$$

If \vec{a} and \vec{b} are \perp to each other then $\vec{a} \cdot \vec{b} = 0$

Now,

$$\vec{a} \cdot \vec{b} = 0$$

$$\lambda(1) + 3(-1) + 2(3) = 0$$

$$\lambda - 3 + 6 = 0$$

$$\lambda + 3 = 0$$

$$\therefore \lambda = -3$$

3. Solution:

Given,

$$|\vec{a}| = 4, |\vec{b}| = 3 \text{ and } \vec{a} \cdot \vec{b} = 6$$

We know that,

$$\vec{a} \cdot \vec{b} = |\vec{a}| \times |\vec{b}| \times \cos\theta$$

So,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$= 6/(4 \times 3)$$

$$= 6/12$$

$$= 1/2$$

$$\theta = \cos^{-1}(1/2)$$

$$\therefore \theta = \pi/3$$

4. Solution:

$$\vec{a} = \hat{i} - \hat{j}$$

$$\vec{b} = -\hat{j} + 2\hat{k}$$

So,

$$\Rightarrow \vec{a} - 2\vec{b} = (\hat{i} - \hat{j}) - 2(-\hat{j} + 2\hat{k})$$

$$\vec{a} - 2\vec{b} = \hat{i} - \hat{j} + 2\hat{j} - 4\hat{k}$$

$$\vec{a} - 2\vec{b} = \hat{i} + \hat{j} - 4\hat{k}$$

$$\Rightarrow \vec{a} + \vec{b} = (\hat{i} - \hat{j}) + (-\hat{j} + 2\hat{k})$$

$$\vec{a} + \vec{b} = \hat{i} - \hat{j} - \hat{j} + 2\hat{k}$$

$$\vec{a} + \vec{b} = \hat{i} - 2\hat{j} + 2\hat{k}$$

Now,

$$(\vec{a} - 2\vec{b}) \cdot (\vec{a} + \vec{b}) = (\hat{i} + \hat{j} - 4\hat{k}) \cdot (\hat{i} - 2\hat{j} + 2\hat{k})$$

$$(\vec{a} - 2\vec{b}) \cdot (\vec{a} + \vec{b}) = 1 \times 1 + 1 \times (-2) + (-4) \times 2$$

$$(\vec{a} - 2\vec{b}) \cdot (\vec{a} + \vec{b}) = 1 - 2 - 8$$

$$(\vec{a} - 2\vec{b}) \cdot (\vec{a} + \vec{b}) = -9$$

5.

(i) Solution:

Given,

$$\vec{a} = \hat{i} - \hat{j}$$

$$\vec{b} = \hat{j} + \hat{k}$$

We know that,

$$\vec{a} \cdot \vec{b} = |\vec{a}| \times |\vec{b}| \times \cos\theta$$

So,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$\cos\theta = \frac{(\hat{i} - \hat{j})(\hat{j} + \hat{k})}{\sqrt{1^2 + 1^2} \times \sqrt{1^2 + 1^2}}$$

$$\cos\theta = \frac{1 \times 0 + (-1) \times 1 + 0 \times 1}{\sqrt{2} \times \sqrt{2}}$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = \cos^{-1}\left(-\frac{1}{2}\right)$$

$$\theta = \pi - \frac{\pi}{3}$$

$$\therefore \theta = \frac{2\pi}{3}$$

Therefore, angle between \vec{a} and \vec{b} is $\frac{\pi}{3}$.

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(ii) Solution:

Given,

$$\vec{a} = 3\hat{i} - 2\hat{j} - 6\hat{k}$$

$$\vec{b} = 4\hat{i} - \hat{j} + 8\hat{k}$$

We know that,

$$\vec{a} \cdot \vec{b} = |\vec{a}| \times |\vec{b}| \times \cos\theta$$

So,

$$\cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \times |\vec{b}|}$$

$$\cos\theta = \frac{(3\hat{i} - 2\hat{j} - 6\hat{k})(4\hat{i} - \hat{j} + 8\hat{k})}{\sqrt{3^2 + (-2)^2 + (-6)^2} \times \sqrt{4^2 + (-1)^2 + 8^2}}$$

$$\cos\theta = \frac{3 \times 4 + (-2) \times (-1) + (-6) \times 8}{\sqrt{9 + 4 + 36} \times \sqrt{16 + 1 + 64}}$$

$$\cos\theta = -\frac{34}{\sqrt{49} \times \sqrt{81}}$$

$$\cos\theta = -\frac{34}{7 \times 9}$$

$$\theta = \cos^{-1}\left(-\frac{34}{63}\right)$$

$$\theta = 122.66^\circ$$

Therefore, angle between \vec{a} and \vec{b} is 122.66° .



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